ABSTRACT
Game-theoretic trade credit models are quite scarce, especially in relation to product promotion. This work examined a trade credit supply chain involving a manufacturer and two retailers in a decentralised supply chain in which the retailers engage in product promotion while the manufacturer financed them through credit provision. It considered a supply chain structure in which the manufacturer provides trade credit to the retailers and a situation in which he does not provide trade credit. It used Stackelberg game theory to determine the optimal promotion efforts, the credit periods and the players’ payoffs, and showed that while the manufacturer is better-off with the retailers’ efforts, the retailers need to consciously determine appropriate optimal effort to avoid getting short-changed. It also showed that while credit period reduces with the manufacturer’s margin, it increases with a retailer’s margin. It further showed that while the retailers’ payoffs reduce continuously with credit period, the manufacturer’s payoff is fixed in the long-run irrespective of the credit period provided by the manufacturer to the retailers. By comparison the players as well as the channel perform better with the adoption of trade credit, however a retailer must avoid placing his price margin at equality with that of the manufacturer if he hopes to enjoy long credit period.

KEYWORDS: Trade credit, Stackelberg game, Supply chain, Credit period

INTRODUCTION
Trade credit TC is usually employed as a supply chain financing strategy where goods are sold to the follower on credit with delayed payment (Cunat and Garcia-Appendini (2012)). It has been established that TC is a funding window for small scale businesses and those still at their tender stage Mach and Wolken (2006). These are related to cooperative advertising where the manufacturer or chain leader provides advertising support to the retailer by paying for a fraction of the cost of advertising the product (Ezimadu and Nwozo (2019), Ezimadu (2020)). Small scale businesses and startups are usually faced with the task of securing a loan with the task of financial resources, and it has been established that TC is usually a means of aiding them with funds (Berger and Udell (1998), Cunat (2007)). We note that outside bank financing, it is considered to be the most employed means of financing business in most countries (Lin and Chou, 2015; Dary and James, 2020). TC models can be categorised into those that are based on empirical data and those that purely are mathematical. Some of these employ simulations to arrive at useful predictions and conclusions. Wu et al. (2018) studied a channel with demand uncertainty involving a manufacturer, a capital-constrained weak retailer and a dominant retailer with favourably wholesale price. To ascertain credit period and the retailer’s cycle, Mahata et al. (2018) built an inventory model on TC involving payment default risk on a supply chain. Palacin-Sanchez et al. (2019) examined the relationship between TC from a supplier and bank credit by a joint determination of the effect of funds from these sources on businesses, and how a country’s institutional establishments affect these funding means. Considering the dependence of demand on price together with product deterioration Das et al. (2021) built a model that combine TC with the reliability of the product, leading to optimization problems. From data collected from listed companies and business enterprises, Machokoto et al. (2022) observed that there is reduced employment of TC in developed economies when compared with those that are still developing. Using some propositions, they found that a country’s institutional factors, the events that occurred within a given decade, and the level or extent to which the financial sector has developed can provide explanation on the reduction in the usage of trade credit. TC links a number of market concepts and variables in a supply chain, and game theory plays an integral role in studying these interactions Ezimadu (2019). This relationship is highlighted by Dary and James Jr (2020). They used TC related literature to study theories on contracts, applications and limitations, and noted that TC is related to game theory. According to them, TC is comparable with game setting with infinite repetitions in which continuous cooperation is required to have a Nash equilibrium. In a study of a manufacturer-buyers channel in which delayed payment is allowed by the manufacturer, Li, et al. (2013) considered a situation where the manufacturer insures his TC deals by transferring the risk of non-payment and secures capital through bank loans. They obtained a newsboy TC insurance model using a loss-averse and Stackelberg game theory. Wu and Zhao (2016) formulated two non-cooperative replenishment trade credit models with product demand risk and payment default risk. They considered Nash and Stackelberg equilibrium. Lei, et al. (2019) considered a supply channel in which a capital-constrained retailer meets his stochastic demand by placing order from a well-funded supplier. They developed a Stackelberg game in which the supplier is the channel leader, and based on dynamic inventory and the flow of capital, they obtained two-period dynamic business financing model. Considering defective items which can be sold at discounted price, Yadav et al. (2020) assumed a situation where demand is product promotion-sensitive. They considered a supplier-purchaser channel model and used a non-cooperative Stackelberg game theory to obtain their results. Zhou et al. (2022) investigated TC and early payment involving a manufacturer, a distributor who is constrained by capital and a retailer platform in which either the manufacturer or the
retail platform finances the distributor. Thus they considered a model in which either the platform or the manufacturer is the Stackelberg leader. In an examination of the effect of business working capital on a retailer’s borrowing decision, Hovelaque et al. (2022) used a non-cooperative game to model a relationship involving a retailer, a supplier and a bank. They considered a situation where either the supplier or the bank is the Stackelberg leader, while the retailer is the follower. In a news-vendor-based study on TC Wang et al. (2022) examined optimal provision of TC from supplier to retailer, and from retailer to end-user, and based on the assumption that credit is a function of demand they also considered order quantity. Using a Stackelberg game-theoretic approach they modelled the relationship between the players, and from their obtained optimal trade credit and order they noted that trade credit is an alternative to price contract.

This work examines a TC setting involving a manufacturer and two competing retailers. The paper employs game theory to consider a situation where the manufacturer who is the Stackelberg leader funds the retailers through TC, with the retailers engaging in promotion of the manufacturer’s product. The paper will X-rays the effect of promotion and credit period on the payoffs for a situation involving provision of TC and where there is no TC. It will also consider the effect of trade credit on promotion and determine an appropriate credit period in a regulated market with equal prince margins by the players.

THE MODEL

This paper considers a market duopoly involving a manufacturer who sells his product through two competing retailers. We assume that the retailers sell only the manufacturer’s brand amidst a product class. The manufacturer provides TC fund $C_i$, $i = \{1, 2\}$ to finance the retailers’ efforts, and allows a credit payment period $t_i$, $i = \{1, 2\}$. The retailers engage in the promotion of the manufacturer’s product using their promotion efforts $\theta_i$, $i = \{1, 2\}$. This is aimed at increasing the product demand by end-users. Thus, the retailers’ decision variables are their efforts $\theta_i$, while the manufacturer’s decision variables are his credits $C_i$ which is a function of his credit period $t_i$.

Promotion Function

We note that advertising is closely related to product promotion. Advertising is a long-term strategy, while promotion is a short-term strategy. Thus, we adopt the widely used advertising-demand function

$$f(\theta) = k\sqrt{\theta_i}$$

used by Xie and Wei (2009), Ezimadu (2019) where $\theta_i$ is the promotion effort and $k$ is the promotion effectiveness.

Clearly $f$ is both increasing and concave in $\theta_i$. Equation (1) is in consonance with the usually observed effect of saturation which is common in advertising which is also applicable to promotion. That is, every additional promotion spending will result in diminishing returns (Ezimadu and Nwozo, 2018; He et al., 2014).

Credit Function

We adopt the credit function

$$T_c = \frac{KSM\sqrt{FP}}{S_t}$$

from Ezimadu and Ezimadu (2022) where $S_m$, $F_p$, $K$ and $S_t$ are supplier’s price margin, promotion effort, stabilizing proportionality constant and credit period respectively. Thus, we have the credit function

$$C_i(M_i, \theta_i, t_i) = \frac{K_M M_i\sqrt{\theta_i}}{t_i}$$

where $K_M$ is stabilizing proportionality constant, $M_i$ = manufacturer’s price margin, $\theta_i$ = promotion effort, $t_i$ = credit period. Clearly, the credit given to a retailer increases with the margin $M_i$ received from him by the manufacturer, his promotion effort $\theta_i$, but reduced with the credit period.

The Game Decision Sequence

We consider this work as a leader-follower Stackelberg game in which the manufacturer who is the channel leader announces or rather informs the retailers of his allowable credit period $t_i$. In reaction the retailers decide on their individual the promotion efforts $\theta_i$. We will determine the Stackelberg equilibrium by backward induction as employed by He et al (2009), He et al. (2014) by first solving the retailers’ problems

$$\max_{\theta_i, t_i} \Pi_i = m_i \alpha \sqrt{\theta_i} - \theta_i + \frac{K_M M_i\sqrt{\theta_i}}{t_i}, \quad i \in \{1, 2\}. \quad (2)$$

In anticipation of the retailers’ reactions, the manufacturer factors their responses into his problem

$$\max_{t_i, t_i} \Pi_M = \sum_{i=1}^{2} \left[ m_i \alpha \sqrt{\theta_i} - \frac{K_M M_i\sqrt{\theta_i}}{t_i} \right], \quad i \in \{1, 2\}. \quad (3)$$

This paper will consider a scenario in which the manufacturer provides trade credit to any of the retailers, and a scenario in which he does not provide of trade credit to neither of them.

Credit Provision Scenario

In this section we consider a situation where the manufacturer provides trade credit to the retailers.

We note that the various terms in (2) are concave in $\theta_i$. Thus (2) is concave. Now, rearranging (2) we have

$$\Pi_i = \left( m_i \alpha + \frac{K_M M_i}{t_i} \right)\sqrt{\theta_i} - \theta_i. \quad (4)$$

By first order condition for concavity we have that

$$\frac{\partial \Pi_i}{\partial \theta_i} = \left[ m_i \alpha + \frac{K_M M_i}{t_i} \right] \left[ \frac{1}{2} \theta_i^{-\frac{1}{2}} \right] - 1 = 0$$

which implies that

$$\theta_i = \left( \frac{a m_i t_i + K_M M_i}{2t_i} \right)^2. \quad (5)$$
Using (5) in (3) we have
\[ \Pi_M = \sum_{i=1}^{2} \left[ aM_i - \frac{K_M M_i}{t_i} \right] \left[ \frac{am_i}{2} + \frac{K_M M_i}{2t_i} \right]. \] (6)

Again by the first order condition we have
\[ \frac{\partial \Pi_M}{\partial t_i} = \sum_{i=1}^{2} \left( \frac{am_i}{2} - \frac{K_M M_i}{t_i^2} \right) \left( \frac{am_i}{2} + \frac{K_M M_i}{2t_i} \right) - \frac{K_M M_i}{t_i}, \]
\[ + \frac{K_M M_i}{t_i} \left( \frac{am_i}{2} + \frac{K_M M_i}{2t_i} \right) = 0, \]

Implying
\[ t_i = \frac{2K_M M_i}{am_i + K_M M_i}. \] (7)

From (5) and (7) we have
\[ \theta_i = \left[ \frac{am_i + K_M M_i}{2} \left( \frac{2K_M M_i}{am_i(m_i - m_j)} \right)^{-1} \right]^2. \] (8)

From (4) and (5) we have
\[ \Pi_i = \left( \frac{am_i + K_M M_i}{t_i} \right) \left( \frac{am_i + K_M M_i}{2t_i} \right) - \left( \frac{am_i + K_M M_i}{2t_i} \right)^2, \]
\[ = \left( \frac{am_i + K_M M_i}{2} \right)^2 - \left( \frac{am_i + K_M M_i}{2t_i} \right)^2. \] (9)

From (7) and (9) we have
\[ \Pi_i = \left( \frac{am_i + K_M M_i}{2} \left( \frac{a(M_i - m_i)}{2K_M M_i} \right) \right)^2, \]
\[ = \left( \frac{a(M_i + m_i)}{4} \right)^2. \] (10)

From (6) and (7) we have
\[ \Pi_M = \sum_{i=1}^{2} \left[ aM_i - \frac{K_M M_i}{t_i} \right] \left[ \frac{am_i}{2} + \frac{K_M M_i}{2t_i} \right], \]
\[ = \frac{1}{8} \sum_{i=1}^{2} (a(M_i + m_i))^2. \]

Allowable Credit Period for Equal Price Margins
We consider a situation with equal price margins. That is \( M_i = m_i, \ i \in \{1, 2\}. \) Now, from (7) we have
\[ M_i = \frac{aM_i t_i}{a t_i - 2K_M}, \]
and
\[ m_i = \frac{(a t_i - 2K_M)M_i}{a t_i}. \]

Since the margins are equal for all the players, we have that
\[ \frac{am_i t_i}{a t_i - 2K_M} = \frac{(a t_i - 2K_M)M_i}{a t_i} \]
\[ \Rightarrow \frac{m_i}{M_i} = \alpha - \alpha^2 t_i^2 + 4K_M a t_i - 4K_M^2 = 0 \]
\[ \Rightarrow t_i = \frac{-4K_M \alpha \pm \sqrt{(4K_M \alpha)^2 - 4 \left( \frac{m_i}{M_i} \alpha - \alpha^2 \right) (4K_M^2)}}{2(m_i/M_i - \alpha^2)}. \] (11)

The equality of \( M_i \) and \( m_i \) and the fact that \( \alpha \in [0, 1] \) imply that \( \alpha > \alpha^2 \) so that (11) becomes
\[ t_i = \frac{-4K_M \alpha + \sqrt{(4K_M \alpha)^2 + 16K_M^2 H}}{2H}, \] (12)
\[ \text{or} \]
\[ t_{i*} = \frac{-4K_M \alpha - \sqrt{(4K_M \alpha)^2 + 16K_M^2 H}}{2H}, \] (13)

where \( H = \alpha - \alpha^2. \)

Obviously \( t_i > 0 \) while \( t_{i*} < 0. \) Since \( t_i < 0, \) it follows that (12) is an appropriate expression for \( t_i \) in a situation where the players are in a regulated market in which equal price is adopted by all parties in the channel coordinated by the manufacturer.

No Credit Scenario
Since no credit is given (2) and (3) can be expressed as
\[ \Pi_i = m_i \alpha \sqrt{\theta_i - \theta_i}, \ i \in \{1, 2\} \] (14)
and
\[ \Pi_M = \sum_{i=1}^{2} [m_i \alpha \sqrt{\theta_i}], \] (15)

respectively.

By the first order condition (14) becomes
\[ \frac{\partial \Pi_i}{\partial \theta_i} = \frac{1}{2} \left( \frac{am_i}{2} \right) - 1 = 0 \]

implying
\[ \theta_i = \left( \frac{am_i}{2} \right)^2. \] (16)

Since credit is inversely proportional to period, it follows that no-credit implies very large \( t_i. \)

Now, from (5) we have
\[ \theta_i = \left\lfloor \frac{am_i}{2} + \frac{K_M M_i}{2t_i} \right\rfloor ^2, \]
so that as \( t_i \to \infty \) we have that
\[ \theta_i = \left( \frac{am_i}{2} \right)^2 \]
which is the same as (16).

From (14) and (16) we have
\[ \Pi_i = m_i \alpha \left( \frac{am_i}{2} \right) - \left( \frac{am_i}{2} \right)^2 = \left( \frac{am_i}{2} \right)^2. \]
Similarly, from (15) and (16) we have
\[
\Pi_M = \sum_{i=1}^{2} \left[ \alpha m_i \alpha m_i \right] = \frac{\alpha^2}{2} \sum_{i=1}^{2} m_i^2. 
\]

RESULTS AND DISCUSSION
We recall that the players are involved in a Stackelberg game in which the manufacturer is the channel leader. Thus, he enjoys a first-mover’s advantage so that \( M_i > m_i, \quad i \in \{1, 2\}. \) As such we let \( M_1 = 4.0, \quad M_2 = 4.5, \quad m_1 = 2, \quad m_2 = 2.5. \) Further, since \( \alpha \) is a measure the promotion effectiveness, we have that \( \alpha \in [0,1]. \) As such we let \( \alpha = 0.2. \) Finally, we let \( K_M = 0.25. \) Let the subscript \( C = 0 \) represent no credit provision situation, and let \( C \neq 0 \) represent a supply chain setting with credit provision.

Effect of Promotion on the Manufacturer’s Payoff
Considering Figure 1 we observe that the manufacturer’s payoff depends on the individual retailer’s promotion efforts. He performs better with trade credit. Of course, this is not unexpected since credit is a kind of expenditure which eventually affects his payoff. On the other hand, Figure 2 shows that the individual retailer’s payoffs are better with provision of credit.

Effect of Credit Periods on the Retailers’ Payoffs
From Figure 3 we observe that the retailers’ payoffs reduce with increasing credit period. Recall that increase in credit period implies reduction in trade credit. Thus, the retailers’ payoffs reduce with reduction in credit. This is quite natural since his firm is financed through credit from the manufacturer. It is pertinent to note that the retailer that “generates” large margin to the manufacturer also enjoys a large payoff with increasing period. This is clear from Figure 4 where \( M_1(= 5) > M_2(= 4.5) \) even if \( m_1 < m_2. \) It is therefore necessary for a retailer to also factor the effect of price margin in bargain with the manufacturer.

Effect of Price Margins on Credit Period
Figure 5 shows that as a retailer’s price margin gets large, the credit period also gets large. Based on the law of price
and demand we note that the increase in price margin creates a situation where the retailer’s product may not be promptly sold-off. Thus, he will require more business time, hence the increase in credit period with increase in price margin. Further we observe that the manufacturer is in a fix between fixing high price margin and allowing long credit payment time. Being the supply chain leader, he can constrain the followers to pay their given credit by reducing the credit period with increasing margin. This can constrain the followers to perform their debt obligation promptly! In addition, from (7) and (12) we observe $t_1 = 5,000$, $t_2 = 5.625$, $t_+ = 0.773$ which shows that if a retailer places his price margin at equality with that of the manufacturer, the manufacturer would give virtually no credit, thus demanding his payment immediately.

**Effect of Credit Period on the Promotion Effort**

![Figure 6: An Illustration of the Effect of Long Credit Period on Promotion](https://dx.doi.org/10.4314/WOJAST.v14i1b.80)

As the credit period increases, the promotion effort reduces. This is quite natural! Since enough credit payment time is allowed, the retailer will be much relaxed and therefore would not need rigorous promotion or financing effort to sell the product.

**The Effect of Credit Period on Manufacturer’s Payoff**

![Figure 7: An Illustration of the Long Run Effect of Credit Period on the Manufacturer’s Payoff](https://dx.doi.org/10.4314/WOJAST.v14i1b.80)

Figure 7 shows that the manufacturer’s payoff increases very rapidly with credit period, attains a maximum, and then exhibits a decline which eventually stabilizes. Thus, he should determine his optimal credit period, and not just adopt unconditional elongation of credit period which is detrimental to him. This is because it will lead to prolongation of credit payment time which can result in getting his financing stagnated or even lead to bad debt. Clearly when the manufacturer generates a larger margin through a retailer, his (the manufacturer’s) payoff is lower with credit period. In essence, a comparison of the manufacturer’s payoff for a situation where he gets a large margin from a retailer and that of a situation where he gets a lower margin from another retailer shows that he is better-off in the long-run with a larger margin scenario.

**Table 1: Comparison of the Payoffs for both Scenarios**

<table>
<thead>
<tr>
<th>Game Scenarios</th>
<th>Payoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Pi_1$</td>
</tr>
<tr>
<td>$C_i \neq 0$</td>
<td>0.0900</td>
</tr>
<tr>
<td>$C_i = 0$</td>
<td>0.0400</td>
</tr>
</tbody>
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From Table 1 we observe that the retailers’ payoffs are better with credit assistance from the manufacturer. Similarly, the manufacturer’s payoff is also better with credit provision to the retailers. That is his support for the retailers is more beneficial to him than not providing support. Further, it is also clear that the channel performs better with credit provision.

**Conclusion**

This work considered a game-theoretic trade credit model on a decentralised supply chain involving a manufacturer and two retailers. The retailers engage in promoting the product, while the manufacturer supports them by providing them with trade credit. Considering two supply chain structures – provision of trade credit and non-provision – the work used Stackelberg game theory to determine the optimal promotion effort, credit periods and payoffs. The work observes that the promotion effort reduces with prolongation of the credit period. In addition, it shows that the retailer that generates a larger margin for the manufacturer also enjoys larger payoff as the credit period increases. Further, while the payoffs stabilize with prolongation of the credit periods, those of the retailers reduce more rapidly than that of the manufacturer. Further we observe that if a retailer sets his price margin at equality with that of the manufacturer, he receives virtually no credit from the manufacturer. Trade credit implementation is a win-win situation for all the players. This work studied a Stackelberg game in which the manufacturer is the supply chain leader. An extension can consider a Nash game in which neither of the players is the supply chain leader. Another extension or modification can incorporate a distributor who will play the role of a middleman between the manufacturer and the retailers as was achieved by Ezimadu (2016). This may be a more realistic situation since most manufacturers do not deal directly with their retailers.

**References**

