https://dx.doi.org/10.4314/WOJAST.v15i1.48

ELLIPTIC INTERVAL TYPE-2 INTUITIONISTIC FUZZY LOGIC SYSTEM FOR NON-LINEAR SYSTEM IDENTIFICATION

ISSN: 2141 – 3290
www.wojast.com

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ABSTRACT

An elliptic membership function has been proposed in the literature for interval type-2 fuzzy logic system. In this paper, elliptic non- membership function is incorporated into the conventional elliptic membership function model to obtain elliptic interval type-2 intuitionistic fuzzy sets for the first time. The elliptic interval type-2 intuitionistic fuzzy logic system so formed is applied for the prediction of two benchmark non-linear systems and results compared with Gaussian interval type-2 intuitionistic fuzzy logic system. Experimental results show that the elliptic interval type-2 intuitionistic fuzzy logic system outperforms the traditional Gaussian interval type-2 intuitionistic fuzzy logic system in the problem instances investigated.

KEYWORD: Interval type-2 intuitionistic fussy sets, elliptic membership function, gradient descent, optimization, system identification

INTRODUCTION

Fuzzy set (FS) of type-1 introduced by Zadeh (1965) has been used extensively for modelling uncertainty in applications until the introduction of type-2 FS in Zadeh (1975), which is an extension of the type-1 FS. Since then, there has been an upsurge in the use of type-2 FS to solve many real world problems. According to Mendel (2007), FS of type-1 has been known to be scientifically incorrect to model uncertainty because the membership function (MF) is precise once it has been obtained. Type-2 FSs are found to outperform type-1 FSs in many applications because type-2 FSs have MFs that are both type-1 FSs (fuzzy). A technology that has boosted and paved the way for effective utilization of fuzzy logic systems is the marriage of fuzzy logic theory and artificial neural network. These combinations have been adopted by researchers to solve many real world problems in diverse domains and magnitude. Type-2 FSs with fuzzy MFs are known to possess additional design degrees of freedom, which give it an edge over the type-1 FSs with the possibility to directly model uncertainties better (Mendel and John, 2002).

Although the classical FSs (T1 and T2) have made great impacts in the modelling of uncertainty in many applications, they however, still use only MFs to define the two states of a concept namely; degree of membership (DoM) and degree of non membership (DoNM), defined by non-MF (NMF). In the case of classical FS, the NMF is complementary to MF, meaning that MF carries a dual representation and this may not be applicable in most real world situations. For certain problem cases, there may be some form of hesitation in trying to assign the MF or NMF values. Hence the need to separately specify NMF of an element such that $0 \leq MF + NMF \leq 1$. Thus, Atanassov (1986) generalized the classical FS and presented intuitionistic fuzzy set (IFS). The IFS has additional component called NMF which is defined separately from MF. Atanassov also defined a hesitation index such that MF and NMF are not always complementary, thus enabling hesitation. According to Atanassov (1986), one of the unique things about IFS is the presence of the hesitation degree. The only condition of IFS is that the sum of MF and NMF is in the range of 0 and 1, $\forall x \in X$. The IFS-based models are often more appropriate in many cases where human opinions are elicited because IFS enables and captures hesitation occasioned by vague perception in human language representation. However, MFs and NMFs of IFS are precise; they may struggle with higher levels of uncertainty. In Atanassov and Gargov (1989), IFS is extended to interval valued intuitionistic fuzzy set (IVIFS) whose MFs and NMFs are fuzzy, thus capturing abundance of information and more fuzziness. For IVIFS, the condition is that the sum of the upper MF and upper NMF must lie between 0 and 1. In Eyoh *et al.*, (2016), a new type of intuitionistic FS called interval type-2 intuitionistic FS (IT2IFS) is introduced where, unlike the IVIFS, the sum of the lower MFs and upper NMFs must not be greater than 1 and the sum of the upper MFs and lower NMFs must not be greater than 1. These present IT2IFS as a different architecture from IVIFS. The interval type-2 intuitionistic fuzzy logic systems (IT2IFLSs) have found applications in many problem domains such as time series (Eyoh *et al.*, 2017, Luo *et al.*, 2019), regression problems (Eyoh *et al.*, 2018a, Yuan and Chao, 2019) and identification and prediction problems (Eyoh *et al.*, 2018b).

An FS (Type-1 or Type-2) can be defined using different forms of MFs. These MFs are regarded as the basic building blocks of FLS and the shapes play important role in the overall performance of FLSs. A MF of type-2 is defined by the shape of the footprint of uncertainties (FOUs) formed by two type-1 MFs such as triangular, trapezoidal, Gaussian, bell-shaped, sigmoid, pi, z-shaped, etc. According to Kayacan *et al.,* (2017), these listed MFs have some parameters that are coupled, especially those that are for the support and width of the MFs. In Khanesar *et al.*, (2011a), a new type of MF called the elliptic MF is introduced for interval type-2 fuzzy sets (IT2FSs). With the elliptic MF, the parameters that define the width of uncertainty are decoupled from the parameters that specify the center and support of the MFs of IT2FSs. This feature presents the elliptic MF as a unique MF amongst existing type-2 fuzzy MFs (Kayacan *et al.*, 2017).

Elliptic type-2 MF is derived from triangular MF and has been defined for interval type-2 fuzzy logic system (IT2FLS). As earlier stated, defining elliptic MF involves decoupling the uncertainty parameters from those that defined the width and center of the elliptic MF. According to Kayacan *et al.*, (2017), uncertainty representation using elliptic MF is closer to the human way of thinking when compared to Gaussian or triangular MFs. This work therefore adopts elliptic MFs and NMFs, otherwise known as elliptic IT2IFSs in the prediction of some non-linear problems and compares its performance with those of Gaussian IT2IFSs. For elliptic IT2IFSs, the degrees of uncertainty are defined by varying parameters a_1 and a_2 while those of the Gaussian are defined by varying the standard deviations. The Gaussian MF being a widely adopted MF for optimization problems is adopted for comparison.

The elliptic MF of IT2FLS was first proposed in Khanesar *et al.*, (2011a). The authors applied the new MF for the analysis of noise reduction property of IT2FLS. The authors noted that elliptic IT2FLS has better noise reduction property when compared to the type-1 FLS. Other type-2 FLSs have been defined using elliptic MFs with applications in several domains. For instance, Kayacan and Maslim (2016), employed elliptic type-2 fuzzy neural network for trajectory tracking control of quadrotor vertical take-off and landing unmanned aerial vehicles. The authors compared the performance of the type-2 elliptic MF with its type-1 and found to be better in terms of accuracy of prediction. Acikgoz and Sekkeli (2019) adopted a type-2 neuro-fuzzy controller defined using elliptic MFs to model solid state transformers. The designed system was compared with its type-1 counterpart and shown to perform better in terms of accelerating the dynamic response of the solid state transformer structure as well as enhancing its durability. Kayacan *et al.*, (2017) applied IT2FS defined with elliptic MF for time series problems with acceptable performance. In Kayacan *et al.*, (2018), the authors employed elliptic IT2FLS for the prediction of oil prices and for 3D trajectory tracking problem of a quadrotor. The authors concluded that the elliptic IT2FLS provided comparable results with triangular and Gaussian IT2FLSs. In Khanesar *et al.*, (2011b), the elliptic IT2FLS trained with Levenberg marquardt algorithm is adopted for the prediction of Mackey Glass time series with acceptable accuracy. Khanesar *et al.*, (2015) proposed an IT2FLS that benefits from elliptic MFs to investigate the trajectory tracking problem of a magnetic rigid spacecraft. Results revealed that the elliptic IT2FLS provided better results in terms of a smaller steady state error and a faster transient response of the system

BASIC DEFINITIONS

Definition 1: Given a finite universe of discourse X, an IFS A^* is a set with MFs μ and non MFs ν with some degree of hesitation π on the set definitions given as:

$$A^* = (< x, \mu_{A^*}(x), \nu_{A^*}(x) > | x \in X) : X \to [0, 1] \qquad (1)$$

Definition 2: A MF denoted as μ is the degree to which an element, x
belongs to a set, A^*, written as $\mu_{A^*}(x) : X \to [0, 1]$

Definition 3: A non-MF ν of an IFS A^* is defined as the degree to which an element does not belong to a set written as $\nu_{A^*}(x) : X \to [0, 1]$

Definition 4: An hesitation index, π of an IFS is the degree of neutrality of an element
x to a set A written as: $\pi_{A^*}(x) = 1 - (\mu_{A^*}(x) + \nu_{A^*}(x))$ such that $X \to [0, 1]$. Obviously
$0 \le \pi_{A^*}(x) \le 1$

Definition 5: An interval type-2 intuitionistic fuzzy set (IT2IFS), $\tilde{A}^*$ on the universe of discourse X is a set with type-2 membership and type-2 non-MFs given as:

$$\tilde{A}^* = (< x, \tilde{\mu}_{\tilde{A}^*}(x), \tilde{\nu}_{\tilde{A}^*}(x) > | x \in X) \qquad (2)$$

Any system that adopts one or more IT2IFS in the antecedent or consequent parts of the rule base is known as interval type-2 intuitionistic fuzzy logic system (IT2IFLS). The IT2IFLS consists of the four modules namely: the intuitionistic fuzzifier, intuitionistic rule base, intuitionistic fuzzy inference engine and intuitionistic output processing. Figure 1 shows the overall structure of IT2IFLS.

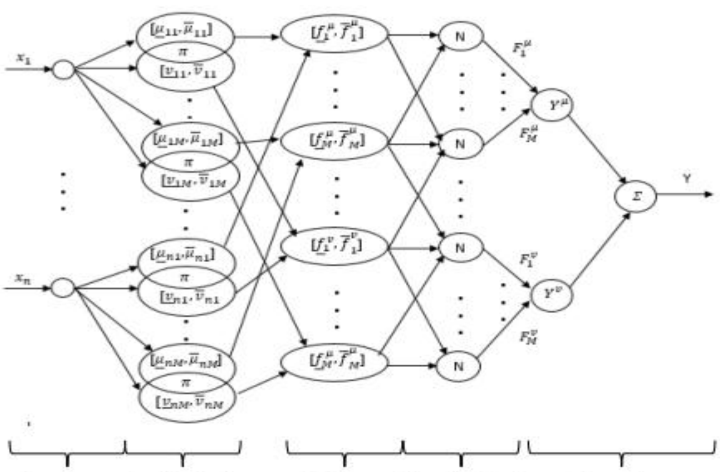

Figure 1: Structure of IT2IFLS (Eyoh *et al*., 2022)

Layer 1: The first layer of the IT2IFLS structure is the input layer that passes input values to the fuzzification layer.

Layer 2: The second layer is the fuzzification layer which translates the inputs values into IT2IFSs. The outputs from this layer are the degrees of membership and non-membership. The intersection points form the hesitation indices.

Layer 3: The third layer is the rule layer of the IT2IFLS which computes the firing strengths of the IF…THEN rules. In layer 4, the firing strengths calculated in Layer 3 are normalized.

The last layer (Layer 5) is the output layer of IT2IFLS. The final output is calculated as the sum of the MF output and NMF output as in Eqn. (3).

$$y = \frac{(1-\beta)\sum_{k=1}^{M}(\underline{f}_k^{\mu}+\overline{f}_k^{\mu})y_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu}+\sum_{k=1}^{M}\overline{f}_k^{\mu}} + \frac{\beta\sum_{k=1}^{M}(\underline{f}_k^{\nu}+\overline{f}_k^{\nu})y_k^{\nu}}{\sum_{k=1}^{M}\underline{f}_k^{\nu}+\sum_{k=1}^{M}\overline{f}_k^{\nu}} \quad (3)$$

k = (1, 2, …, M) are the number of IF…THEN rules.

The output is a weighted average of each IF-THEN rule's output and as such does not require any defuzzification procedure. $\underline{f}_k^{\mu}$, $\overline{f}_k^{\mu}$, $\underline{f}_k^{\nu}$, $\overline{f}_k^{\nu}$, are the lower membership, upper membership, lower non-membership and upper non-membership firing strengths respectively. β is the user defined parameter that determines the contribution of MF and NMF to the final outputs while y_k^{μ} and y_k^{ν} are the consequent outputs for MFs and NMFs respectively.

Elliptic Interval Type-2 Intuitionistic Fuzzy Set

The elliptic MF so far has been defined for IT2FLSs with only MFs. For interval type-2 intuitionistic fuzzy sets, two elliptic fuzzy functions are defined: one for MF and the other for non-MF. For the MF: the upper and the lower MFs are specified.

The elliptic upper MF is defined as follows:

$$\overline{\mu} = \left(1 - \left|\frac{x-c}{d}\right|^{a_1}\right)^{\frac{1}{a_1}}(1-\pi) \quad (4)$$

The elliptic lower MF is defined as follows:

$$\underline{\mu} = \left(1 - \left|\frac{x-c}{d}\right|^{a_2}\right)^{\frac{1}{a_2}}(1-\pi) \quad (5)$$

Applying the same principles for Gaussian NMF, the elliptic NMF is obtained as follows:

The elliptic upper NMF is defined as follows:

$$\overline{\nu} = (1-\pi) - \left(1 - \left|\frac{x-c}{d}\right|^{a_2}\right)^{\frac{1}{a_2}} \quad (6)$$

The elliptic lower MF is defined as follows:

$$\underline{\nu} = (1-\pi) - \left(1 - \left|\frac{x-c}{d}\right|^{a_1}\right)^{\frac{1}{a_1}} \quad (7)$$

where c and d are the center and the width of the MF and NMF respectively. The parameters a_1 and a_2 determine the width of the uncertainty of the proposed MF and NMF and should be selected such that $a_1 > 1$ and $0 < a_2 < 1$. Figure 2 shows intuitionistic elliptic MF and NMFs when parameter $a_1 = 1.2$ and $a_2 = 0.9$ while Figure 3 shows intuitionistic elliptic MF and NMFs when parameter $a_1 = 1.6$ and $a_2 = 0.6$.

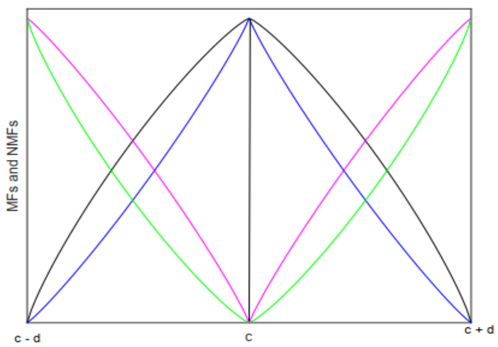

Figure 2: Elliptic IT2IFS when $a_1 = 1.2$ and $a_2 = 0.9$

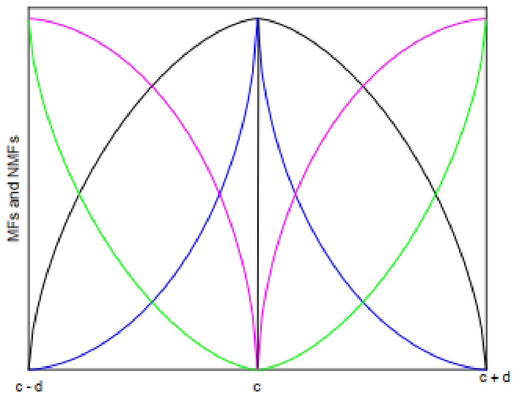

Figure 3: Elliptic IT2IFS when $a_1 = 1.6$ and $a_2 = 0.6$

When a_1 and a_2 are equal to 1, a triangular intuitionistic MF and NMF is formed as shown in Figure 4.

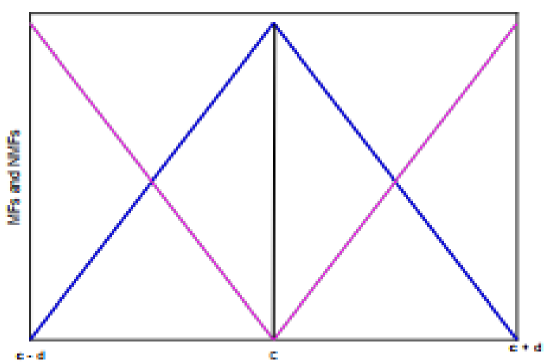

Figure 4: IFS formed when $a_1 = 1$ and $a_2 = 1$

https://dx.doi.org/10.4314/WOJAST.v15i1.48

COMPUTATIONAL APPROACH

Elliptic IT2IFLS Parameter Update using Gradient Descent

In this section, the rules for the learning of IT2IFLS parameters using gradient descent (GD) approach are presented. GD-based approaches have been widely adopted in the literature for the update of the parameters of FLSs. The problems under investigation are optimization (minimization) problems and the purpose is to minimize the cost function. For a single output of IT2IFLS, the cost function is computed as:

$$E = \frac{1}{2}(y^a - y)^2 \tag{8}$$

where y^a is the measured output and y is the output generated by the IT2IFLS model. The update rules based on GD are represented as in (9) to (14):

$$w_{ik}(t+1) = w_{ik}(t) - \gamma \frac{\partial E}{\partial w_{ik}} \tag{9}$$

$$b_k(t+1) = b_k(t) - \gamma \frac{\partial E}{\partial b_k} \tag{10}$$

$$c_{ik}(t+1) = c_{ik}(t) - \gamma \frac{\partial E}{\partial c_{ik}} \tag{11}$$

$$a_{1,ik}(t+1) = a_{1,ik}(t) - \gamma \frac{\partial E}{\partial a_{1,ik}}$$

$$a_{2,ik}(t+1) = a_{2,ik}(t) - \gamma \frac{\partial E}{\partial a_{2,ik}} \tag{12}$$

$$d_{ik}(t+1) = d_{ik}(t) - \gamma \frac{\partial E}{\partial d_{ik}} \tag{13}$$

(14)

where γ is the learning rate, w, b are the parameters of the consequent parts and c, d, a_1, a_2 are parameters of the antecedent parts. The consequent parameters of IT2IFLS are adjusted as follows:

$$\frac{\partial E}{\partial w_{ik}} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial w_{ik}} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial w_{ik}} + \frac{\partial y}{\partial y_k^{\nu}}\frac{\partial y_k^{\nu}}{\partial w_{ik}}\right]$$

$$= (y(t) - y^a(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\bar{f}_k^{\mu}} + \frac{\bar{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\bar{f}_k^{\mu}}\right) + \beta\left(\frac{\underline{f}_k^{\nu}}{\sum_{k=1}^{M}\underline{f}_k^{\nu} + \sum_{k=1}^{M}\bar{f}_k^{\nu}} + \frac{\bar{f}_k^{\nu}}{\sum_{k=1}^{M}\underline{f}_k^{\nu} + \sum_{k=1}^{M}\bar{f}_k^{\nu}}\right)\right] * x_i \tag{15}$$

$$\frac{\partial E}{\partial b_k} = \frac{\partial E}{\partial y}\frac{\partial y}{\partial y_k}\frac{\partial y_k}{\partial b_k} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial y_k^{\mu}}\frac{\partial y_k^{\mu}}{\partial b_k} + \frac{\partial y}{\partial y_k^{\nu}}\frac{\partial y_k^{\nu}}{\partial b_k}\right] = (y(t) - y^a(t)) * \left[(1-\beta)\left(\frac{\underline{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\bar{f}_k^{\mu}} + \frac{\bar{f}_k^{\mu}}{\sum_{k=1}^{M}\underline{f}_k^{\mu} + \sum_{k=1}^{M}\bar{f}_k^{\mu}}\right)\right.$$

$$\left. + \beta\left(\frac{\underline{f}_k^{\nu}}{\sum_{k=1}^{M}\underline{f}_k^{\nu} + \sum_{k=1}^{M}\bar{f}_k^{\nu}} + \frac{\bar{f}_k^{\nu}}{\sum_{k=1}^{M}\underline{f}_k^{\nu} + \sum_{k=1}^{M}\bar{f}_k^{\nu}}\right)\right] * 1 \tag{16}$$

where y_k is the output of the k^{th} rule. The premise parameters for IT2IFLS are computed as follows:

$$\frac{\partial E}{\partial c_{ik}} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial \underline{f}_k^{\mu}}\frac{\partial \underline{f}_k^{\mu}}{\partial \underline{\mu}_{ik}}\frac{\partial \underline{\mu}_{ik}}{\partial c_{ik}} + \frac{\partial y}{\partial \bar{f}_k^{\mu}}\frac{\partial \bar{f}_k^{\mu}}{\partial \bar{\mu}_{ik}}\frac{\partial \bar{\mu}_{ik}}{\partial c_{ik}} + \frac{\partial y}{\partial \underline{f}_k^{\nu}}\frac{\partial \underline{f}_k^{\nu}}{\partial \underline{\nu}_{ik}}\frac{\partial \underline{\nu}_{ik}}{\partial c_{ik}} + \frac{\partial y}{\partial \bar{f}_k^{\nu}}\frac{\partial \bar{f}_k^{\nu}}{\partial \bar{\nu}_{ik}}\frac{\partial \bar{\nu}_{ik}}{\partial c_{ik}}\right] \tag{17}$$

$$\frac{\partial E}{\partial d_{ik}} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial \underline{f}_k^{\mu}}\frac{\partial \underline{f}_k^{\mu}}{\partial \underline{\mu}_{ik}}\frac{\partial \underline{\mu}_{ik}}{\partial d_{ik}} + \frac{\partial y}{\partial \bar{f}_k^{\mu}}\frac{\partial \bar{f}_k^{\mu}}{\partial \bar{\mu}_{ik}}\frac{\partial \bar{\mu}_{ik}}{\partial d_{ik}} + \frac{\partial y}{\partial \underline{f}_k^{\nu}}\frac{\partial \underline{f}_k^{\nu}}{\partial \underline{\nu}_{ik}}\frac{\partial \underline{\nu}_{ik}}{\partial d_{ik}} + \frac{\partial y}{\partial \bar{f}_k^{\nu}}\frac{\partial \bar{f}_k^{\nu}}{\partial \bar{\nu}_{ik}}\frac{\partial \bar{\nu}_{ik}}{\partial d_{ik}}\right] \tag{18}$$

(19)

$$\frac{\partial E}{\partial a_{2,ik}} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial \underline{f}_k^{\mu}}\frac{\partial \underline{f}_k^{\mu}}{\partial \underline{\mu}_{ik}}\frac{\partial \underline{\mu}_{ik}}{\partial a_{2,ik}} + \frac{\partial y}{\partial \underline{f}_k^{\nu}}\frac{\partial \underline{f}_k^{\nu}}{\partial \underline{\nu}_{ik}}\frac{\partial \underline{\nu}_{ik}}{\partial a_{1,ik}}\right] \tag{20}$$

$$\frac{\partial E}{\partial a_{1,ik}} = \sum_k \frac{\partial E}{\partial y}\left[\frac{\partial y}{\partial \bar{f}_k^{\mu}}\frac{\partial \bar{f}_k^{\mu}}{\partial \bar{\mu}_{ik}}\frac{\partial \bar{\mu}_{ik}}{\partial a_{1,ik}} + \frac{\partial y}{\partial \bar{f}_k^{\nu}}\frac{\partial \bar{f}_k^{\nu}}{\partial \bar{\nu}_{ik}}\frac{\partial \bar{\nu}_{ik}}{\partial a_{2,ik}}\right]$$

The derivatives for the MFs are as follows:

$$\frac{\partial \bar{\mu}_{ik}}{\partial a_{1,ik}} = (\pi - 1)\left(\frac{1}{a_{1,ik}^2}\ln\left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right) \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}} + \frac{1}{a_{1,ik}}\ln\left|\frac{x_i - c_{ik}}{d_{ik}}\right| \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\left(1 \quad \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1}\right) \tag{21}$$

$$\frac{\partial \bar{\mu}_{ik}}{\partial c_{ik}} = (\pi - 1)\frac{1}{|d_{ik}|}\operatorname{sign}(x_i - c_{ik})\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1} \quad (22$$

$$\frac{\partial \bar{\mu}_{ik}}{\partial d_{ik}} = -\left((\pi - 1)\frac{1^2}{|d_{ik}|}\operatorname{sign}(d_{ik})|x_i - c_{ik}|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1}\right) \tag{23}$$

$$\frac{\partial \underline{\mu}_{ik}}{\partial a_{2,ik}} = (\pi - 1)\left(\frac{1}{a_{2,ik}^2}\ln\left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right) \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}} + \frac{1}{a_{2,ik}}\ln\left|\frac{x_i - c_{ik}}{d_{ik}}\right| \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1}\right) \tag{24}$$

$$\frac{\partial \underline{\mu}_{ik}}{\partial c_{ik}} = (\pi - 1)\frac{1}{|d_{ik}|}\operatorname{sign}(x_i - c_{ik})\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1} \quad (25)$$

$$\frac{\partial \underline{\mu}_{ik}}{\partial d_{ik}} = -\left((\pi - 1)\frac{1}{|d_{ik}|}\operatorname{sign}(d_{ik})|x_i - c_{ik}|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1}\right) \quad (26)$$

The derivatives for the non-MFs are obtained in the same way thus:

$$\frac{\partial \bar{v}_{ik}}{\partial a_{2,ik}} = -(\pi - 1)\left(\frac{1}{a_{2,ik}^2}\ln\left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right) \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}} + \frac{1}{a_{2,ik}}\ln\left|\frac{x_i - c_{ik}}{d_{ik}}\right|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1} \quad (27)$$

$$\frac{\partial \bar{v}_{ik}}{\partial c_{ik}} = -\left((\pi - 1)\frac{1}{|d_{ik}|}\operatorname{sign}(x_i - c_{ik})\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1}\right) \quad (28)$$

$$\frac{\partial \bar{v}_{ik}}{\partial d_{ik}} = \left((\pi - 1)\frac{1^2}{|d_{ik}|}\operatorname{sign}(d_{ik})|x_i - c_{ik}|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{2,ik}}\right)^{\frac{1}{a_{2,ik}}-1}\right) \quad (29)$$

$$\frac{\partial \underline{v}_{ik}}{\partial a_{1,ik}} = -(\pi - 1)\left(\frac{1}{a_{1,ik}^2}\ln\left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right) \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}} + \frac{1}{a_{1,ik}}\ln\left|\frac{x_i - c_{ik}}{d_{ik}}\right|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1}\right) \quad (30)$$

$$\frac{\partial \underline{v}_{ik}}{\partial c_{ik}} = -\left((\pi - 1)\frac{1}{|d_{ik}|}\operatorname{sign}(x_i - c_{ik})\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1}\right) \quad (31)$$

$$\frac{\partial \underline{v}_{ik}}{\partial d_{ik}} = \left((\pi - 1)\frac{1^2}{|d_{ik}|}\operatorname{sign}(d_{ik})|x_i - c_{ik}|\left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}-1} \times \left(1 - \left|\frac{x_i - c_{ik}}{d_{ik}}\right|^{a_{1,ik}}\right)^{\frac{1}{a_{1,ik}}-1}\right) \quad (32)$$

EXPERIMENTAL STUDIES

The elliptic IT2IFLS is proposed for non linear systems prediction and its performance is compared with Gaussian IT2IFLS with uncertain standard deviation. The parameters and the intervals adopted for the studies are as depicted in Table 1. The performance criterion adopted is the RMSE in order to assess the performance of both systems and is defined as:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y^{\alpha} - y)^2} \quad (33)$$

Table 1: Parameter boundaries for the Prediction Problems

Elliptic IT2IFS MF /NMF		Gaussian IT2IFS MF/NMF	
Parameter	Boundary	Parameter	Boundary
Center (c)	[0, 1]	Center (c)	[0, 1]
Width (d)	[0.5, 1.30]	Standard deviation (σ)	[0, 1]
a_1	[1.1, 1.5]	π	[0, 1]
a_2	[0.2, 1]		
π	[0, 1]		

Mackey Glass Prediction Problem

In this study, a benchmark Mackey-Glass prediction problem is analyzed which is as defined in equation (34).

$$\frac{dx(t)}{dt} = \frac{a * x(t-\tau)}{1 + x(t-\tau)^n} - b * x(t) \quad (34)$$

where a, b and n are real numbers, t is the current time and τ is a non-negative time delay constant. At $\tau \leq 17$, the Mackey-Glass tends to display a deterministic behaviour which turns chaotic when $\tau > 17$. For this analysis, the inputs vector is given as: $[x(t\text{-}18), x(t\text{-}12), x(t\text{-}6), x(t)]$ with $\tau = 17$ while the target output is $x(t+6)$. For each input, two elliptic and Gaussian IT2IFSs are utilized.

Table 2: Performance of elliptic and Gaussian IT2IFLSs on the prediction of Mackey-Glass

IT2IFLS MF/NMF	Train RMSE	Test RMSE
Elliptic IT2IFLS	0.0190	0.0130
Gaussian IT2IFLS	0.0194	0.0163

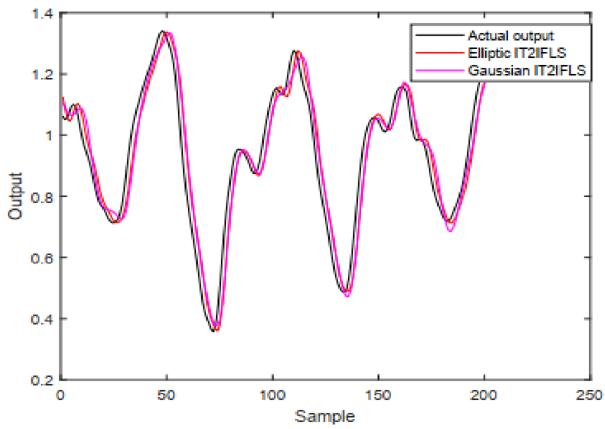

Figure 5: Actual and predicted outputs of Mackey-Glass time series data

Lorenz time series

The Lorenz time series has been introduced by Edward Lorenz (1963) and has contributed to the development of Chaotic time series prediction. The Lorenz equation is given in equation (35). The Lorenz model is a nonlinear dynamic system that demonstrates a long-term behavior of the Lorenz oscillator. The Lorenz model mathematical equation is as expressed below:

$$dx(t)\,dt = \rho[y(t) - x(t)]$$
$$dy(t)\,dt = x(t)\,[r - z(t)] - y(t)$$
$$dz(t)\,dt = x(t)y(t) - bz(t) \quad (35)$$

The commonly used initial conditions for parameters, σ, r and b take the values 10, 28 and 8/3 respectively. Data for the experiment is generated using the x, y and z-components (see Figure 6) of the Lorenz time series. Figure 7 is the 3-dimensional phase space of Lorenz chaotic time series. A one step ahead prediction is analyzed using the input-output generating vector which consists of the present value and three last values: [x(t), x(t-1), x(t-2), x(t-3); x(t+1)], where $x(t+1)$ is the target. A total of 2500 data points are generated with 1500 used for training and the remaining 1000 instances for testing.

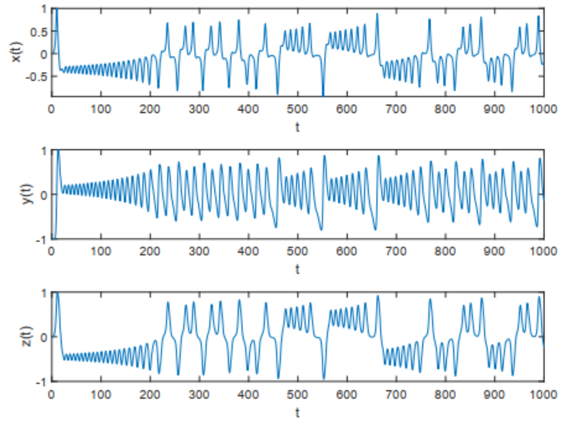

Figure 6: Lorenz time series on x, y and z-axes.

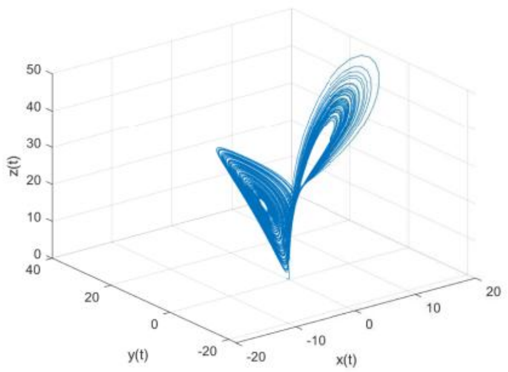

Figure 7: 3D-Lorenz attractor phase diagram

Table 3: Performance comparison of IT2IFLS-Elliptic (Gaussian) on Lorenz time series (x - axis)

IT2IFLS MF/NMF	Train RMSE	Test RMSE
Elliptic IT2IFLS	0.0799	0.0515
Gaussian IT2IFLS	0.1064	0.0691

Table 4: Performance comparison of IT2IFLS-Elliptic (Gaussian) on Lorenz time series (y-axis)

IT2IFLS MF/NMF	Train RMSE	Test RMSE
Elliptic IT2IFLS	0.1368	0.0878
Gaussian IT2IFLS	0.1830	0.1190

Table 5: Performance comparison of IT2IFLS-Elliptic (Gaussian) on Lorenz time series (z-axis)

IT2IFLS MF/NMF	Train RMSE	Test RMSE
Elliptic IT2IFLS	0.0189	0.0508
Gaussian IT2IFLS	0.0951	0.0618

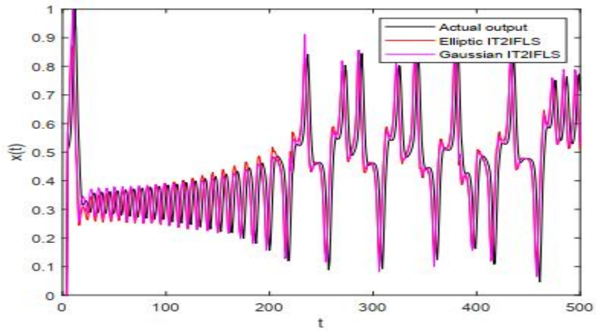

Figure 8: x-axis actual and predicted outputs using elliptic and Gaussian IT2IFLSs

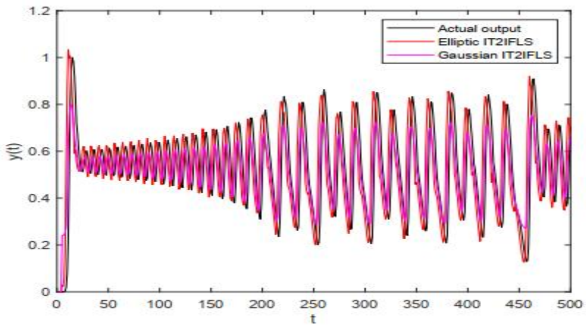

Figure 9: y-axis actual and predicted outputs using elliptic and Gaussian IT2IFLSs

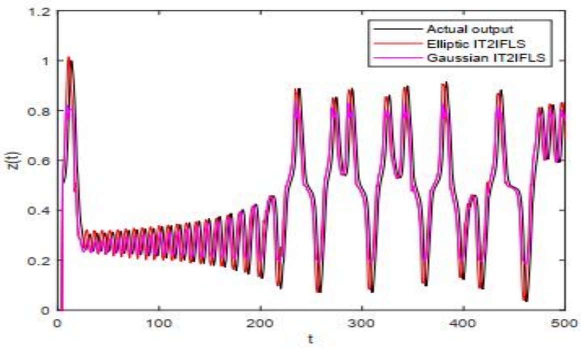

Figure 10: z-axis actual and predicted outputs using elliptic and Gaussian IT2IFLSs.

DISCUSSION AND CONCLUSION

Gaussian membership and non-MFs have been used extensively in fuzzy logic systems to solve problems involving parameter optimization in uncertain environments. In this paper, an elliptic membership and non-MFs are adopted for the prediction of two non-linear systems and the results compared with Gaussian membership and non MFs. Table 2 shows the RMSE of the prediction of Mackey Glass time series using elliptic and Gaussian IT2IFLSs while Figure 5 shows the actual and the predicted outputs. From Table 2, the training RMSE of elliptic IT2IFLS is comparable with that of the Gaussian IT2IFLS, however, for the test RMSE, the elliptic IT2IFLS shows improved performance, an indication that elliptic IT2IFLS has lower error in terms of accuracy of prediction in the test set which is not biased. Analysis is also conducted using the Lorenz time series, another widely used benchmark dataset. Shown in Tables 3, 4 and 5 are the results of prediction based on elliptic and Gaussian IT2IFLSs on the x, y and z-axes data of the Lorenz time series. As seen from Tables 3 to 5, the elliptic IT2IFLS provides lower RMSE for all the axes predictions. Figures 8, 9 and 10 are the actual and predicted outputs of Lorenz time series for x, y and z-axes respectively of the first 500 instances. A closer look at the three figures shows that the elliptic IT2IFLS predictions follow the actual outputs more closely than those of the Gaussian IT2IFLS. In the overall, this study demonstrates that the elliptic IT2IFLS is more efficient in terms of error reduction and can be applied in any uncertain environment where accuracy is paramount.

In the future, we intend to adopt other learning algorithms (extended Kalman filter, simulated annealing) and real world data to test the efficiency of the two fuzzy models.

ACKNOWLEDGMENT: The authors wish to acknowledge the supports of the Tertiary Education Trust fund (TETFUND) through the TETFUND Center of Excellence in Computational Intelligence Research, University of Uyo and the University of Uyo Management.

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