Adjusting for the Incidence of Measurement Errors in Multilevel Models Using Bootstrapping and Gibbs Sampling Techniques

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Abstract

In the face of seeming dearth of objective methods of estimating measurement error variance and realistically adjusting for the incidence of measurement errors in multilevel models, researchers often indulge in the traditional approach of arbitrary choice of measurement error variance and this has the potential of giving misleading inferences. This paper employs bootstrapping and Gibbs Sampling techniques to systematically estimate measurement error variance of selected error-prone predictor variables and adjusts for measurement errors in 2 and 4 level model frameworks. Five illustrative data sets, partly supplemented through simulation, were drawn from an educational environment giving rise to the multilevel structures needed. Adjusting for the incidence of measurement errors using these techniques generally revealed coefficient estimates of error-prone predictors to have increased numerical value, increased standard error, reduced overall model deviance and reduced coefficient of variation. The techniques, however, performed better for error-prone predictor(s) having random coefficients. It is opined that the bootstrapping and Gibbs Sampling techniques for adjusting for the incidence of measurement errors in multilevel models is systematic and realistic enough to employ in respect of error-prone predictors that have random coefficients and adjustments that are meaningful should be appraised taking into cognizance changes in the coefficient of variation alongside other traditionally expected changes that should follow measurement error adjustments.

Key words: Multilevel models, Measurement error adjustment, Coefficient of variation, Predictor variables, Bootstrapping, Gibbs sampling.

1.0 Introduction

In many of the variables used in the physical, biological, social and medical science, measurement errors are found. The errors are essentially random or systematic. Both types of errors could be problematic in statistical inference. In fixed effects models such as linear and generalized linear models, the incidence and effects of measurement errors on the response and explanatory variables has been well documented in the literature [4], [9], [2], [8], [1], [12], [4]. Generally, the consequences of ignoring measurement errors for independent observations and response values are well understood in linear models.

The efficacy of mixed models such as multilevel linear models is also adversely affected by a failure to properly account for measurement errors in their formulation and estimation. In particular, the behaviour of biases associated with measurement error in covariates or the response for multilevel hierarchical linear models is, up to date, not well known and can be complex [7]. In handling the incidence of measurement errors in multilevel modeling methodology, one of the daunting challenges that often confront researchers is that of estimating realistically measurement error variances and reliabilities of error-prone variables in a multilevel model. Most of the current techniques for estimating measurement error variance are, in general deficient**;** there is inability to sufficiently justify independence of measurement errors and the so called unidimensionality assumption as required in educational mental testing; accuracy and consistency of the estimates of the measurement error variance could not be guaranteed [3]. The method of instrumental variables strongly recommended for certain situations as in mental testing (see [3]) requires, however, that several different instrumental variables be considered for comparison. There is also often the difficulty of establishing that measurement errors are independent of instrumental variables [11]. Some other researchers often simply assume measurement error variance and reliability values for error-prone variables in the multilevel models at the risk of obtaining unrealistic estimates. This paper employs bootstrapping and Gibbs sampling techniques to realistically estimate measurement error variances of selected error-prone explanatory variables and adjusts for the incidence of these errors giving rise to more adequate multilevel models.

2.0 Methodology

2.1 Data Structure

The illustrative data employed was drawn from an educational environment. There were five data sets(Data 1-5) utilized. Data 1-3 were derived from 50 randomly selected secondary schools in Benue State of Nigeria while Data 4 and 5 included data supplemented by simulated values. Data 1 constituted a 4-level data structure in which there were 9,999 level 1 units (here students), 450 level 2 units (here subjects or subject groups), 150 level 3 units (here classes) and 50 level 4 units (here schools). The clustering was such that for any original sample n_j ($20 \leq n_j \leq 30$) of the students from each school j, the n_j was "replicated" into 9 clusters giving rise to $9n_j$ level 1 units for school j (j = 1,2, …,50). In other words, the same n_j students in school j were mirrored in 9 clusters or groups and, in particular, for each school j, we had $9n_j$ level 1 units nested in 9 level 2 units that were further nested in 3 level 3 units . Data 2 also constituted a 4-level data structure but here there were 6,666 level 1 units(students), 300 level 2 units(subjects or subject groups), 150 level 3 units(classes) and 50 level 4 units(schools); in this dataset, the seeming confounding characteristics in Data 1 were reduced by removing the level 2 unit or cluster relating to Common Entrance (CE) and variables based on it. Data 3 is a 2-level data structure with students nested in schools; any sample drawn in a school constituted a "statistical cohort" of students from whom Mathematics (M) and Science and Technology (ST) scores in JSS1, JSSCE and SSCE/WAEC between 2002 and 2008 were captured. Data 3 had 1,111 level 1 units and 50 level 2 units. Additional levels 1 and 2 units were further generated via simulation to supplement needed data for further exploration. These gave rise to Data 4 (having 2,222 level 1 units with same 50 level 2 units) and Data 5(having 4022 level 1 units and 110 level 2 units).

2.2 Description of Variables

Variable name	Description of Variable	Data set where used
$Navgstem_{ij}$	Student's Final STM score ; a level 1 response variable.	1, 2
$Ncescore_{ij}$	Student's entrance score; a level 1 predictor variable	1
$Normscore_{ij}$	JSS1 school STM score student's subject score per class; a level 1 predictor variable.	1, 2
$Navg1stem_{l}$	Final School STM score; a level 4 predictor variable	1, 2

$Navgce_l$	School common entrance score; a level 4 predictor variable.	1
$Navg2stem_l$	JSSCE school STM score ; a level 4 predictor variable.	1, 2
$Navg3stem_j$	Final School STM score; a level 4 predictor variable	1, 2
$Navgsub_j$	Score per subject; a level 2 predictor variable.	1, 2
$Navgincls_k$	Score in class ; a level 3 predictor variable.	1, 2
$Navgscore_{ij}$	STM score per student in all classes; a level 1 response variable.	3-5
$NJS1avg_{ij}$	STM score per student in JSS1 subjects; a level 1 predictor variable	3-5
$NJCEavg_{ij}$	STM score per student in JSSCE subjects; a level 1 predictor variable.	3-5
$Schstatus_l$	school status(i.e whether school is owned as private or public); a categorical predictor variable.	3-5
$Schsystem_l$	school system; it is a categorical predictor variable with the systems categorized into "Boardsytem", " Daysystem" or "Bothsystem".	1-5
$Schgender_l$	School gender ; it is categorical predictor variable with school gender categorized into Boys school(Boysch), Girls school (Girlsch) or Mixed(Mixedsch).	1-5
$Nrsqindex_l$	School staff quality index (an indication of academic staff quality or strength in any particular school. This is estimated by dividing the total number of qualified academic staff by the entire estimated student population in the school; it is a predictor variable.	1-5
$PSStatus_l$	An indication of Electric Power Supply status in a school ; it is a categorical predictor variable categorized into school generator, PHCN, Both or one.	3, 4
$Labav_l$	An indication of the availability of Science Laboratories in a school; it is a categorical predictor variable categorized into "no science lab", " one science lab " or " two or more science labs" .	3

3 Multilevel Models and Measurement Errors

A k-level model may be expressed in the compact form:

$$Y = X\gamma + ZU + Z^{(1)}e \tag{2.1}$$

where, Y is a column vector of true unobservable responses each assumed continuous.

$Z = [Z^{(k)}, Z^{(k-1)}, \ldots, Z^{(2)}]$
and
$U' = [u^{(k)}\ u^{(k-1)}, \ldots, u^{(2)}]$.
The $Z^{(k)}$'s are block diagonal matrices having diagonal elements as $Z_j^{(k)}$ ($j = 1,2,\ldots,m_k$) while $u^{(k)}$, X and γ are column matrices with elements, respectively, $u_j^{(k)}$, X_j ($j=1,2,\ldots,m_k$),and γ_{h0} ($h=0,1,\ldots,p$).
We assume that $Z^{(1)}e$ and U are normally distributed with zero mean and we, symbolically, write:

$$Z^{(1)}e = r \sim N(O, \sigma^2 \ddot{I}^*) \quad \ldots (2.2)$$

and

$$U \sim N(O, T^*) \quad \ldots (2.3)$$

where $\ddot{I}^*$ and T^* are appropriate block diagonal matrices comprising, respectively, the blocks of unit matrices and blocks of variance-covariance matrices of the residual vectors associated with the k-level model (that is the residual contributions from the levels 2, 3, …, k in the k-level model).

We infer from (2.1),(2.2) and (2.3) that Y is normally distributed with $E(Y) = X\gamma$ and variance-covariance matrix, $V_k = V = E[\widetilde{E}\ \widetilde{E}'] = \sum_l \{V_{k(l)}\}$,where $\widetilde{E} = ZU + Z^{(1)}e$. The notation V_k here referring to the covariance(or variance-covariance) matrix associated with the response vector for the k-level model and $V_{k(l)}$($l = 1,2,\ldots k$), respectively, denote the contributions to the covariance matrix of the response vector from levels k, k-1,…,1 in a k-level model. The level 1 residuals are assumed to be independent across level 1 units. Similarly, levels 2, 3,…,k residuals are assumed to be independent across levels 2 ,3,…, k units respectively. It should be noted also that V_k is a block diagonal matrix with block diagonal elements $V_{k(l)}$($l = 1,2,\ldots k$) and each of these elements is also block diagonal comprising blocks in their composition.

If the collection or measurement of explanatory or response variables incorporated in (2.1) are susceptible to errors then the estimated coefficient parameters will be asymptotically biased and consequently incorrect inferences can result in respect of the relevance or otherwise of some model variables. In practice explanatory or response variables utilized to fit models in social or educational environments are subject to some degree of measurement error.

A basic model for measurement errors in a 2-level continuous response linear model for p explanatory variables and responses takes the form.

$$\widetilde{Y} = \widetilde{y} + \widetilde{q} \quad (2.4)$$

$\widetilde{Y} = [Y_{11}\ Y_{21} \ldots Y_{n1} \ldots Y_{1j}\ Y_{2j} \ldots Y_{njj} \ldots Y_{1J}\ Y_{2J} \ldots\ Y_{nJJ}]'$
$\widetilde{y} = [y_{11}\ y_{21} \ldots y_{n1} \ldots y_{1j}\ y_{2j} \ldots y_{njj} \ldots y_{1J}\ y_{2J} \ldots\ y_{nJJ}]'$
$\widetilde{q} = [q_{11}\ q_{21} \ldots q_{n1} \ldots q_{1j}\ q_{2j} \ldots q_{njj} \ldots q_{1J}\ q_{2J} \ldots\ q_{nJJ}]'$
$X_{ij} = [x_{0ij}\ x_{1ij} \ldots\ x_{hij} \ldots x_{pij}]$ and $x_{0ij}=1$

In respect of the explanatory variables or predictors, we have

$$\widetilde{X} = \widetilde{x} + \widetilde{m} \quad (2.5)$$

where
$\widetilde{X} = [X_1\ X_2,\ldots X_J]'$
$\widetilde{x} = [x_0\ x_1 \ldots\ x_h \ldots x_p]'$, x_0 = a column of ones.
$\widetilde{m} = [m_1\ m_2 \ldots m_h \ldots m_p]'$
With
$x_h = [x_{h11}\ x_{h21} \ldots x_{hn11} \ldots x_{h1j}\ x_{h2j1}, \ldots, x_{h2jJ} \ldots\ x_{hnJJ}]'$
$m_h = [m_{h11}\ m_{h21} \ldots m_{hn11} \ldots m_{h1j}\ m_{h2j1}, \ldots, m_{h2jJ} \ldots\ m_{hnJJ}]'$
and for each j we can write
$m_{hj} = [m_{h1j}\ m_{h2j} \ldots m_{hnjj}]'$
$x_{hj} = [x_{h1j}\ x_{h2j} \ldots x_{hnjj}]'$

The measurement error vectors $\tilde{m}$ and $\tilde{q}$ are assumed independent and normally distributed with zero mean vectors. The measurement error models reflected by (2.4) and (2,5) can be analogously expressed in matrix form for any k-level model. The concern of researchers and statisticians is to seek ways of adjusting for the incidence of these measurement errors and to do this entails a an estimation of (or the use of known value(s) of) measurement error variances of perceived error-prone variables and there after use the estimated values to modify the affected model and estimate same.

Assuming the variables measured with error do not have random coefficients, then following Goldstein(2003), the ME corrected fixed coefficients estimate for any k- level model is

$$\widehat{M}_{xx} = M_{XX} - C_{\Omega_1} - C_{\Omega_2} - \ . \ . \ . \ - C_{\Omega_k} \quad (2.6)$$

where

$$C_{\Omega_k} = \sum_{\alpha}(J_{(1,n_\alpha)} V_\alpha^{-1} J_{(n_\alpha,1)})\Omega_{k\alpha m} \ ,$$

k = 1,2,…

are correction matrices for measurement errors, $\Omega_{k\alpha m}$ is the covariance matrix of measurement errors for the αth level k block, V α is the αth block of V, the variance-covariance matrix of residuals in the k-level model.

For the random components, based upon the model with observed variables, we write the residual for a unit in a level k as

$$_{ij..l} = Z_l^{(k)} u_l^{(k)} + \ ... \ + Z_k^{(3)} u_k^{(3)} + Z_j^{(2)} u_j^{(2)} + Z_{ijk..l}^{(1)} e_{ijk..l} + q_{ij..l} - m'\beta \quad (2.7)$$

. The estimation of the variance or variance-covariance components(i.e the random components) are all estimated iteratively and, for a k-level model, the measurement error corrected estimate of these components, assuming the coefficients of the variables measured with error do not have random coefficients, is obtained at each iteration as

$$E[\tilde{Y}\tilde{Y}'] - [\bigoplus_{ij..l} \sigma^2_{ij..lq} + T_1 + T_2 + \ ... \ T_k]$$

(2.8)

Where $T_1 = \bigoplus_{ij}\{\hat{\beta}'\Omega_{1ijm}\hat{\beta})$ and $T_k = \bigoplus_{\alpha}(\hat{\beta}'\Omega_{k\alpha m}\hat{\beta})J_{(n_\alpha,n_\alpha)}$ for k ≥ 2.

We note that Ω_{1ij} is the covariance of measurement errors for the *ij*th measurements of level 1 while $\sigma^2_{ij..lq}$ is the measurement error variance for the *ij..l*th response measurement.

If the coefficients of the variables measured with error have random coefficients then the formulae in (2.6) and (2.8) do not apply and in particular $\tilde{m}' V^{-1} \tilde{m}$ has measurement errors in all its components and, following the suggestions made by Woodhouse [13], Moment-based techniques are not appropriate but rather the Bayesian technique of Gibbs sampling(an MCMC technique) is employed. Some of the selected predictor variables perceived error-prone in this paper have random coefficients and so Gibbs sampling technique rather than moment-based technique shall be employed to adjust for the incidence of these errors and, for the estimation of measurement error variances and reliabilities of the error-prone variables, the bootstrapping technique shall be employed.

2.4 The Multilevel Models Examined

The multilevel models formulated in respect of each of the data sets (1-5) are respectively given by (2.9), (2.10), (2.11), (2.12) and (2.13) below.

$$Navgstem_{ijkl} = \beta_{0i} + \beta_{1j}\left(Normscore - m(Subject)\right)_{ijkl} + \beta_{2l}\left(Ncescore - m(Subject)\right)_{ijkl} + \beta_3(Navg3stem - gm)_l + \beta_4(Navgce - gm)_l +$$
$$\beta_5 DaySystem_l + \beta_6 BothSystem_l + \beta_7 \text{Girlsch}_l + \beta_8 \text{Mixedsch}_l + \beta_9 (\text{Nrsqindex- gm})_l + e_{ijkl}$$

$$\beta_{0l} = \beta_0 + f_{0l}$$
$$\beta_{ij} = \beta_1 + u_{1jkl}$$
$$\beta_{2l} = \beta_2 + f_{2l}$$

$$\begin{bmatrix} f_{0l} \\ f_{2l} \end{bmatrix} \sim N(0, \Omega_f) : \Omega_f = \begin{bmatrix} \sigma^2_{f0} & \\ \sigma_{f02} & \sigma^2_{f2} \end{bmatrix}$$
$$u_{0jkl} \sim N\left(0, \sigma^2_{u0}\right)$$
$$e_{ijkl} \sim N\left(0, \sigma^2_e\right)$$

(2.9)

$$Navgstem_{ijkl} = \beta_{0l} + \beta_{1l}(Normscore - m(Subject))_{ijkl} + \beta_2(Nrsqindex - gm)_l$$
$$+\beta_3(Navg3stem - gm)_l + \beta_4 Daysystm_l + \beta_5 Bothsystm_l + \beta_6 Schstatus\_1_l + e_{ijkl}$$

$$\beta_{0l} = \beta_0 + f_{0l}$$
$$\beta_{1l} = \beta_1 + f_{1l}$$
$$\begin{bmatrix} f_{0l} \\ f_{1l} \end{bmatrix} \sim N(0, \Omega_f) : \Omega_f = \begin{bmatrix} \sigma^2_{f0} & \\ \sigma_{f01} & \sigma^2_{f1} \end{bmatrix}$$
$$e_{ijkl} \sim N\left(0, \sigma^2_e\right)$$

(2.10)

$$Navgscore_{ij} = \beta_{0j} + \beta_{1j}(NJS1avg - m(School))_{ij} + \beta_{2j}(NJCEavg - m(School))_{ij}$$
$$+\beta_3(Navg3stem - gm) + \beta_4 Schstatus\_1_{ij} + \beta_5 Girlsch_j + \beta_6 Mixedsch_j + \beta_7 Labav\_1_j$$
$$+ \beta_8 Psstatus\_2_j + \beta_9 Psstatus\_3_j + \beta_{10} Psstatus\_4_j + \beta_{11}(Nrsqindex - gm)_j$$
$$+ \beta_{12} Daysytm_j + \beta_{13} Bothsystm_j + e_{ij}$$

with

$$\beta_{0j} = \beta_0 + u_{0j}$$
$$\beta_{1j} = \beta_1 + u_{1j}$$
$$\beta_{2j} = \beta_2 + u_{2j}$$

and

$$\begin{bmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{bmatrix} \sim \mathrm{N}(0,\Omega_{\mathrm{u}}) : \Omega_{\mathrm{u}} = \begin{bmatrix} \sigma^2_{uo} & & \\ \sigma_{uo1} & \sigma^2_{u1} & \\ \sigma_{uo2} & \sigma_{u12} & \sigma^2_{u2} \end{bmatrix}$$

$$\mathrm{e}_{\mathrm{ij}} \sim \mathrm{N}(0,\sigma^2_{\mathrm{e}}).$$

(2.11)

$$Navgscore_{ij} = \beta_{0j} + \beta_{1j}(NJS1avg - m(SCHOOL))^t_{ij} + \beta_{2j}(NJCEavg - m(SCHOOL))_{ij}$$
$$+\beta_3(Navg3stem - gm)_j + \beta_4 Schstatus\_1_{ij} + \beta_5 Girlsch_j + \beta_6 Mixedsch_j + \beta_7(Nrsqindex - gm)^t_j +$$
$$\beta_8 Labav\_1_j + \beta_9 Daysystm_j + \beta_{10} Bothsystm_j + \beta_{11} Psstatus\_2_j + \beta_{12} Psstatus\_3_j + \beta_{13} Psstatus\_4_j + e_{ij}$$
$$\beta_{0j} = \beta_0 + u_{0j}$$
$$\beta_{1j} = \beta_1 + u_{1j}$$
$$\beta_{2j} = \beta_2 + u_{2j}$$
$$\begin{bmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{bmatrix} \sim \mathrm{N}(0,\Omega_{\mathrm{u}}) : \Omega_{\mathrm{u}} = \begin{bmatrix} \sigma^2_{uo} & & \\ \sigma_{uo1} & \sigma^2_{u1} & \\ \sigma_{uo2} & \sigma_{u12} & \sigma^2_{u2} \end{bmatrix}$$

$$\mathrm{e}_{\mathrm{ij}} \sim \mathrm{N}(0,\sigma^2_{\mathrm{e}}).$$

(2.12)

$$Navgscore_{ij} = \beta_{0j} + \beta_{1j}(\mathrm{NJS1avg - m(SCHOOL)})_{\mathrm{ij}} + \beta_{2j}(NJCEavg - m(SCHOOL))_{ij}$$
$$+\beta_3(\mathrm{Navg3stem - gm})_{\mathrm{j}} + \beta_4 Daysytm_j + \beta_5 \mathrm{Bothsystm}_{\mathrm{j}} + \beta_6(\mathrm{NJS1avg - m(SCHOOL)}).\mathrm{Daysystm}_{\mathrm{ij}}$$
$$+\beta_7(NJS1avg - m(SCHOOL)).Bothsystm_{ij} + \beta_8(Nrsqindex - gm)_j + e_{ij}$$
$$\beta_{0j} = \beta_0 + u_{0j}$$
$$\beta_{1j} = \beta_1 + u_{1j}$$
$$\beta_{2j} = \beta_2 + u_{2j}$$
$$\begin{bmatrix} u_{0j} \\ u_{1j} \\ u_{2j} \end{bmatrix} \sim \mathrm{N}(0,\Omega_{\mathrm{u}}) : \Omega_{\mathrm{u}} = \begin{bmatrix} \sigma^2_{uo} & & \\ \sigma_{uo1} & \sigma^2_{u1} & \\ \sigma_{uo2} & \sigma_{u12} & \sigma^2_{u2} \end{bmatrix}$$

$$\left[\mathrm{e}_{0\mathrm{ij}}\right] \sim \mathrm{N}(0,\Omega_e)$$

(2.13)

2.5.1 The Measurement Error AdjustmentApproach Using Bootstrapping and Gibbs Sampling Techniques.

The approach essentially entails re-sampling repeatedly from each of the clusters or subgroups in a data structure to estimate the variance of the error-prone predictor variable, its measurement error variance, reliability and ultimately adjusting for the incidence of measurement errors and re-estimating the k-level model accordingly. The steps are:

(i) From each group (or subgroup) of the multilevel model obtain an estimate of the Explanatory variable mean , $\overline{X_{\bullet}}j$, based on sample sizes of at least 30 in each group.

(ii) Average these $\overline{X_{\bullet}}j$'s (using arithmetic mean) across the entire groups to obtain a value, say $\overline{X^*}$.

(iii) Estimate the measurement error (ME) variance, σ^2_{hm}, as the mean of the squares of deviations of $\overline{X_{\bullet}}j$'s from $\overline{X^*}$.

(iv) Estimate σ^2_{hX} as in the first paradigm approach. Estimate R_h accordingly.

(v) Use the values σ^2_{hm} and σ^2_{hX} to adjust for measurement error in the variable (s) of interest and hence re-estimate the k-level model accordingly via Gibbs sampling ; a Markov Chain Monte Carlo(MCMC) method.

(vi) Check for possible attenuation and/or inconsistency of the estimated multilevel parameters

(vii) If there is attenuation (reduced or no increase in predictive power of corresponding predictor) and/or inconsistency of the estimated multilevel parameters then repeat steps (i) to (vi) , possibly increasing re-sampling size per cluster and/or increasing number of samples.

3.0 Analysis and Discussion

Four issues were addressed in the analysis of the multilevel models associated with each of the five data sets:

(a) estimation of the measurement error variances and reliabilities of STM score per student in JSS1 subjects(NJS1avg) or its proxies(such as Normscore variable for Data 1 and 2) as well as the School staff quality index(Nrsqindex) predictor variables.

(b) coefficient estimates of the perceived error-prone predictors (and their standard errors) prior to adjustments in measurement error.

(c) coefficient estimates of the perceived error-prone predictors (and their standard errors) following adjustments in measurement error.

(d) examination of coefficient of variation values of coefficient estimates of the error-prone predictors.

Bootstrapping with a minimum of 2000 replicates for each of the NJS1avg and Nrsqindex variables in each of the data sets and following steps (i) to (iv) we obtain measurement error variance and reliability values as reflected in table 1 below.

Table 1: Estimated Variances, Measurement Error (M.E) Variances and Reliabilities in respect of the 'student's subject score per class' Predictor Variable or their proxies in various datasets.

Table 1: Estimated Variances, Measurement Error

Data	Variable	Variance	M.E Variance	Reliability
1	NJS1avg**	0.44393	0.222942	0.666872
2	NJS1avg*	0.541682	0.250571	0.683724
3	NJS1avg	0.735439	0.255298	0.742315
4	NJS1avg	0.674021	0.253647	0.726576
5	NJS1avg	0.822923	0.635357	0.564311

NJS1avg** and NJS1avg* actually refer to the Normscore variables used for Data 1 and 2 and are realistic proxies of the NJS1avg variable as they are already associated with the JSS1 scores. We find that for Data 2, 3 and 4 the variable NJS1avg indicates a reasonably constant measurement error variance; an average of 0.25. In Data 1, the NJS1avg variable gave measurement error variance estimate slightly lower (i.e 0.22) than what obtained in Data 2, 3 and 4 but the average for the Data 1-4 measurement error variance of the NJS1avg variable is still 0.25. The rather high measurement error variance estimate for the NJS1avg variable (here 0.64) for Data 5 may be attributable to weaknesses associated with the normal probability distribution model and the accompanying assumptions that were employed to simulate additional units for levels 1 and 2. The near-absence within group variation in respect of School staff quality index (Nrsqindex) predictor variable may have probably accounted for the high measurement error variance estimate of 0.85 associated with it.

Using iterative generalized least squares(IGRLS) that is implemented in MLWiN package 2.20 [10] to estimating models (2.9) -(2.13) prior to measurement error adjustments, we obtain coefficient estimates(with standard errors) of the NJS1avg variable or its proxy along with coefficient of variation(CV) of these estimates as reflected in Table 2 below.

Table 2: Coefficient Estimates of Student's STM score in JSS1(NJS1avg) and School staff quality Index(Nrsqindex), their standard errors,
coefficients of variation(CV) and model deviance(D) for the measurement error unadjusted scenarios.

Table 2: Coefficients of Student STM

Data	Variable	Coefficient estimate	Standard error	Coefficient of variation (CV)	Model deviance (D)
1	NJS1avg **	0.273	0.020	0.073	21967
	Nrsqindex	-0.014	0.009	-0.643	
2	NJS1avg*	0.314	0.033	0.105	15206
	Nrsqindex	-0.053	0.032	-0.604	
3	NJS1avg	0.680	0.017	0.025	808
	Nrsqindex	0.010	0.062	6.20	
4	NJS1avg	0.671	0.020	0.030	1188
	Nrsqindex	0.011	0.06 1	5.54	
5	NJS1avg	0.744	0.018	0.024	3456
	Nrsqindex	0.030	0.082	2.73	

Employing Gibbs Sampling technique implemented in MLWiN package 2.20 [10], we adjust for the incidence of measurement errors to obtain estimate results of the two predictor variables under investigation as in Table 3 below.

Table 3: Coefficient Estimates of Student's STM score in JSS1 (NJS1avg) and School Staff quality Index (Nrsqindex), their standard errors, coefficients of variation (CV) and model deviance (D) for the measurement error adjusted scenarios.

Table 3: Predictor Variables

Data	Variable	M.E Variance	Coefficient estimate	Standard error	Coefficient of variation (CV)	Model deviance (D)
1	NJS1avg **	0.22	0.394	0.029	0.074	20023
	Nrsqindex	0.85	-0.101	0.074	-0.732	
1	NJS1avg **	0.25	0.416	0.029	0.070	19904

	Nrsqindex	None	-0.015	0.009	-0.600	
1	NJS1avg **	0.22	0.398	0.028	0.070	20068
	Nrsqindex	None	-0.015	0.009	-0.600	
2	NJS1avg*	0.25	0.468	0.10	0.218	12732
	Nrsqindex	0.85	-0.260	0.156	-0.600	
2	NJS1avg*	0.25	0.480	0.094	0.196	12928
	Nrsqindex	None	-0.039	0.027	-0.692	
3	NJS1avg	0.25	0.871	0.029	0.033	-4476
	Nrsqindex	0.85	0.035	0.070	2.00	
3	NJS1avg	0.25	0.860	0.019	0.022	-4406
	Nrsqindex	None	0.043	0.050	1.163	
4	NJS1avg	0.25	0.828	0.021	0.025	-9170
	Nrsqindex	0.85	-0.007	0.041	-5.86	
4	NJS1avg	0.25	0.869	0.038	0.044	-9325
	Nrsqindex	None	-0.003	0.044	14.67	
5	NJS1avg	0.64	0.893	0.017	0.019	-16900
	Nrsqindex	0.85	-0.029	0.030	-1.03	
5	NJS1avg	0.25	0.901	0.016	0.018	-17433
	Nrsqindex	0.85	-0.025	0.028	-1.12	
5	NJS1avg	0.25	0.909	0.014	0.015	-18359
	Nrsqindex	None	0.005	0.012	2.40	
5	NJS1avg	0.064	0.899	0.014	0.016	-17855
	Nrsqindex	None	0.002	0.013	6.50	

Following measurement error adjustments, Data 1, 3, 4 and 5 all reflected an average CV of the coefficient estimate of NJS1avg to be equal to or less than what obtained in the measurement error unadjusted scenarios. In the case of the Nrsqindex variable, the measurement error adjustment did not seem as impressive as what obtains in the NJS1avg variable; Data 1 and 2 did not reveal a drop in the numerical value of the CV of the coefficient estimate of the Nrsqindex variable. Data 2, 3 and 5 however reflected numerical CV values of the coefficient estimates of Nrsqindex for the measurement error adjusted cases to be, on average, less than or equal to what obtained in the measurement error unadjusted cases. We observe that , apart from the near-absence between cluster variations in so far as the Nrsqindex variable was concerned, the variable also has a fixed coefficient. It is discernable that, in general, measurement error adjustments done gave rise to increase in numerical size of perceived error-prone predictors, increased standard error and reduced model deviance as expected. It is also found that, in the measurement error adjustments where measurement error variance values were assumed(rather than estimated) , coefficients tended to have been inaccurately determined with exaggerated estimates and lower standard errors. Ignoring the likelihood of measurement errors in some predictors and adjusting for error in some other predictors tend to also yield much higher coefficient estimate values with the overall model deviance not necessarily being lower. Deviations from expected post-measurement error adjustment effects are also discernable for variables with low reliability (i.e high measurement error variance); thus assuming a low measurement error variance value (i.e. high reliability) for a variable is likely to result in a coefficient estimate value indicating a higher predictive power than what obtains when we assume a higher measurement error variance (i.e. low reliability) for such a variable.

Regardless of some inadequacies arising from supplementary data generation approaches that gave rise to Data 4 and 5 above and hence seeming unimpressive results in some measurement error adjustments done, the hypothesis that using estimated measurement error variance as input into model estimation process, as done here using bootstrapping and Gibbs sampling, is more objective, more logical and realistic than using an assumed value.

4.0 Conclusion

Although the incidence and effects of measurement errors on the response and explanatory variables in fixed effects models such as linear and generalized linear models , has been well documented in the literature(see, for example,[4],[9], [[2], [[8], [1], [12], [5]), studies on the behaviour of biases associated with measurement error in covariates or the response for mixed models such as multilevel hierarchical linear models is, up to date, not well known and can be complex [7]. One of the daunting challenges that often confront researchers is that of realistically estimating measurement error variances and reliabilities of error-prone variables in a multilevel model to enable realistic measurement error adjustment. An iterative measurement error adjustment technique entailing bootstrapping and Gibbs Sampling is applied on an educational illustrative data (i.e Data 1-5) to which levels two or four models are associated.

Employing the iterative measurement error adjustment technique on the STM score per student in JSS1 subjects (NJS1avg) variable generally indicated numerical increase in the coefficient estimate, increased standard error of the coefficient estimate, decreased overall model deviance, decreased estimate of the coefficient of variation(CV) of the coefficient estimate. The near-absence between cluster variance coupled with possible weaknesses in the supplementary data generating simulation method employed in respect of predictors with fixed coefficients(such as Nrsqindex) and some data sets however revealed slightly differing trends. It is opined in this paper that the bootstrapping and Gibbs Sampling measurement error adjustment approach for addressing incidence of measurement errors in multilevel models is more efficacious in a situation where the error-prone predictor variables under consideration have random coefficients. It is a suggested that a realistic appraisal of the effectiveness or otherwise of a measurement error variance estimation and measurement error adjustment approach should, apart from examining the general expectations of increase in numerical value of coefficient estimate, increased standard error, reduced level 1 residual and reduced over all model deviance, also take into cognizance the coefficient of variation(CV) values of the coefficient estimates associated with the perceived error-prone predictors.

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