Water Pipeline Network Analysis Using Simultaneous Loop Flow Correction Method

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Abstract

A water pipeline network analysis with a case study of Owerri in Imo State, Nigeria municipal water reticulated system has been undertaken. What prompted this study is that the case study has a lot of fluctuations in its head loss. Also, the discharge is not proportional to the pipe diameter. The study therefore adopted simultaneous loop flow correction method because it computes simultaneous flows corrections for all loops, hence, the best since computational procedures takes into account the iterative influence of flow corrections between loops which have common pipes. After applying the simultaneous loop flow correction analyzer in a twenty-four sampled pipeline network, a drastic reduction in head loss and regular line along the axis was observed. Besides, the rate at which the water flows was observed to be proportional to the pipe diameter. Hence, the method is a useful aid in planning, designing and operating of reticulated pipeline network for higher efficiency and improved economy.

KEYWORDS: Water Reticulated System, Simultaneous Loop Flow, Iteration, Flow Rate, Pipeline Network, Head Loss

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1.0 Introduction

Water is an essential natural resource for industrial and natural process, for example, it is used for oil refining, liquid extraction in hydrometallurgical process, cooling, scrubbing in the iron and steel industry and several operations in food processing facilities. Water is an essential input to achieve some desired outcomes, including health and income. Water affects sanitation and hygiene because lack of access to water leads to unhygienic behavior. Water supply is the provision of water by public utilities, commercial organizations, community endeavors or by individuals, usually via a system of pumps and pipes. Water supply systems get water from various locations, including ground water (aquifers) surface water (lakes and rivers) conservation and the sea through desalination. The water is then in most cases purified, disinfected through chlorination and sometimes fluoridated. Treated water then either flows by gravity or pumped reservoirs, which can be elevated such as water towers.

Analysis and design of pipe networks create a relatively complex problem, particularly if the network consists of a range of pipe as frequently occurs in water distribution systems. In the absence of significant fluid acceleration, the behavior of a network can be determined by a sequence of steady state conditions, which form a small but vital component for assessing the adequacy of a network.

In 2010, about 86% of the global population 96.74 billion people had access to piped water supply through house connections or to an improved water source through other means than house, including standpipes, water kiosks, protected springs and protected wells. However, about 14% (884 million people) did not have access to an improved water sources and had to use unprotected wells or springs, canals, lakes or rivers for their water needs [13].

Greater number of people having access to pipe water receive poor quality of service, especially in developing countries where about 80% of the world population lives, the United Nations (2006) revealed that total water availability per capita in Nigeria decreased from 2514 m^3 per year in 2000 to 2250 m^3 per year in 2005. In Nigeria, only water as at 2010 as reported in ***Thisday*** News paper of August 19, 2010 under the heading, Nigeria, only 17.2 percent have access to potable water. In light of this, this year's world water day centers on water and food security as decline access to water affects agricultural produces. Therefore, there is a dire need for adequate planning, design and provision of reticulated water to every nook and cranny of this nation to ensure a healthy living standard for all, boost agricultural produces and provide enough water for industrial purposes.

Materials and Method

A pipe network is a set of pipes which are interconnected in such a way that flow from a given input get to a given outlet. An attempt to apply Bernoulli's equation and the continuity of flow equation to the various elements in the network would lead to a very large number of simultaneous equations which would be cumbersome to solve.

In this work, simultaneous loop flow technique was used to determine the discharge (flow rate) connections in each pipe; head losses in each pipe and Node pressures. Minor losses due to pipe fittings such as valves; pipe bends, elbows, etc can be accounted for by using the equivalent length of pipe method.

The waterworks of Owerri Municipal Area was adopted as a case study. It covers some parts of Owerri Municipal council and Owerri West Local Government Area. The pipe lengths, various pipe diameters, volume flow rates, piping materials, types of joints, and other relevant information was collected from the water board. These values were used in this analytical work. The network diagram of ISWC distribution is shown in Figure 1.

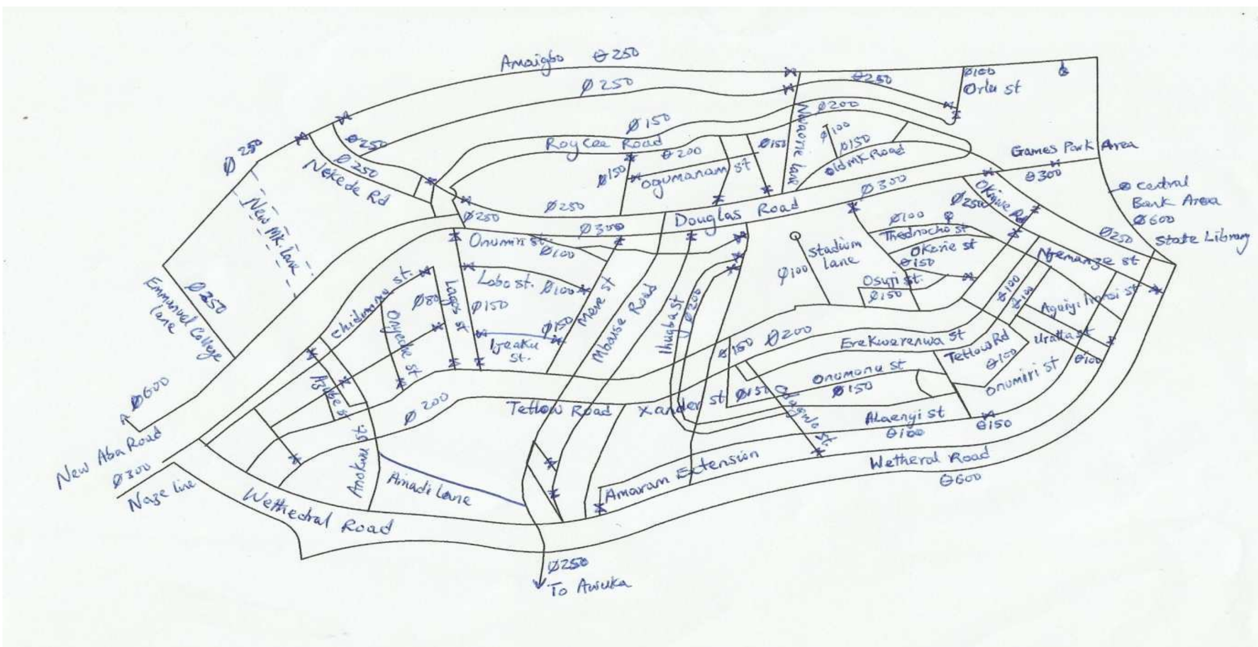


Figure 1 (the network diagram of ISWC)

Solution of Network Equations

Direct solution of systems of non-linear simultaneous equation is not feasible; hence, it is necessary to use iterative solution methods.

Generally, these methods start with an estimated solution which is interactively refined by repeated corrections until the deviation from the true solution is reduced to an acceptable tolerance value. Hardy Cross (HC) method is the most widely used technique for solving for the unknown in water network analysis. It is based on a loop iterative computation.

Newton-Raphson method is a better technique for solving the network problems; however, the method adopted here computes simultaneous flow corrections for all loops, hence, the best since the computational procedure takes into account the iterative influence of flow corrections between loops which have common pipes. In HC method, an initial flow distribution, which satisfies flow continuity at nodes, is assumed. Nevertheless, the simultaneous loop flow correction method converges more rapidly to the true solution than the loop by loop Hardy Cross method.

The first order estimates of the loop flow corrections, which would reduce the loop out-of-balance heads to zero, are found from the following Newton – Raphson approximations.

$$-h_1 = \frac{\partial h_1}{\partial q_1}\Delta q_1 + \frac{\partial h}{\partial q_2}\Delta 1 q_2 + \ldots\ldots + \frac{\partial h}{\partial q_n}\Delta q_n$$

$$-h_2 = \underline{\partial h_2}\,\Delta q_1 + \underline{\partial h_2}\,\Delta 1 q_2 + \ldots\ldots + \underline{\partial h}\,\Delta q_n$$

$$-h_n = \frac{\partial h_n}{\partial q_1}\Delta q_1 + \frac{\partial h_n}{\partial q_2}\Delta 1 q_2 + \ldots\ldots + \frac{\partial hn}{\partial q_n}\Delta q_n$$

Twenty Four (24) Pipeline Network Case Study

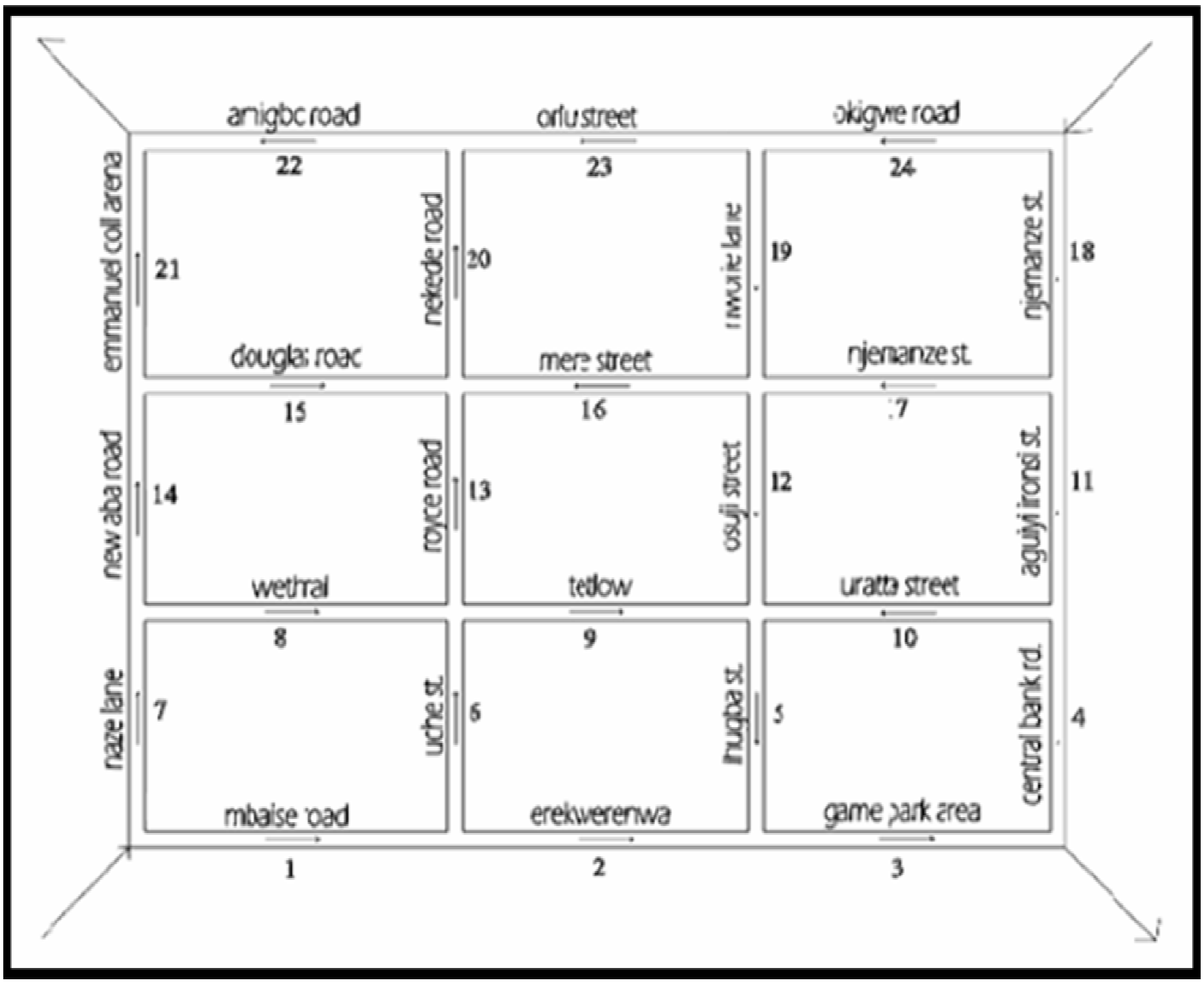


Figure 2: (Network Diagram Of The Study Area)

The Network Diagram

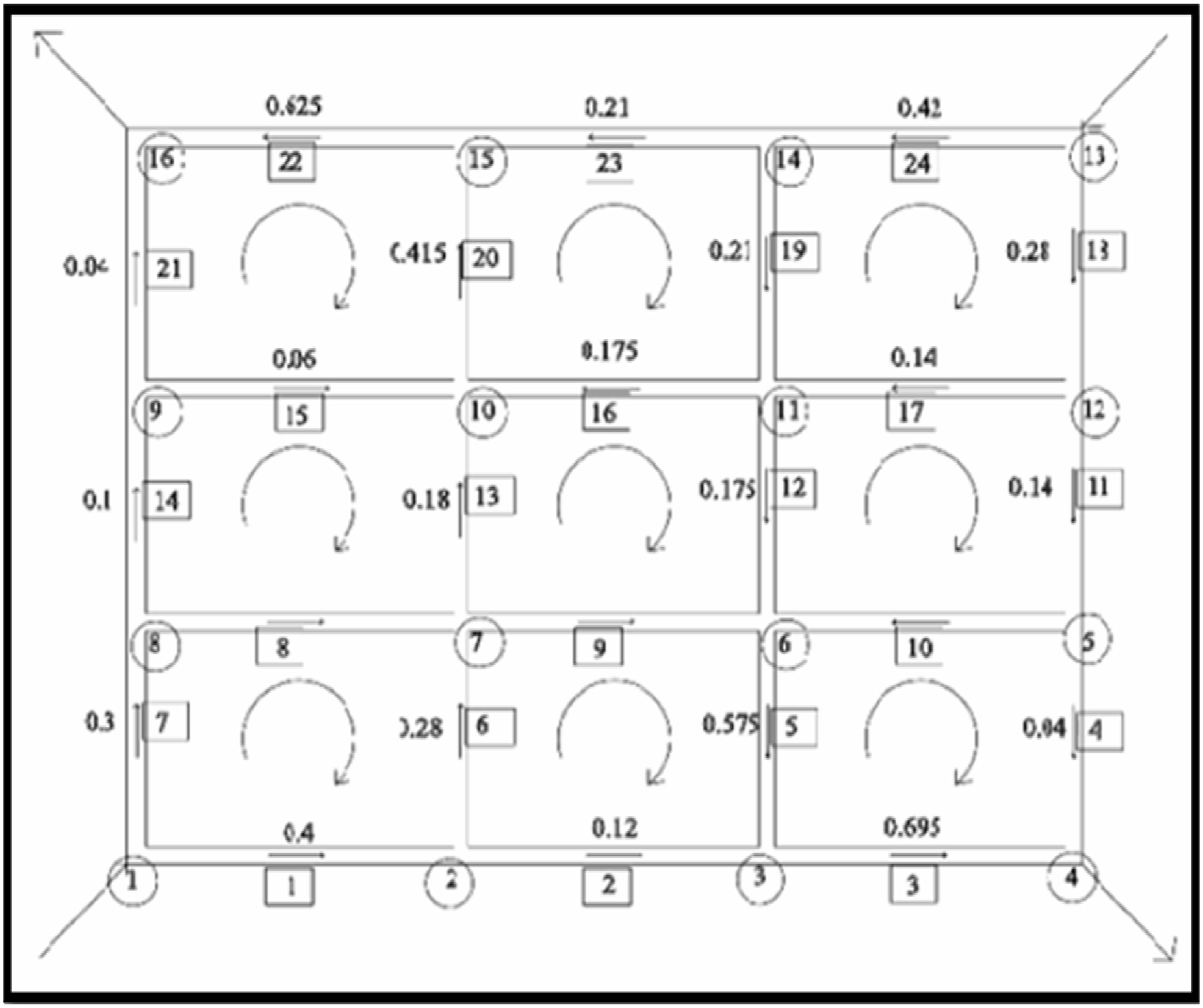


Figure 3: (Network Diagram With Pipe Length, Discharge, Nodes, And The Loops

The figure above show the network diagram of the study area, showing the nodes, the pipes, the loops, and the discharge in all the pipes involved. The network has sixteen fixed grade nodes *(NF=16)*, and twenty four pipes (NP=24) which are connected to form nine loop *(NL=9)* and nine path (or pseudo loop) *(NF-l=9)*. The geometric data for the network is shown in Table 1. In order to simplify the network/demands at the nodes, pumps and pressure reducing valves were not included. This enabled an exact solution for flows in all pipes and heads at the nodes to be determined analytically. Results from simultaneous loop flow techniques can then be obtained.

Table 1: (Table values)

Pipe	1	2	3	4	5	6	7	8
D	0.30	0.15	0.30	0.30	0.20	0.20	0.30	0.60
L	700	300	400	350	350	200	300	500
K	576	7,901	329.22	288.06	2,187.5	1,250	246.91	38.58
Q_0	0.40	0.12	0.695	0.04	0.575	0.28	0.30	0.20
hL	92.18	113.77	159.02	0.46	723.24	98.00	22.22	1.54
Nh/Q	460.9	1,896	457	23	2,512	700	148	15

Pipe	9	10	11	12	13	14	15	16
D	0.15	0.15	0.15	0.15	0.15	0.30	0.30	0.15
L	200	200	200	200	200	350	800	200
K	5,267.5	5,267.5	5,267.5	5,267.5	5,267.5	288.06	658.43	5,267.49
Q_0	0.10	0.14	0.14	0.175	0.18	0.10	0.06	0.175
hL	253.12	52.67	103.24	161.32	170.67	2.88	2.37	161.32
Nh/Q	1,687	1,053	1,476	1,844	1,896	58	79	1,855

Pipe	17	18	19	20	21	22	23	24
D	0.15	0.15	0.15	0.30	0.30	0.30	0.15	0.30
L	200	300	200	350	500	600	300	600
K	5,267.49	7,901.23	5,267.49	288.06	411.52	493.83	7,901.23	493.83
Q_0	0.14	0.28	0.21	0.415	0.04	0.625	0.21	0.42
hL	103.24	619.45	232.30	49.61	0.66	192.90	348.44	87.11
Nh/Q	737	4,425	2,212	239	33	617	3,318	415

Here, $h_L = KQ^2$ where K (Weisbach constant) = $\frac{8\,fL}{\pi^2 g D^5}$

f is Weisbach friction factor which is approximately 0.0242 and g is gravitational constant taken to be 9.81.

This set of linear simultaneous equation in Δ can be written in matrix form as follows:-

$$\begin{pmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \cdots\cdots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \cdots\cdots & \frac{\partial h_2}{\partial q_n} \\ \frac{\partial hn}{\partial q_1} & \frac{\partial hn}{\partial q_2} & \cdots\cdots & \frac{\partial q_n}{\partial q_n} \end{pmatrix} \begin{pmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_n \end{pmatrix} = \begin{pmatrix} -h_1 \\ -h_2 \\ -h_n \end{pmatrix}$$

LOOP 1

From the Network diagram in figure 3, the Head Loss in each loop can be determined thus;

$K_7Q^2_7 + K_8 Q_8^2 - K_6 Q_6^2 - K_1Q_1^2 = H_1 - H_1 = 0$

LOOP 2

$K_6 Q^2_6 + K_9 Q_9^2 + K_5 Q_5^2 - K_2 Q_2^2 = H_2 - H_2 = 0$

LOOP 3

$- K_5 Q^2_5 - K_{10} Q_{10}^2 + K_4 Q^2_4 - K_3 Q^2_3 = H_3 - H_3 = 0$

LOOP 4

$K_{10} Q_{10}^2 - K_{12} Q_{12}^2 - K_{17} Q_{17}^2 + K_{11} Q_{11} = H_5 - H_5 = 0$

LOOP 5

$-K_9 Q_9^2 + K_{13} Q_{13}^2 - K_{16} Q^2_{16} + K_{12} Q_{12}^2 = H_6 - H_6 = 0$

LOOP 6

$-K_8 Q_8^2 + K_{14} Q_{14}^2 + K_{15} Q_{15}^2 - K_{13} Q^2_{13} = H_7 - H_7 = 0$

$J\Delta Q=-H$

LOOP 7

$K_{21} Q_{21}^2 - K_{22} Q_{22}^2 - K_{20} Q_{20}^2 - K_{15} Q_{15}^2 = H_9 - H_9 = 0$

LOOP 8

$K_{20} Q_{20}^2 - K_{23} Q_{23}^2 + K_{19}^2 + K_{16} Q^2_{16} = H_{10} - H_{10} = 0$

LOOP 9

$-K_{19} Q_{19}^2 - K_{24}Q_{24}^2 + K_{18} Q_{18}^2 + K_{17} Q_{17}^2 = H_{11} - H_{11} = 0$

This gives the first computation for the head loss in loops.

22.22 + 1.54 – 98 – 92.18 = – 166.42
98 + 253.12 + 723.24 –113. 77 = 960.59
–723.24 – 52.67 + 0.46 – 159.02 = – 934.47
52.67 – 161.32 – 103.24 + 103.24 = – 108.65
–253.12 + 170.67 – 161.32 + 161.32 = – 82.45
–1.54 + 2.88 + 2.88 + 2.37 – 170.67 = –166.96
0.66 – 192.90 – 49.61 – 2.37 = – 244.22
49.61 – 348.44 + 232.30 + 161.32 = 94.79
–232.30 – 87.11 + 619.45 + 103.24 = 403.28

$$
\begin{pmatrix}
\frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \frac{\partial h_1}{\partial q_3} & \frac{\partial h_1}{\partial q_4} & \frac{\partial h_1}{\partial q_5} & \frac{\partial h_1}{\partial q_6} & \frac{\partial h_1}{\partial q_7} & \frac{\partial h_1}{\partial q_8} & \frac{\partial h_1}{\partial q_9} \\
\frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \frac{\partial h_2}{\partial q_3} & \frac{\partial h_2}{\partial q_4} & \frac{\partial h_2}{\partial q_5} & \frac{\partial h_2}{\partial q_6} & \frac{\partial h_2}{\partial q_7} & \frac{\partial h_2}{\partial q_8} & \frac{\partial h_1}{\partial q_9} \\
\frac{\partial h_3}{\partial q_1} & \frac{\partial h_3}{\partial q_2} & \frac{\partial h_3}{\partial q_3} & \frac{\partial h_3}{\partial q_4} & \frac{\partial h_3}{\partial q_5} & \frac{\partial h_3}{\partial q_6} & \frac{\partial h_3}{\partial q_7} & \frac{\partial h_3}{\partial q_8} & \frac{\partial h_3}{\partial q_9} \\
\frac{\partial h_4}{\partial q_1} & \frac{\partial h_4}{\partial q_2} & \frac{\partial h_4}{\partial q_3} & \frac{\partial h_4}{\partial q_4} & \frac{\partial h_4}{\partial q_5} & \frac{\partial h_4}{\partial q_6} & \frac{\partial h_4}{\partial q_7} & \frac{\partial h_4}{\partial q_8} & \frac{\partial h_4}{\partial q_9} \\
\frac{\partial h_5}{\partial q_1} & \frac{\partial h_5}{\partial q_2} & \frac{\partial h_5}{\partial q_3} & \frac{\partial h_5}{\partial q_4} & \frac{\partial h_5}{\partial q_5} & \frac{\partial h_5}{\partial q_6} & \frac{\partial h_5}{\partial q_7} & \frac{\partial h_5}{\partial q_8} & \frac{\partial h_5}{\partial q_9} \\
\frac{\partial h_6}{\partial q_1} & \frac{\partial h_6}{\partial q_2} & \frac{\partial h_6}{\partial q_3} & \frac{\partial h_6}{\partial q_4} & \frac{\partial h_6}{\partial q_5} & \frac{\partial h_6}{\partial q_6} & \frac{\partial h_6}{\partial q_7} & \frac{\partial h_6}{\partial q_8} & \frac{\partial h_6}{\partial q_9} \\
\frac{\partial h_7}{\partial q_1} & \frac{\partial h_7}{\partial q_2} & \frac{\partial h_7}{\partial q_3} & \frac{\partial h_7}{\partial q_4} & \frac{\partial h_7}{\partial q_5} & \frac{\partial h_7}{\partial q_6} & \frac{\partial h_7}{\partial q_7} & \frac{\partial h_7}{\partial q_8} & \frac{\partial h_7}{\partial q_9} \\
\frac{\partial h_8}{\partial q_1} & \frac{\partial h_8}{\partial q_2} & \frac{\partial h_8}{\partial q_3} & \frac{\partial h_8}{\partial q_4} & \frac{\partial h_8}{\partial q_5} & \frac{\partial h_8}{\partial q_6} & \frac{\partial h_8}{\partial q_7} & \frac{\partial h_8}{\partial q_8} & \frac{\partial h_8}{\partial q_9} \\
\frac{\partial h_9}{\partial q_1} & \frac{\partial h_9}{\partial q_2} & \frac{\partial h_9}{\partial q_3} & \frac{\partial h_9}{\partial q_4} & \frac{\partial h_9}{\partial q_5} & \frac{\partial h_9}{\partial q_6} & \frac{\partial h_9}{\partial q_7} & \frac{\partial h_9}{\partial q_8} & \frac{\partial h_9}{\partial q_9}
\end{pmatrix}
\begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix}
=
\begin{bmatrix} -h_1 \\ -h_2 \\ -h_3 \\ -h_4 \\ -h_5 \\ -h_6 \\ -h_7 \\ -h_8 \\ -h_9 \end{bmatrix}
$$

The first matrix formation that is drawn from the table above (table 1) is shown in matrix 1.

The matrix values

$$
J_1 =
\begin{pmatrix}
5{,}472 & -700 & 0 & 0 & 0 & -15 & 0 & 0 & 0 \\
-700 & 6{,}798 & -2{,}515 & 0 & -1{,}687 & 0 & 0 & 0 & 0 \\
0 & -2{,}515 & 4{,}048 & -1{,}052 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1{,}053 & 5{,}110 & -1{,}844 & 0 & 0 & 0 & -737 \\
0 & -1{,}687 & 0 & -1{,}844 & 7{,}271 & -1{,}896 & 0 & -1{,}844 & 0 \\
-15 & 0 & 0 & 0 & -1{,}896 & 2{,}048 & -79 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -79 & 968 & -239 & 0 \\
0 & 0 & 0 & 0 & -1{,}844 & 0 & -239 & 7{,}613 & -2{,}212 \\
0 & 0 & 0 & -737 & 0 & 0 & 0 & -2{,}212 & 7{,}789
\end{pmatrix}
\begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix}
=
\begin{bmatrix} 166.42 \\ -960.59 \\ 934.47 \\ 108.65 \\ 82.45 \\ 166.96 \\ 244.22 \\ -94.79 \\ -403.28 \end{bmatrix}
$$

$\Delta q_1 = 0.0259$, $\Delta q_2 = -0.0384$, $\Delta q_3 = 0.2290$, $\Delta q_4 = 0.0846$

$\Delta q_5 = 0.0624$ $\Delta q_6 = 0.1497$, $\Delta q_7 = 0.2640$, $\Delta q_8 = -0.0019$ $\Delta q_9 = -1.0443$

Matrix 1

First iteration tables – Table 2

Pipe	1	2	3	4	5	6	7	8
Q_0	0.3741	0.1584	0.466	0.269	0.3076	0.2157	0.3259	0.0762
h_L	80.63	198.25	71.49	20.84	206.98	58.16	26.22	0.224
nh/Q	431.06	2503.16	306.82	154.94	1,345.77	539.27	160.91	5.88

Pipe	9	10	11	12	13	14	15	16
Q_0	0.1992	–0.0444	0.2246	0.1528	0.0927	0.2597	–0.054	0.1107
h_L	111.60	–10.38	265.72	122.98	45.27	17.96	–1.94	64.55
nh/Q	1,120.48	467.57	2,366.16	1,609.69	976.70	143.85	71.45	1,166.21

Pipe	17	18	19	20	21	22	23	24
Q_0	0.0111	0.2357	0.2524	0.1491	0.3040	0.3610	0.2119	0.4643
h_L	0.649	438.95	335.57	6.40	38.03	64.36	354.78	106.46
nh/Q	116.94	3,724.65	2,659.03	85.85	250.20	356.57	3,348.56	458.58

Computation of $Q^{(i)}$ (from the 1st iteration)

$Q_1 = Q_1^{(0)} - \Delta QL_1 = 0.40 - 0258 = 0.3741$

$Q_2 = Q_2^{(0)} - \Delta QL_2 = 0.12 - (-0.0384) = 0.1584$

$Q_3 = Q_3^{(0)} - \Delta QL_3 = 0.695 - 0.2290 = 0.4660$

$Q_4 = Q_4^{(0)} - \Delta QL_3 = 0.04 + 0.2290 = 0.2690$

$Q_5 = Q_5^{(0)} - \Delta QL_2 - \Delta QL_3 = 0.575 - 0.0384 - 0.2290 = 0.3076$

$Q_6 = Q_6^{(0)} - \Delta QL_1 - \Delta QL_2 = 0.28 - 0.0259 - 0.0384 = 0.2157$

$Q_7 = Q_7^{(0)} + \Delta QL_1 = 0.3 + 0.0259 = 0.3259$

$Q_8 = Q_8^{(0)} + \Delta QL_1 - \Delta QL_6 = 0.2 + 0.0259 - 0.1497 = 0.0762$

$Q_9 = Q_9^{(0)} + \Delta QL_2 - \Delta QL_5 = 0.3 - 0.0384 - 0.0624 = 0.1992$

$Q_{10} = Q_{10}^{(0)} - \Delta QL_{10} + \Delta QL_4 = 0.10 - 0.2290 + 0.0846 = -0.0444$

$Q_{11} = Q_{11}^{(0)} + \Delta QL_3 + \Delta QL_4 = 0.14 + 0.0846 = 0.2246$

$Q_{12} = Q_{12}^{(0)} - \Delta QL_4 + \Delta QL_5 = 0.175 - 0.0846 + 0.0624 = 0.1528$

$Q_{13} = Q_{13}^{(0)} + \Delta QL_5 - \Delta QL_6 = 0.18 + 0.0624 - 0.1497 = 0.0927$

$Q_{14} = Q_{14}^{(0)} + \Delta QL_6 = 0.1 + 0.1497 = 0.2497$

$Q_{15} = Q_{15}^{(0)} + \Delta QL_6 - \Delta QL_7 = 0.06 + 0.1497 - 0.2640 = 0.0543$

$Q_{16} = Q_{16}^{(0)} - \Delta QL_5 - \Delta QL_8 = 0.175 - 0.0624 - 0.0019 = 0.1107$

$Q_{17} = Q_{17}^{(0)} - \Delta QL_4 + \Delta QL_9 = 0.14 - 0.0846 - 0.0443 = 0.0111$

$Q_{18} = Q_{18}^{(0)} + \Delta QL_9 = 0.28 - 0.0443 = 0.2357$

$Q_{19} = Q_{19}^{(0)} - \Delta QL_8 + \Delta QL_9 = 0.21 - 0.0019 + 0.0443 = 0.2524$

$Q_{20} = Q_{20}^{(0)} - \Delta QL_7 + \Delta QL_8 = 0.415 - 0.2640 - 0.0019 = 0.1491$

$Q_{21} = Q_{21}^{(0)} + \Delta QL_7 = 0.04 + 02640 = 0.3040$

$Q_{22} = Q_{22}^{(0)} - \Delta QL_7 = 0.625 - 0.\ 2640 = 0.3610$

$Q_{23} = Q_{23}^{(0)} - \Delta QL_8 = 0.21 + 0.0019 = 0.2119$

$Q_{24} = Q_{24}^{(0)} - \Delta QL_9 = 0.42 + 00.21 + 0.0443 = 0.4643$

The second computations for the head loss in loops.

$h_1 = 26.22 + 0.224 - 58.16 - 265.72 = -297.43$

$h_2 = 58.16 + 111.60 + 206.98 - 198.25 = 178.49$

$h_3 = -206.98 + 10.38 + 20.84 - 71.49 = -247.25$

$h_4 = -10.38 - 122.98 - 0.649 + 265.72 = 131.71$

$h_5 = -111.60 + 45.27 - 64.55 + 122.98 = -7.90$

$h_6 = -0.224 + 17.96 - 1.94 - 45.27 = -29.47$

$h_7 = 38.03 - 64.36 - 6.40 + 1.94 = -30.79$

$h_8 = 6.40 - 354.78 + 335.57 + 64.55 = 51.74$

$h_9 = -335.57 - 106.46 + 438.95 + 0.649 = -2.43$

The second matrix formation that is drawn from the first iteration table (table 2) is shown in matrix 2 below.

$$J_2 = \begin{pmatrix} 1{,}137.12 & -539.27 & 0 & 0 & -5.88 & 0 & 0 & 0 & 0 \\ -539.27 & 5{,}508.68 & -1345.77 & 0 & -1.120.48 & 0 & 0 & 0 & 0 \\ 0 & -1345.72 & 2{,}275.11 & -467.57 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -467.57 & 4{,}560.36 & -1{,}609.69 & 0 & 0 & 0 & -116.97 \\ 0 & -1{,}120.48 & 0 & -1{,}609.69 & 4{,}873.08 & -976.70 & 0 & -1{,}166.21 & 0 \\ -5.88 & 0 & 0 & 0 & -976.70 & 1{,}197.88 & -71.45 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -71.457 & 64.07 & -85.85 & 0 \\ 0 & 0 & 0 & 0 & -1{,}166.21 & 0 & -85.85 & 7{,}259.65 & -2{,}659.03 \\ 0 & 0 & 0 & -116.94 & 0 & 0 & 0 & -2{,}659.03 & 6{,}959.2 \end{pmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix} = \begin{bmatrix} 297.436 \\ -178.49 \\ 247.25 \\ -13.711 \\ 7.90 \\ 29.474 \\ 30.79 \\ -51.74 \\ 2.431 \end{bmatrix}$$

$\Delta q_1 = 0.2740$, $\Delta q_2 = 0.0258$; $\Delta q_3 = 0.1212$, $\Delta q_4 = -0.0134$

$\Delta q_5 = 0.0089$, $\Delta q_6 = 0.0358$ $\Delta q_7 = 0.0430$ $\Delta q_8 = -0.0060$ $\Delta q_9 = -0.0022.$

Matrix 2

2nd Iteration Tables – Table 3

Pipe	1	2	3	4	5	6	7	8
$Q^{(2)}$	0.1001	0.1326	0.3448	0.3902	0.2122	–0.032	0.5999	0.3144
hL	5.77	138.93	39.14	43.86	98.50	–1.32	88.86	3.81
Nhl/Q	115.28	2,095.48	227.03	225.61	928.37	81.23	296.25	24.24

Pipe	9	10	11	12	13	14	15	16
$Q^{(2)}$	0.2161	–0.1790	0.2112	0.1751	0.0658	0.2855	– 0.062	0.0958
hL	131,34	–168.78	234.96	161.50	22.81	23.48	–2.49	48.34
Nhl/Q	1,215.55	1,885.81	2,2251	1844.66	693.31	164.48	80.98	1,009.19

pipe	7	8	9	20	1	2	3	4
$(^2)$	.0223	.2335	.2486	0. 1001	.3470	.3180	.2179	.4665
L	.62	30.79	25.54	2. 89	9.55	9.94	75.15	07.47
				57				

hl/Q	34.98	,689.85	,618.99	.74	85.59	14.09	.443.32	60.75

Computation of Q $^{(2)}$ (from 2nd iteration)

$Q_1 = Q_1^{(1)} – \Delta QL_1 =$ 0.3741 – 02740 = 0.1001
$Q_2 = Q_2^{(1)} – \Delta QL_2 =$ 0.1584 –0.0258 = 0.1326
$Q_3 = Q_3^{(1)} – \Delta QL_3 =$ 0.4660 – 0.1212 = 0.3448
$Q_4 = Q_4^{(1)} – \Delta QL_3$ = 0.2690 + 0.1212 = 0.3902
$Q_5 = Q_5^{(1)} – \Delta QL_2 – \Delta QL_3$ = 0.3076 + 0.0258 – 0.1212 = 0.2122
$Q_6 = Q_6^{(1)} – \Delta QL_1 – \Delta QL_2$ = 0.2157 – 0.2740 + 0.0258 = –0.0325
$Q_7 = Q_7^{(1)} + \Delta QL_1$ = 0.3259 + 0.2740 = 0.5999
$Q_8 = Q_8^{(1)} + \Delta QL_1 – \Delta QL_6$ = 0.0762 + 0.2740 – 0.0358 = 0.3144
$Q_9 = Q_9^{(1)} + \Delta QL_2 – \Delta QL_5$ = 0.1992 + 0.0258 – 0.0089 = 0.2161
$Q_{10 = Q10(1)} – \Delta QL_3 {}_+ \Delta QL_4$ = 0.0444 – 0.1212 – 0.0134 = – 0.1790
$Q_{11} = Q_{11}^{(1)} + \Delta QL_4$ = 0.2246 – 0.0134 = 0.2112
$Q_{12} = Q_{12}^{(1)} – \Delta QL_4 {}_+ \Delta QL_5$ = 0.1528 + 0.0134 + 0.0089 = 0.1751
$Q_{13} = Q_{13}^{(1)} + \Delta QL_5 – \Delta QL_6$ = 0.0927 + 0.0089 – 0.0358 = 0.0658
$Q_{14} = Q_{14}^{(1)} + \Delta QL_6$ = 0.2497 + 0.0358 = 0.2855
$Q_{15} = Q_{15}^{(1)} + \Delta QL_6 – \Delta QL_7$ = –0.0543 + 0.0358 – 0.0430 = – 06150
$Q_{16} = Q_{16}^{(1)} – \Delta QL_5 – \Delta QL_8$ = 0.1107 – 0.0089 – 0.0060 = 0.0958
$Q_{17} = Q_{17}^{(1)} – \Delta QL_4 + \Delta QL_9$ = 0.0111 + 0.0134 – 0.0022 = 0.0223
$Q_{18} = Q_{18}^{(1)} + \Delta QL_9$ = 0.2357 – 0.0022 = 0.2335
$Q_{19} = Q_{19}^{(1)} – \Delta QL_8 + \Delta QL_9$ = 0.2524 – 0.0060 + 0.0022 = 0.2486
$Q_{20} = Q_{20}^{(1)} – \Delta QL_7 + \Delta QL_8$ = 0.1491 – 0.0430 – 0.0060 = 0.1001
$Q_{21} = Q_{21}^{(1)} + \Delta QL_7$ = 0.3040 + 0.0430 = 0.3470
$Q_{22} = Q_{22}^{(1)} – \Delta QL_7$ = 0.3610 – 0.0430 = 0.3180
$Q_{23} = Q_{23}^{(1)} – \Delta QL_8$ = 0.2119 + 0.0060 = 0.2179
$Q_{24} = Q_{24}^{(1)} – \Delta QL_9$ = 0.4643 + 0.0022 = 0.4665

The third computation of the head loss in each loop.

h_1 = 88.86 + 3. 81 + 1.3 – 5.77 = 88.22
h_2 = –1.32 + 131.34 + 98.50 – 138.93 = 89.59
h_3 = – 98.50 + 168.78 + 43.86 – 39.14 = 75.00
h_4 = – 168.78 – 161.50 – 2.62 + 234.96 = 97.94
h_5 = – 131.34 + 22.81 – 48.34 + 161.50 = 4.63
h_6 = – 3.81 + 23.48 – 2.49 – 22.81 = – 5.63
h_7 = 49.55 – 49.94 – 2.89 + 2.49 = – 0.08
h_8 = 2.89 – 375.15 + 325.54 + 48.34 = 1.62
h_9 = – 325.54 – 107.47 + 430.79 + 2.62 = 0.40

The third matrix formation that is drawn from the second iteration table (table 3) is shown in matrix 3

$$J_3= \begin{pmatrix} 517 & -81.23 & 0 & 0 & 0 & -24.24 & 0 & 0 & 0 \\ -81.23 & 4{,}320.63 & -928.37 & 0 & -1{,}215.55 & 0 & 0 & 0 & 0 \\ 0 & -928.37 & 3{,}266.02 & -1{,}885.81 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1{,}885.81 & 6{,}190.45 & -1{,}844.66 & 0 & 0 & 0 & -234.98 \\ 0 & -1{,}215.55 & 0 & -1{,}844.66 & 4{,}762.71 & -693.31 & 0 & -1{,}009.19 & 0 \\ -24.24 & 0 & 0 & 0 & -693.31 & 963.01 & -80.98 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -80.98 & 738.40 & -57.74 & 0 \\ 0 & 0 & 0 & 0 & -1{,}009.19 & 0 & -57.74 & 7{,}129.2 & -2{,}618.99 \\ 0 & 0 & 0 & -234.98 & 0 & 0 & 0 & -2{,}618.99 & 7{,}004.57 \end{pmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix} = \begin{bmatrix} -88.22 \\ -89.59 \\ -75.00 \\ 97.94 \\ -4.63 \\ 5.63 \\ 0.79 \\ -1.62 \\ -0.40 \end{bmatrix}$$

$q_1 = -0.1761$, $q_2 = -0.0330$, $q_3 = -0.0301$, $q_4 = 0.0040$,
$q_5 = -0.0089$, $q_6 = -0.0050$, $q_7 = 0.0004$, $q_8 - 0.0017$, $q_9 = -0.0006$

Matrix 3

3rd Iteration Tables – Table 4

Pipe	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$Q(^2)$	0.2762	0.1656	0.3749	0.3601	0.2093	0.1106	0.4238	0.1433
hL	43.95	216.68	46.27	37.35	95.83	15.29	44.35	0.79
Nhl/Q	318.25	2,616.91	246.84	207.44	915.72	276.49	209.30	11.03

Pipe	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$Q(^2)$	0.1920	–0.1449	0.2152	0.1622	0.0619	0.2805	– 0.0669	0.1030
hL	103.68	–110.60	243.94	138.58	20.18	22.66	–2.95	55.88
Nhl/Q	1,080.00	1,526.57	2,267.10	1,708.75	652.02	161.57	88.19	1,085.05

Pipe	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
Q_0	0.0177	0.2329	0.2475	0.0980	0.3474	0.3176	0.2196	0.4971
hL	1.65	428.58	322.67	2.77	49.67	49.81	381.03	122.03
nh/Q	186.44	3,680.38	2.60.43	56.53	285.95	313.66	3,470.22	490.97

Computation of Q3 (from 3rd iteration)

$Q_1 = Q_1(^2) - \Delta QL_1 = 0.1001 + 0.1761 = 0.2762$
$Q_2 = Q_2(^2) - \Delta QL_2 = 0.1326 + 0.0330 = 0.1656$
$Q_3 = Q_3(^2) - \Delta QL_3 = 0.3448 + 0.0301 = 0.3749$
$Q_4 = Q_4(^2) - \Delta QL_3 = 0.3902 - 0.0301 = 0.3601$
$Q_5 = Q_5(^2) - \Delta QL_2 - \Delta QL_3 = 0.2122 - 0.0330 + 0.0301 = 0.2093$
$Q_6 = Q_6(^2) - \Delta QL_1 - \Delta QL_2 = 0.0325 + 0.1761 - 0.0330 = 0.1106$

$Q_7 = Q_7(^2) + \Delta QL_1 = 0.5999 - 0.1761 = 0.4238$
$Q_8 = Q_8(^2) + \Delta QL_1 - \Delta QL_6 = 0.3144 - 0.1761 + 0.0050 = 0.1433$
$Q_9 = Q_9(^2) + \Delta QL_2 - \Delta QL_5 = 0.2161 - 0.0330 + 0.0089 = 0.1920$
$Q_{10} = Q_{10}(^2) - \Delta QL_3 + \Delta QL_4 = -0.1790 + 0.0301 + 00040 = -0.1449$
$Q_{11} = Q_{11}(^2) + \Delta QL_4 = 0.2112 + 0.0040 = 0.2152$

$Q_{12} = Q_{12}(^2) - \Delta QL_4 + \Delta QL_5 = 0.1751 - 0.0040 - 0.0089 = 0.1622$
$Q_{13} = Q_{13}(^2) + \Delta QL_5 - \Delta QL_6 = 0.0658 - 0.0089 + 0.0050 = 0.0619$
$Q_{14} = Q_{14}(^2) + \Delta QL_6 = 0.2855 - 0.0050 = 0.2805$
$Q_{15} = Q_{15}(^2) + \Delta QL_6 - \Delta QL_7 = - 0.0615 - 0.0050 - 0.0004 = - 0.0669$
$Q_{16} = Q_{16}(^2) - \Delta QL_5 - \Delta QL_8 = 0.0958 + 0.0089 - 0.0017 = 0.1030$
$Q_{17} = Q_{17}(^2) - \Delta QL_4 + \Delta QL_9 = 0.0223 - 0.0040 - 0.0006 = 0.0177$
$Q_{18} = Q_{18}(^2) + \Delta QL_9 = 02335 - 0.0006 = 0.2329$
$Q_{19} = Q_{19}(^2) - \Delta QL_8 + \Delta QL_9 = 0.2486 - 0.0017 + 0.0006 = 0.2475$
$Q_{20} = Q_{20}(^2) - \Delta QL_7 + \Delta QL_8 = 0.1001 - 0.0004 - 0.0017 = 0.0980$
$Q_{21} = Q_{21}(^2) + \Delta QL_7 = 0.3470 + 0.0004 = 0.3474$
$Q_{22} = Q_{22}(^2) - \Delta QL_7 = 0.3180 - 0.0004 = 0.3176$
$Q_{23} = Q_{23}(^2) - \Delta QL_8 = 0.2179 + 0.0017 = 0.2196$
$Q_{24} = Q_{24}(^2) - \Delta QL_9 = 0.4665 + 0.0006 = 0.4671$

The fourth head loss determination in each loop.

$h_1 = 44.35 + 0.79 - 15.29 - 43.95 = -14.1$
$h_2 = 15.29 + 103.68 + 95.83 - 216.68 = -1.88$
$h_3 = - 95.83 + 110.60 + 37.35 - 46.27 = 5.85$
$h_4 = -110.60 - 138.58 - 1.65 + 243.94 = -6.89$
$h_5 = -103.68 + 20.18 - 55.88 + 138.58 = -0.80$
$h_6 = - 0.79 + 22.6 - 2.95 - 20.18 = -1.26$
$h_7 = 49.67 - 49.81 - 2.77 + 2.95 = 0.04$
$h_8 = 2.77 - 281.03 + 322.67 + 55.88 = 0.29$
$h_9 = -322.67 - 122.03 + 428.58 + 1.65 = -14.47$

The fourth matrix formation that is drawn from the third iteration table (table 4) is shown in matrix 4

$$J_4 = \begin{pmatrix} 815.07 & -276.49 & 0 & 0 & 0 & -11.03 & 0 & 0 & 0 \\ -276.49 & 4{,}889.12 & -915.72 & 0 & -1{,}080 & 0 & 0 & 0 & 0 \\ 0 & -915.72 & 2{,}896.57 & -1{,}526.57 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1{,}526.57 & 5{,}688.86 & -1{,}708.75 & 0 & 0 & 0 & -186.44 \\ 0 & -1080 & 0 & -1{,}708.75 & 4{,}525.82 & -652.02 & 0 & -1{,}085.05 & 0 \\ -11.03 & 0 & 0 & 0 & -652.02 & 912.81 & -88.19 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -88.19 & 744.33 & -56.53 & 0 \\ 0 & 0 & 0 & 0 & -1{,}085.05 & 0 & -56.53 & 7{,}219.23 & -2{,}607.43 \\ 0 & 0 & 0 & -186.44 & 0 & 0 & 0 & -2{,}607.43 & 6{,}965.22 \end{pmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix} = \begin{bmatrix} 14.10 \\ 1.88 \\ -5.85 \\ 6.89 \\ 0.80 \\ 1.26 \\ -0.04 \\ -0.29 \\ 14.47 \end{bmatrix}$$

$q_1 = 0.0179$, $q_2 = 0.0017$, $q_3 = -0.0006$, $q_4 = 0.0017$, $q_5 = 0.0020$
$q_6 = 0.0030$, $q_7 = 0.0004$, $q_8 = 0.0012$, $q_9 = 0.0026$

Matrix 4

4th Iteration Tables – Table 5

Pipe	1	2	3	4	5	6	7	8
$Q(^4)$	0.2583	0.1639	0.3755	0.3595	0.2116	0.0944	0.4417	0.1582
hL	38.44	212.25	46.42	37.23	97.94	11.14	48.17	0.97
Nhl/Q	297.64	2,589,99	247.24	207.12	925.71	236.02	218.11	12.26

Pipe	9	10	11	12	13	14	15	16
$Q(^4)$	0.1917	–0.1426	0.2169	0.1625	0.0609	0.2835	–0.064	0.1022
hL	103.36	–107.11	247.81	139.09	19.54	23.15	–2.72	55.02
Nhl/Q	1,078.35	1,502.24	2,285.02	1,711.88	641.71	163.32	84.60	1,076.71

Pipe	17	18	19	20	21	22	23	24
$Q(^4)$	0.0186	0.2355	0.2461	0.0988	0.3478	0.3172	0.2184	0.4945
hL	1.82	438.20	319.03	2.81	49.78	49.69	376.88	120.76
Nhl/Q	195.70	372.44	2592.69	56.88	286.26	313.30	3,451.28	488.41

Computation of $Q(^4)$ (from the 4th iteration)

$Q_1 = Q_1(^3) – \Delta QL_1 = 0.2762 – 0.0179 = 0.2583$
$Q_2 = Q_2(^3) – \Delta QL_2 = 0.1656 – 0.0017 = 0.1639$
$Q_3 = Q_3(^3) – \Delta QL_3 = 0.3749 + 0.0006 = 0.3755$
$Q_4 = Q_4(^3) + \Delta QL_3 = 0.3601 – 0.0006 = 0.3595$
$Q_5 = Q_5(^3) – \Delta QL_2 – \Delta QL_3 = 0.2093 + 0.0017 + 0.0006 = 0.2116$
$Q_6 = Q_6(^3) – \Delta QL_1 – \Delta QL_2 = 0.11.06 – 0.0179 + 0.0017 = 0.0944$
$Q_7 = Q_7(^3) + \Delta QL_1 = 0.4238 + 0.0.0179 = 0.4417$
$Q_8 = Q_8(^3) + \Delta QL_1 – \Delta QL_6 = 0.1433 + 0.0179 – 0.0030 = 0.1582$

$Q_9 = Q_9((^3) +\Delta DQL_2 – \Delta QL_5 = 0.1920 + 0.0017 – 0.0020 = 0.1917$
$Q_{10} = Q_{10}(^3) – \Delta QL_3 + \Delta QL_4 = –0.1449 + 0.0006 + 0.0617 = –0.1426$
$Q_{11} = Q_{11}(^3)+ \Delta QL_4 = 0.2152 + 0.0017 = 0.2169$

$Q_{12} = Q_{12}(^2) – \Delta QL_4 + \Delta QL_5 = 0.1622 – 0.0017 + 0.0020 = 0.1625$

$Q_{13} = Q_{13}(^3) + \Delta QL_5 - \Delta QL_6 = 0.0619 + 0.0020 – 0.0030 = 0.0609$
$Q_{14} = Q_{14}(^3) + \Delta QL_6 = 0.2805 + 0.0030 = 0.2835$
$Q_{15} = Q_{15}(^3) + \Delta QL_6 – \Delta QL_7 = – 0.0669 + 0.0030 – 0.0004 = –0.0643$
$Q_{16} = Q_{16}(^3) – \Delta QL_5 – \Delta QL_8 = 0.1030 – 0.0020 +0.0012 = 0.1022$
$Q_{17} = Q_{17}(^3) – \Delta QL_4 + \Delta QL_9 = 0.0177 – 0.0017 + 0.0026 = 0.0186$
$Q_{18} = Q_{18}(^3) + \Delta QL_9 = 0.2329 + 0.0026 = 0.2355$
$Q_{19} = Q_{19}(^3) + \Delta QL_8 + \Delta QL_9 = 0.2475 + 0.0012 – 0.0026 = 0.2461$
$Q_{20} = Q_{20}(^3) – \Delta QL_7 + \Delta QL_8 = 0.0980 – 0.0004 + 0.0012 = 0.0988$
$Q_{21} = Q_{21}(^3) + \Delta QL_7 = 0.3474 + 0.0004 = 0.3478$
$Q_{22} = Q_{22}(^3) - \Delta QL_7 = 0.3176 – 0.0004 = 0.3172$
$Q_{23} = Q_{23}(^3) - \Delta QL_8 = 0.2196 – 0.0012 = 0.2184$
$Q_{24} = Q_{24}(^3) - \Delta QL_9 = 0.4971 – 0.0026 = 0.4945$

The fifth computation of the head loss in each loop.

$h_1 = 48.17 + 0.97 – 11.14 – 38.44 = – 0.44$
$h_2 = 11.14 + 103.36 + 97.94 – 212.25 = 0.19$
$h_3 = –97.94 – 107.11 + 37.23 – 46.42 = – 0.02$
$h_4 = –107.11 – 139.09 – 1.82 + 247.81 = – 0.21$
$h_5 = –103.36 + 19.54 – 55.02 + 139.09 = 0.25$
$h_6 = –97 + 23.15 – 2.72 – 19.54 = – 0.08$
$h_7 = 49.78 – 49.69 – 2.81 + 2.72 = 0.00$
$h_8 = 2.81 – 376.88 + 319.03 + 55.02 = – 0.02$
$h_9 = 319.03 – 120.76 + 438.20 + 1.82 = 0.23$

The fifth matrix formation that is drawn from the fourth iteration table (table 5) is shown in matrix 5

$$J_5 = \begin{pmatrix} 764.03 & -236.02 & 0 & 0 & 0 & -12.26 & 0 & 0 & 0 \\ -236.02 & 4{,}830.07 & -925.7 & 0 & -1078.35 & 0 & 0 & 0 & 0 \\ 0 & -925.71 & 2{,}882.31 & -1502.24 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1502.24 & 5{,}694.84 & -1{,}711.88 & 0 & 0 & 0 & -195.7 \\ 0 & -1{,}078.35 & 0 & -1711.88 & 4{,}508.65 & -641.71 & 0 & -1076.71 & 0 \\ -12.26 & 0 & 0 & 0 & -641.71 & 901.89 & -84.60 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -84.60 & 741.04 & -56.88 & 0 \\ 0 & 0 & 0 & 0 & -1076.71 & 0 & -56.88 & 7{,}178.08 & -2592.69 \\ 0 & 0 & 0 & -195.70 & 0 & 0 & 0 & -2592.69 & 6{,}998.24 \end{pmatrix} \begin{bmatrix} \Delta q_1 \\ \Delta q_2 \\ \Delta q_3 \\ \Delta q_4 \\ \Delta q_5 \\ \Delta q_6 \\ \Delta q_7 \\ \Delta q_8 \\ \Delta q_9 \end{bmatrix} = \begin{bmatrix} 0.44 \\ -0.19 \\ 0.02 \\ 0.21 \\ -0.25 \\ 0.08 \\ -0.00 \\ 0.02 \\ -0.23 \end{bmatrix}$$

$q_1 = 0.5711$ $q_2 = -0.0188$ $q_3 = 0.0143$ $q_4 = 0.0257$ $q_5 = -0.0452$ $q_6 = 0.0648$

$q_7 = 0.0060$ $q_8 = -0.0180$ $q_9 = -0.0388$

5th iteration table – Table 6

Pipe	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
$Q(^4)$	-0.3128	0.1827	0.3612	0.3738	0.1785	-0.4955	1.0128	0.6645
hL	-1.52	263.69	42.94	40.24	69.69	-306.85	253.23	17.03
Nhl/Q	9.72	2886.69	215.3	215.30	780.84	1238.6	500.06	51.26

Pipe	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**
$Q(^4)$	0.2249	-0.1312	0.2421	0.0916	-0.0491	0.3483	-0.0055	0.1294
hL	142.22	-90.66	3.090	44.19	-12.69	34.94	-1.99	88.19
Nhl/Q	1264.7	1382.01	25.53	564.45	516.90	200.63	723.64	1363.06

Pipe	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
$Q(^4)$	-0.0459	0.1967	0.2669	0.0868	0.3538	0.3112	0.2364	0.5333
hL	-11.10	305.66	375.17	2.17	51.50	47.82	441.49	140.43
Nhl/Q	483.66	3107.88	2811.32	50.00	291.12	307.33	3735.11	526.65

Computation of Q (^{5}) (from the 5th iteration)

$Q_1 = Q_1(^4) - \Delta QL_1 = 0.2583 - 0.5711 = -0.3128$

$Q_2 = Q_2(^4) - \Delta QL_2 = 0.1639 + 0.0188 = 0.1827$

$Q_3 = Q_3(^4) - \Delta QL_3 = 0.3755 - 0.0143 = 0.3612$

$Q_4 = Q_4(^4) + \Delta QL_3 = 0.3595 + 0.0143 = 0.3738$

$Q_5 = Q_5(^4) - \Delta QL_2 - \Delta QL_3 = 0.2116 - 0.0188 - 0.0143 = 0.1785$

$Q_6 = Q_6(^4) - \Delta QL_1 - \Delta QL_2 = 0.0944 - 0.5711 - 0.0188 = -0.4955$

$Q_7 = Q_7(^4) + \Delta QL_1 = 0.4417 + 0.0.5711 = 1.0128$

$Q_8 = Q_8(^4) + \Delta QL_1 - \Delta QL_6 = 0.1582 + 0.5711 - 0.0648 = 0.6645$

$Q_9 = Q_9((^4) + \Delta DQL_2 - \Delta QL_5 = 0.1917 - 0.0188 + 0.0452 = 0.2181$

$Q_{10} = Q_{10}(^4) - \Delta QL_3 + \Delta QL_4 = -0.1426 - 0.0143 + 0.0257 = -0.1312$

$Q_{11} = Q_{11}(^4) + \Delta QL_4 = 0.2169 + 0.0252 = 0.2421$

$Q_{12} = Q_{12}(^4) - \Delta QL_4 + \Delta QL_5 = 0.1625 - 0.0257 - 0.0452 = 0.0916$

$Q_{13} = Q_{13}(^4) + \Delta QL_5 - \Delta QL_6 = 0.0609 - 0.0452 - 0.0648 = -0.0491$

$Q_{14} = Q_{14}(^4) + \Delta QL_6 = 0.2835 + 0.0648 = 0.3483$

$Q_{15} = Q_{15}(^4) + \Delta QL_6 - \Delta QL_7 = -0.0643 + 0.0648 - 0.0060 = -0.0055$

$Q_{16} = Q_{16}(^4) - \Delta QL_5 - \Delta QL_8 = 0.1022 + 0.0452 - 0.0180 = 0.1294$

$Q_{17} = Q_{17}(^4) - \Delta QL_4 + \Delta QL_9 = 0.0186 - 0.0257 - 0.0388 = -0.0459$

$Q_{18} = Q_{18}(^4) + \Delta QL_9 = 0.2355 - 0.0388 = 0.1967$

$Q_{19} = Q_{19}(^4) + \Delta QL_8 + \Delta QL_9 = 0.2461 - 0.0180 + 0.0388 = 0.2669$

$Q_{20} = Q_{20}(^4) - \Delta QL_7 + \Delta QL_8 = 0.0988 - 0.0060 - 0.0180 = 0.0868$

$Q_{21} = Q_{21}(^4) + \Delta QL_7 = 0.3478 + 0.0060 = 0.3538$

$Q_{22} = Q_{22}(^4) - \Delta QL_7 = 0.3172 - 0.0060 = 0.3112$

$Q_{23} = Q_{23}(^4) - \Delta QL_8 = 0.2184 + 0.0180 = 0.2364$

$Q_{24} = Q_{24}(^4) - \Delta QL_9 = 0.4945 + 0.0388 = 0.5330$

Results and Discussion

The simultaneous loop flow correction analyzer was applied to the water pipeline network in Owerri municipal area, a great reduction of the head loss was achieved in the system. This is pictured in the graphs below.

Graph 1 compares the original head loss(H_o) with the corrected one (H_f), the original head loss (as calculated from the distribution network from the data collected) shows an irregular head loss across the wax (hops) from the graph, the blue line indicates the original while the red line indicates the corrected head loss. (as calculated after the fifth iteration). The result shows a drastic reduction in head loss which is approximately zero in all the loops involves and the line is regular along the axis (H_f).

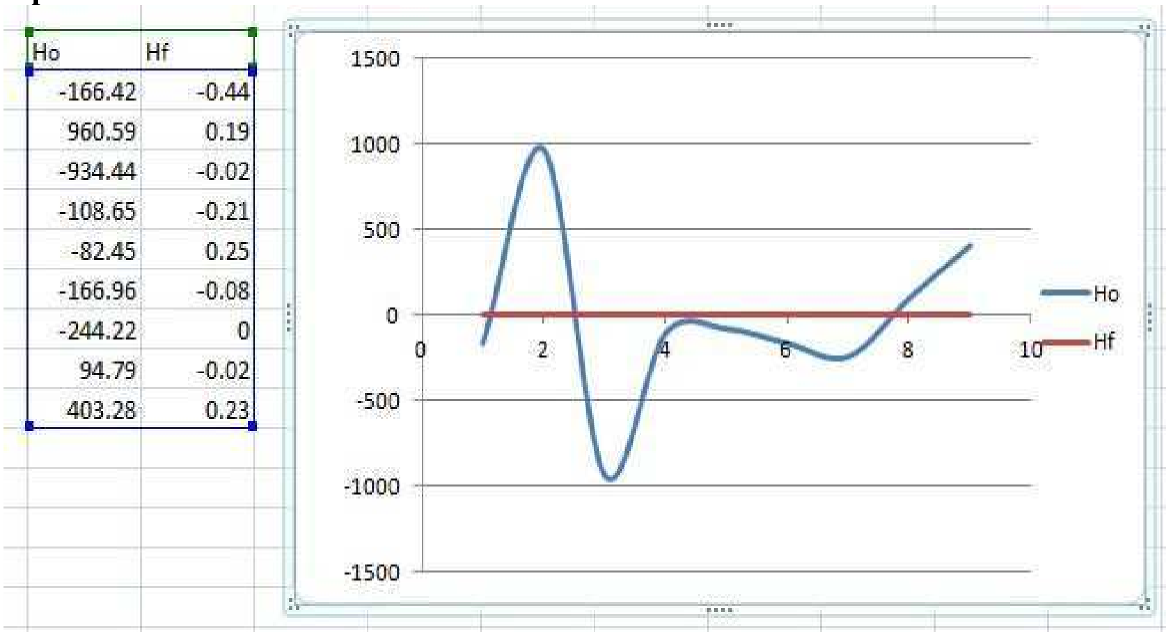
Graphs 2 and 3 compare the original and the corrected result using the discharge Q and the pipe diameter D. Practically, the rate of flow per minute is directly proportional to the pipe diameter (the longer the diameter, the greater the flow and *vise versa*). The original flow Q from the data collected shows proportionality with the pipe diameter which is fixed and this in contrary is an indication of bad network design that leads to losses as shown in graph 2. Furthermore, graph 3 pictures the corrected flow rate as it varies with the pipe diameter, thus, this graph indicates a direct proportionality of the flow rate, Q and the pipe diameter, D.

Graph 4 and 5 compares the pipe length and the head loss along the pipes. From definition, the loss is the energy per unit weight of fluid. In other words, it is the rate of the product of force and the length to the weight of fluid. This definition of head loss indicates that the head loss increases with increase in pipe length. Thus, Graph 4 pictures the original variation of the head loss with the pipe length. The graph lines counter each other, showing the level of non proportionality between the pipe length and head loss while the corrected result produced after the fifth iteration shows the level of proportionality between the two constraints since the line did not counter each other along the axis as shown in graph 5.

This method (simultaneous loop flow correction) produced an accurate result with a great reduction in the head losses in each loop after the fifth iteration.

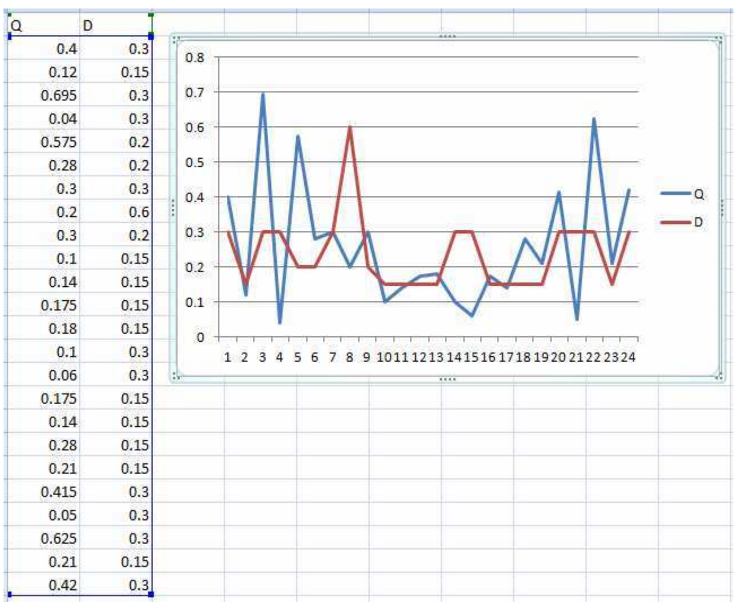
In addition to the above achievement, the rate at which the water flows (Q) was observed to be proportional to the pipe diameter as shown in the graph 5. Here, the initial flow rates were compared with the corrected flow rates with respect to the pipe diameters. With the initial discharge, which is not proportional to the diameter, the deformation in pipes will be high. Hence, the simultaneous flow correction technique adopted in this work in analyzing pipeline network of water reticulation system has proven to be the best numerical technique.

Graphs and Result Curves



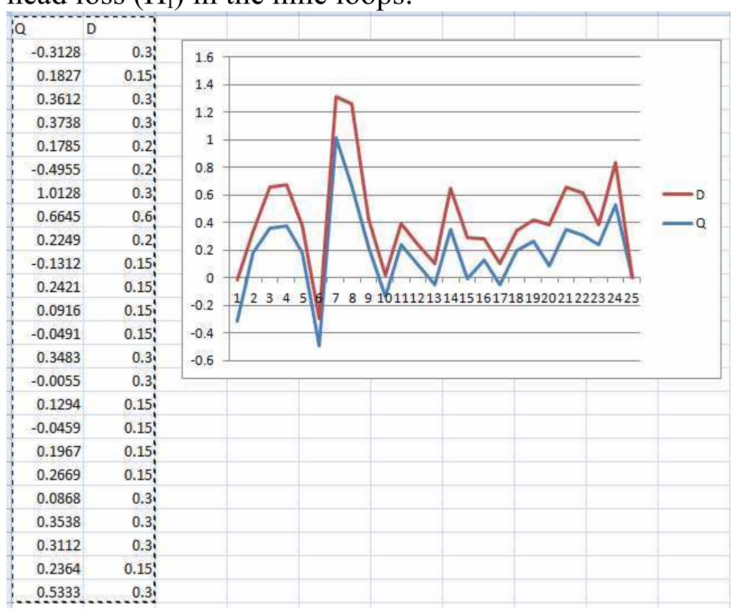
Ho	Hf
-166.42	-0.44
960.59	0.19
-934.44	-0.02
-108.65	-0.21
-82.45	0.25
-166.96	-0.08
-244.22	0
94.79	-0.02
403.28	0.23

Graph 1- The graph of initial (H_o) and final head loss (H_f) in the nine loops.



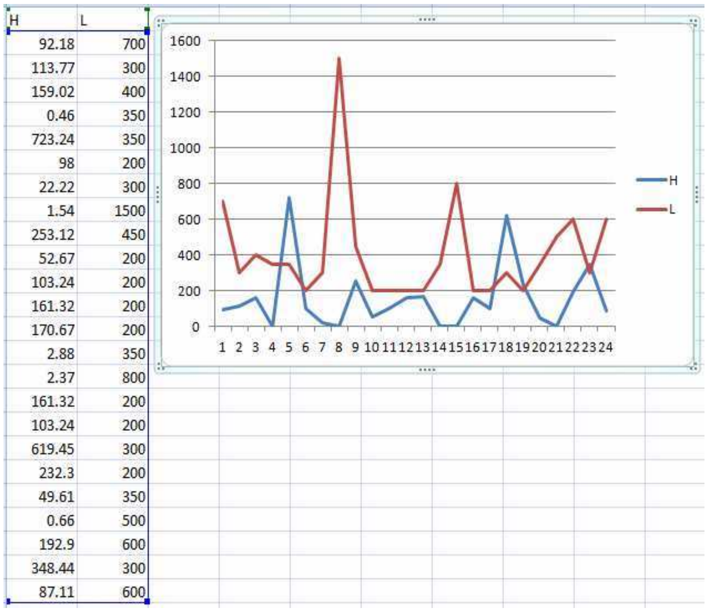
Q	D
0.4	0.3
0.12	0.15
0.695	0.3
0.04	0.3
0.575	0.2
0.28	0.2
0.3	0.3
0.2	0.6
0.3	0.2
0.1	0.15
0.14	0.15
0.175	0.15
0.18	0.15
0.1	0.3
0.06	0.3
0.175	0.15
0.14	0.15
0.28	0.15
0.21	0.15
0.415	0.3
0.05	0.3
0.625	0.3
0.21	0.15
0.42	0.3

Graph 2- The graph that shows the existing discharge, Q/the pipe diameter, D.



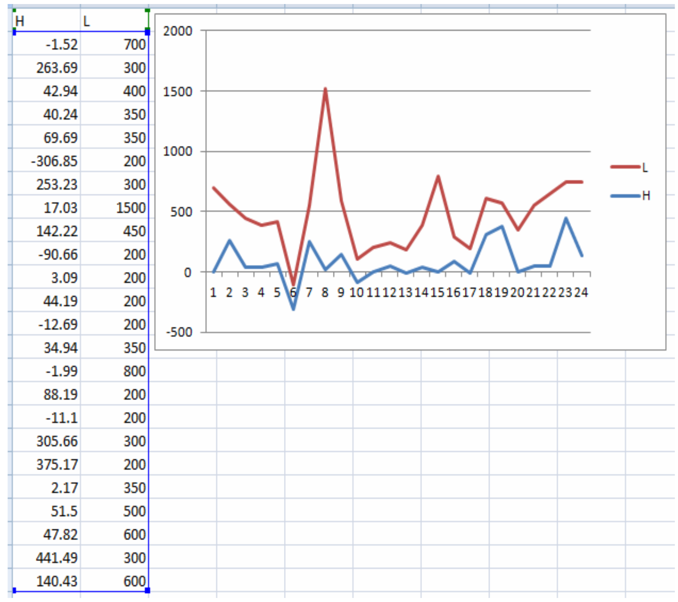
Q	D
-0.3128	0.3
0.1827	0.15
0.3612	0.3
0.3738	0.3
0.1785	0.2
-0.4955	0.2
1.0128	0.3
0.6645	0.6
0.2249	0.2
-0.1312	0.15
0.2421	0.15
0.0916	0.15
-0.0491	0.15
0.3483	0.3
-0.0055	0.3
0.1294	0.15
-0.0459	0.15
0.1967	0.15
0.2669	0.15
0.0868	0.3
0.3538	0.3
0.3112	0.3
0.2364	0.15
0.5333	0.3

Graph 3-The graph that shows the corrected discharge, Q/the pipe diameter, D.



H	L
92.18	700
113.77	300
159.02	400
0.46	350
723.24	350
98	200
22.22	300
1.54	1500
253.12	450
52.67	200
103.24	200
161.32	200
170.67	200
2.88	350
2.37	800
161.32	200
103.24	200
619.45	300
232.3	200
49.61	350
0.66	500
192.9	600
348.44	300
87.11	600

Graph 4- The graph that shows the initial head loss H_o/the pipe length, L.



H	L
-1.52	700
263.69	300
42.94	400
40.24	350
69.69	350
-306.85	200
253.23	300
17.03	1500
142.22	450
-90.66	200
3.09	200
44.19	200
-12.69	200
34.94	350
-1.99	800
88.19	200
-11.1	200
305.66	300
375.17	200
2.17	350
51.5	500
47.82	600
441.49	300
140.43	600

Graph 5- The graph that shows the corrected head loss, H_f/the pipe length, L.

Conclusion

The simultaneous loop flow correction method were applied to the network distribution that involves twenty four pipes and nine loops, which is one of the highest as regards to the number of pipes analyzed so far.

It has been observed that the existing system has a lot of fluctuations in its head loss. Also, the discharge (Q) is not proportional to the pipe diameter (D). With the application of simultaneous loop flow correction method, the result produced shows a great improvement as it has to do with head loss and the proportionality between the discharge (Q) and the pipe diameter (D) was highly appreciated in this paper.

Hence, the method is useful aid in designing network for maximum economy. It is believed that the findings from this work will be a great asset in planning, designing and operation of reticulated water pipeline network. The results obtained could also be applied in any part of the world since environmental and climatic conditions do not affect the operation of the waterworks. It is hoped that if the results are applied adequately, all Nigerians should have access to potable water.

Nomenclature

Q_0 = Initial assumed flow rate (m^3/s),
ΔQ = Corrective discharge (m^3/s),
D = Pipe diameter (m),
H_L or H = Head losses in pipe (m of fluid),
Q = Flow rate through pipe, (in and out of the node) (m^3/s),
C_F = Unite conversion factor (English = 4.66, SI = 10.29),
f = Darcy-Weisbach friction factor,
C = Hazen-Williams coefficient,
L = length of pipe,
n = Manning roughness coefficient,
i = Subscript indicating location,
j = Subscript indicating location,
ΔH = Head difference between reservoirs in a loop

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