An example of a Design of small Gravity Dams

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1. Introduction

The purpose of this paper is to consider and discuss the merits of the methods employed in the consideration and design of a concrete diversion dam designed by the writer with the purpose of diverting a maximum flow of 1.5 m³/sec from the Culfo River for the agricultural irrigation of approximately 1250 hectares at the Arba Minch Farm in Gemu Gofa Province.

It is of course fully appreciated that there are numerous and wide variations in the criteria which can be employed to design such type of gravity dams as carried out in various parts of the world and by numerous engineers.

However, it is suggested that the criteria borne in mind in the design of the small dam which is the subject of the following notes may be of value in considering the design of similar diversion or gravity dams of varying sizes.

2. Hydrogeological data

Five investigation boreholes have been drilled along the proposed axis of the dam to a depth ranging between 15 and 31.5 m.

On the left side of the Valley the soil consists of basalt boulders with gravel sand and silt forming a sort of soft conglomerate.

On the right side sandy silty formations prevail, occasionally including some gravel.

The frequent alternation of thin layers (sandy — silts and silty — sands in general) did not suggest the opportunity of laboratory tests of undisturbed samples, but the visual inspection of the latter showed fairly good degree of relative density or consistency.

The intermediate borehole at the centre of the axis of the dam has the same features as the boreholes on the left side of the valley down to 5 m depth, while the underlying formations are similar to those found on the right side.

According to the water tests carried out in the abovementioned boreholes the permeability coefficient $K$ is generally low from the seepage point of view (order of $10^{-4}$ to $10^{-6}$ cm/sec). Values of $K$ in the order of $10^{-6}$ cm/sec often indicate that the strata is practically impervious.

On the other hand the dissipation of excess pore pressures produced by loading will be a short term phenomenon, and therefore the stability analysis against shear failure may be performed in terms of effective stresses, assuming an average value for the internal angle of friction:

$$\phi = 30^\circ$$

with no cohesion assumed.

3. Design data

On the basis of the limited hydrological observations of the river and the irrigation water requirement ($Q_{max} = 1.5$ m³/sec) for the above mentioned area of 1250 ha to be commanded the following design data have been established:

- dam crest elevation: 108.70 m
- length of spillway: 33 m
- maximum head of spillway: 1 m

(Elev. 109.70)

- maximum estimated flood: 66 m³/sec

With the foregoing data the following was proposed for the preliminary design of the dam:

- foundation width of the dam: 4.40 m
- foundation depth: 105.50 m elev.
- upstream ground elevation: 107.00 m
- downstream ground elevation: 106.65 m

(rock rip rap)

- system of piles: 5 ranges

After obtaining the results of the geohydrological investigation of the dam site the preliminary design of the diversion dam shown in Fig. 1 was revised for safety against piping, shear failure, excessive seepage and up lift under the dam and also checked against over turning and sliding effects.
4. Safety against Piping

The exit gradient generally provides the most significant criterion design for the factor of safety with respect to piping.

For a depressed structure on a permeable base of infinite extent the exit gradient may be analytically calculated by Harr's expression:

\[ l_e = \frac{\alpha H}{\pi D_f} \]  

where:

- \( H \) = hydraulic head loss
- \( D_f \) = Embedment depth of structure
- \( \alpha \) = coefficient to be determined according to Harr as a function of the ratio \( D_f/B \)

where \( B \) is the width of structure (Fig. 2).
The weighted-creep theory as developed by Lane could also be employed as criteria for the design of low concrete dams on pervious foundations to be safe against piping and uplift pressures. Although this is an empirical method, considerable confidence has been placed in it by many engineers and has been successfully used for the design of many structures.

We consider the following expressions for the calculation of hydraulic gradient:

\[ I_e = \frac{H}{\sum B} \quad \text{Bligh's equation} \]

\[ C = F_s = \frac{I_{cr}}{I_e} \quad \text{(2)} \]

\[ I_e = \frac{H}{\frac{1}{3}B + \sum t} \quad \text{Lane's equation} \]

where:

- \( H \) = hydraulic head loss
- \( B \) = Horizontal section of the line of creep
- \( t \) = Vertical section of the line of creep
- \( C \) = safe weighted creep ratio or expressed as coefficient of safety factor \( F_s \)
- \( I_{cr} \) = critical gradient, as per Terzaghi = 1.

### TABLE 1 — Weighted-Creep Ratio

<table>
<thead>
<tr>
<th>Type of Material</th>
<th>Safe Weighted Creep Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lane's Value of ( C = F_s )</td>
</tr>
<tr>
<td>Very fine sand or silt</td>
<td>8.5</td>
</tr>
<tr>
<td>Fine sand</td>
<td>7.0</td>
</tr>
<tr>
<td>Medium sand</td>
<td>6.0</td>
</tr>
<tr>
<td>Coarse sand</td>
<td>5.0</td>
</tr>
<tr>
<td>Fine gravel</td>
<td>4.0</td>
</tr>
<tr>
<td>Medium gravel</td>
<td>3.5</td>
</tr>
<tr>
<td>Gravel and sand</td>
<td>—</td>
</tr>
<tr>
<td>Coarse gravel including cobbles</td>
<td>—</td>
</tr>
<tr>
<td>Boulders with some cobbles &amp; gravel</td>
<td>2.5</td>
</tr>
<tr>
<td>Boulders, gravel &amp; sand</td>
<td>—</td>
</tr>
<tr>
<td>Soft clay</td>
<td>—</td>
</tr>
<tr>
<td>Medium clay</td>
<td>—</td>
</tr>
<tr>
<td>Hard clay</td>
<td>—</td>
</tr>
<tr>
<td>Very hard clay or hard pan</td>
<td>—</td>
</tr>
</tbody>
</table>

From the preliminary design data (Fig. 1) we have:

- \( D_f = 1.15 \) m downstream side
- \( B = 4.40 \) m
- \( H = 2.70 \) m (max. upstream water elevation 109.70 m minus downstream water level supposed at elevation 107.60 m).

Employing Harr’s criteria from (1) and from (Fig. 2) \( \alpha = 0.5 \) and therefore:

\[ I_e = \frac{\alpha H}{\pi D_f} = \frac{0.5 \times 2.70}{3.14 \times 1.15} = 0.374 \]

The factor of safety in our case is:

\[ F_s = \frac{I_{cr}}{I_e} = \frac{1}{0.374} = 2.67 \]

This is too low for any type of pervious soil, compared with the minimum allowable value of 4 according to the criteria of Lane (Table 1). When fine sand is present, as in our case, Khosla, Bose and Taylor recommend a safety factor of at least 6 and it may be even raised to 10.

From our preliminary design \( F_s \) is very low and it was required to increase the safety factor. An increase of \( F_s \) can be obtained by enlarging and deepening the structure. The enlargement alone does not bring about convenient solution as shown in Table 2.

Maintaining \( D_f \) constant and varying \( B \) we calculate \( F_s \):

<table>
<thead>
<tr>
<th>( B ) (m)</th>
<th>( F_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.40</td>
<td>2.67</td>
</tr>
<tr>
<td>6.00</td>
<td>3.04</td>
</tr>
<tr>
<td>8.00</td>
<td>3.45</td>
</tr>
<tr>
<td>10.00</td>
<td>3.72</td>
</tr>
<tr>
<td>12.80</td>
<td>4.00</td>
</tr>
<tr>
<td>23.00</td>
<td>6.00</td>
</tr>
</tbody>
</table>

More advantage can be obtained by increasing the depth \( D_f \) as shown by Table 3. In this case two alternatives were considered:

\( B = 4.40 \) m and \( B = 6.00 \) m

<table>
<thead>
<tr>
<th>( B ) (m)</th>
<th>( F_s )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.15</td>
<td>2.67</td>
</tr>
<tr>
<td>1.65</td>
<td>3.56</td>
</tr>
<tr>
<td>2.15</td>
<td>4.24</td>
</tr>
<tr>
<td>2.65</td>
<td>4.95</td>
</tr>
<tr>
<td>3.15</td>
<td>5.82</td>
</tr>
</tbody>
</table>

The above calculations refer to structures without cut-offs or sheet pilings but the result is not likely to be much different when the width of the structure and the depth of vertical elements are not very important; anyway the safety factor would be underestimated, if \( D_f \) represents the depth of vertical elements.

In our case a sheet piling may meet with some difficulty due to the presence of boulders, and the execution of two cutoff walls (one upstream and one downstream) will be more suitable.
Fig. 3 — Stability Analysis Case A

Fig. 4 — Stability Analysis Case B
On the basis of the results of the foregoing analysis the following elements have been fixed for the final design of the diversion dam (Figs. 3 & 4)

base width \((b)\) = 6.00 m
base elevation = 105.00 m
Cutoff depth below downstream ground level \((D_f)\) = 3.15 m.

According to figures in Table 3, calculated following Harr's criteria, the factor of safety is 6.2 for the most unfavourable case of maximum spillway head. This can be considered satisfactory. Following Eligh's criteria the safety factor,

\[
F_s = \frac{15.05}{2.70} = 5.57
\]

And according to Lane,

\[
F_s = \frac{11.65}{2.70} = 4.3
\]

Both of these values are greater than four. Considering \(\alpha = 0.51\) when the water is at dam crest level:

\[
I_e = \frac{\alpha H}{\pi D_f} = \frac{0.510 \times 2.05}{3.14 \times 3.15} \approx 0.106
\]

\[
F_s = \frac{I_e}{I_e} = \frac{1}{0.106} = 9.5
\]

F.s rises to about 10.

5. Stability against Shear Failure

The ultimate bearing capacity of a foundation with inclined resultant load applied to a granular subsoil, is given by the Erinch Hansen equation:

\[
q_u = 0.5 \gamma N_r B \left(1 - 0.3 \frac{B}{L}\right) \left[1 - 1.5 \frac{H}{N}\right] + \gamma D_f N_q \left[1 + 0.2 \frac{B}{L} + 0.1 \frac{D_f}{B}\right] \left[1 - 1.5 \frac{H}{N}\right] \tag{3}
\]

where:

\(B\) = width of foundation
\(L\) = length of foundation
\(H\) = horizontal forces acting on the soil
\(N\) = vertical forces acting on the soil
\(D_f\) = depth of foundation
\(\gamma\) = submerged density of soil
\(N_r, N_q\) = bearing capacity factors, functions of the internal angles of friction of the soil \((\phi)\), according to Terzaghi.

When the resultant, inclined force \(R = f(H, N)\) is eccentric, \(B\) must be substituted with the reduced effective width:

\[
B' = B - 2.e \tag{4}
\]

where \(e\) represents the eccentricity.

Considering the final adopted design of the dam (Section 4) and the nature of the soil (Section 2) we arrive at:

\[
B' = 6.00 - 2e \quad L = 33 m
D_f = 3.15 m \quad N_r = 20
\phi = 30^\circ \text{ (determined in Section 2)} \quad N_q = 22
\gamma = 1 \text{ ton/m}^2
\]

The bearing capacity factors \(N_r, N_q\) are determined from Fig. 5.

As concerns the stress conditions, two cases have been considered:

CASE “A" (Fig. 3)

Upstream water elevation = 108.70 m (dam crest),
Downstream water elevation = 106.65 m (rock riprap top level).

Full water thrust is considered on both upstream and downstream sides \((S_w \text{ and } S'_w)\); earth pressures compensating mutually.

Vertical resultant \(N\) including saturated weight of soil between cutoff walls and full uplift force “\(U\)” at cutoff base level (trapezoidal diagram with upstream pressure of 5.20 ton/m² and downstream value of 3.15 ton/m²):

Total load (structure + soil between cutoffs) \(P = 51.2\) ton
Uplift force at cutoffs base level \(U = 25.1\) ton
Resultant vertical force \(N = 26.1\) ton
Water thrust on upstream side \(S_w = 13.5\) ton
Water thrust on downstream side \(S'_w = 5.0\) ton
Resultant horizontal force \(H = 8.5\) ton

Taking moment of forces about the upstream cutoff wall toe, moment arm of the resultant inclined force \(R:\)

\[
a = \frac{M_1 N + M_2 H}{N} = 3.20 \text{ m.}
\]

Eccentricity of the resultant inclined force \(R:\)

\[
e = a - \frac{B}{2} = 3.20 - 3.00 = 0.20 \text{ m.}
\]

According to equations, (3) and (4) and to the magnitude and position of forces shown in (Fig. 3) the following values of ultimate bearing capacity have been obtained:

\[
q_u = 52.8 \text{ ton/m}^2/\text{m.}
\]

The applied average pressure to be compared with \(q_u\) is given by the expression:

\[
P_o = \frac{N}{B'} = \frac{N}{B - 2e} \tag{5}
\]

According to Equations (4) and (5), we have

\[
P_o = 4.65 \text{ ton/m}^2/\text{m.}
\]
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and the following safety factor is obtained:

$$F_s = \frac{q_t}{P_o} = 11.40$$

The stability against shear failure is then ample even in the most unfavourable conditions.

The stability against shear failure is also ample in the above indicated Case (B).

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**Fig. 5** — Chart showing relation between $\phi$ and bearing capacity factors

**CASE "B"** (Fig. 4)

Upstream water elevation = 109.70 m (maximum spillway head),
Downstream water elevation = 107.00 m (0.35m above riprap top).

Horizontal water thrust is calculated according to the new conditions and Vertical resultant force $N$ as in case (A).

It is very unlikely that the maximum head condition may last for such a long time to produce uplift pressure worse than that considered in case (A). Therefore $U$ is taken as in case (A).

Vertical forces

- $P = 51.2$ ton
- $U = 25.1$ ton
- $N = 26.1$ ton

Horizontal forces

- $S_w = 18.8$ ton
- $S'_w = 5.0$ ton
- $H = 12.7$ ton

Proceeding as in the previous Case (A)

Eccentricity of resultant force $R$:

$$e = 0.68 \text{ m}$$

Applied pressure at cutoffs base level:

$$P_o = \frac{N}{B - 2e} = 5.63 \text{ ton/m}^2.$$
7. Overturning

Since the resultant of vertical forces is much higher than the resultant of horizontal forces in both cases A & B (Section 5) the structure is safe against overturning.

Taking moment about the downstream cut-off wall toe:

\[ \frac{H}{N} \text{ is equal to or less than } "f" \]

Case A: \( "f" = \frac{8.5}{26.1} = 0.325 \)

Case B: \( "f" = \frac{12.7}{26.1} = 0.485 \)

8. Sliding

The allowable sliding factor is the coefficient of static friction between two sliding surfaces reduced by an appropriate factor of safety. If \( "f" \) represents the allowable sliding factor, a dam is considered safe against sliding when

\[ F_s = \frac{MN}{MH} \]

Case A: \( F_s = \frac{2.15 \times 8.5}{3.50 \times 26.1} = 0.2, \text{ less than 1} \)

Case B: \( F_s = \frac{2.40 \times 12.7}{3.50 \times 26.1} = 0.33, \text{ less than 1} \)

The factor of safety against overturning is sometimes defined as the ratio of the tightening moments to the overturning moments about the toe of the dam. When so defined all excepting the direct foundation reaction are deemed active. Ordinarily the factor of safety against overturning is between 2 and 3.

Case A: \( F_s = \frac{MN}{MH} = 5, \text{ greater than 3} \)

Case B: \( F_s = \frac{MN}{MH} = 3.3, \text{ greater than 3} \)

Our case represents for permissible \( "f" \) — concrete against gravel, coarse sand and boulders.

Furthermore the cut-off walls prevent displacement as could be observed from Figs. 3, 4 and 6.

Exact values for coefficients of static friction cannot be determined without the benefit of laboratory tests, but the allowable sliding factors given below which include ample factors of safety for concrete against sliding on the various foundation materials may be used as a general guide:

<table>
<thead>
<tr>
<th>Material</th>
<th>( f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sound rock, clean and irregular surface</td>
<td>0.8</td>
</tr>
<tr>
<td>Rock, some jointing and laminations</td>
<td>0.7</td>
</tr>
<tr>
<td>Gravel and course sand</td>
<td>0.4</td>
</tr>
<tr>
<td>Sand</td>
<td>0.3</td>
</tr>
<tr>
<td>Shale</td>
<td>0.3</td>
</tr>
<tr>
<td>Silt and clay</td>
<td>testing required</td>
</tr>
</tbody>
</table>

9. Conclusions

The final design of the diversion dam is safe against piping, shear failure, excessive seepage, uplift, overturning and sliding effects.
Piles are not necessary for stability against sliding or shear failure as discussed in section 5.

The presence of a double cutoff wall and an adequate compacted mixture of clay, sand and gravel at the upstream side of the cutoff wall and the graded inverted filter at the downstream side of the cutoff wall will prevent piping, excessive seepage and uplift effect under the dam. The downstream rock riprap must be sufficiently thick and long in order to prevent erosion near the dam by the over flowing water and for the purpose gabions have been recommended. But erosion may still take place beyond the protected zone. In this case the toe of the valley slopes should also be protected by rockfills and periodically repaired as necessary.

REFERENCES: