APPLICATION OF MICROSTRIP LINES FOR THE REALIZATION OF PATCH ANTENNAS

Mihret Tesfaye and Mohammed Abdo
Department of Electrical and Computer Engineering
Addis Ababa University

ABSTRACT
This paper presents a simple analytical analysis method for electrically thick rectangular probe fed microstrip patch antenna. The method is a hybrid analysis method which is a combination of the simple cavity model and full wave modal expansion analysis. First, a cavity model is developed and then a full wave modal expansion of both the resonant and non-resonant mode is derived. Based on this expansion, radiation pattern, directivity, resonant frequency, input impedance, bandwidth, and radiation efficiency are computed and good agreement with reported experimental results is achieved. Design procedure of thick rectangular probe-fed microstrip patch antenna is presented based on a simple analytical analysis method developed. Design curves are plotted for directivity, resonant frequency, input impedance, bandwidth, and radiation efficiency.

INTRODUCTION
Microstrip lines were first proposed in 1952 and were increasingly used in the late 1960's and 1970's to realize circuits, generally called Microwave integrated circuits (MICs). The application of Microstrip lines to design antenna has been suggested in 1953 by Deschamps [10]. More than 20 years after the original suggestion was made, the first actual microstrip antenna appeared in 1974 [1], [11]. It is then of a considerable interest in microstrip antennas developed a steady flow of research and publications. The microstrip antenna consists of a radiating structure spaced a small fraction of a wavelength above a ground plane (Fig. 1). Often microstrip antennas are also referred to as patch antennas. The radiating patch may be square, rectangular, thin strip (dipole), circular, elliptical, triangular, or any other configuration [2], [11]. Actually, microstrip antennas are low profile, conformal to planar and non-planar surfaces, simple, light, inexpensive, mechanically robust and compatible with MMIC designs. Because of these attractive advantages, uses of microstrip antennas are particularly increasing in high performance aircraft, spacecraft, missile applications, mobile phones and wireless communications [2].

An efficient use of these antennas requires the knowledge of radiation pattern, directivity, resonant frequency, input impedance, and bandwidth and radiation efficiency. However, analysis of microstrip antenna is very difficult because of the presence of a dielectric substrate [6], [11]. A number of papers have appeared on the subject of impedance and pattern predictions for microstrip antenna. These papers have presented basically four different approaches to the problem with differing degrees of flexibility, accuracy, and computational effort.

i) The simplest approach, applicable to only rectangular microstrip antennas involves treating the radiative properties of the latter as two parallel radiating slots interconnected by a low impedance transmission line [2], [11]. This approach will give fair agreement with the experiment as long as the feed point is chosen near one of the two radiating edges.

ii) The second model is cavity model. In this model, it is generally assumed that the field inside the
Mihret Tesfaye and Mohammed Abdo

antenna is approximated by the dominant mode of the cavity derived from the antenna by enclosing its periphery by a magnetic wall [2], [9], [11]. This approach also yields loci which are symmetric about the real axis of the Smith chart and hence do not accurately represent the impedance of the antenna at all feed points.

iii) A third method models the microstrip as a grid of wires and solves the resulting structure numerically. The numerical method includes FEM and FDTD methods [4]. While this method seems to be somewhat more accurate and general, it is more complex and costly to apply than necessary for most practical microstrip and provides little physical insight into the operation of these antennas.

iv) A fourth method replaces the antenna, initially with a cavity whose fields are computed using a full mode-expansion. It is the neglecting of the non-resonant modes that, causes much of the error in the first two methods. Inclusion of the effects of these modes yields computed loci which are shifted into the inductive side of the Smith chart, in agreement with experimental results. This method can be adapted in analysis of thick rectangular patch antennas as it is presented in this paper.

For a long time, microstrip antennas were characterized by narrow bandwidth and low efficiency. Many researchers and users were considering these properties as inherent to it. Recently, experimental and theoretical studies were made to develop broadband microstrip antennas. It is then discovered that broadband microstrip antennas can be achieved by using impedance matching, parasitic element, multimode operation, multilayer patches, and by increasing substrate thickness [1]. But the simplest and most effective way to reach broadband operation is to increase the substrate thickness [1]. In this case, however, the simple models developed for thin microstrip antenna are no longer applicable; hence, work has started on the development of more sophisticated integral equation techniques.

More recently, the use of thicker substrates was combined with that of very low permeability materials as low as 1.7, thus makes it possible to prevent the appearance of surface wave while enhancing radiation and widening the frequency bandwidth [3], [7].

Thus, the objective of this study is developing a simpler analytical analysis and design methods of electrically thick patch antennas. It should be noted that a number of theoretical and experimental analyses of thick microstrip patch antennas appeared recently, but most of which are full wave analysis using numerical methods [4]. These numerical methods for thick substrate patches are too complex to apply, and hence a simpler analytical methods based on the cavity model and full wave modal expansion is developed and presented in this paper.

THICK RECTANGULAR PROBE-FED MICROSTRIP ANTENNAS

Recent interest has developed in microstrip antenna etched on electrically thick substrates \((h > 0.05\lambda_0)\). This interest is primarily due to two reasons. First, as these antennas are used for applications with increasingly higher operating frequencies (in GHz ranges), and consequently shorter wavelengths, even antennas with physically thin substrate become thick when compared to a wavelength [3], [11]. Second, microstrip antennas have inherently narrow bandwidths normally not suitable for wideband application. Thus it is desirable to find methods to increase their bandwidth. It is, however, well known that by increasing the thickness of the dielectric substrate, the bandwidth of a patch antenna can be increased. Wider bandwidth, as high as 20 percent, could be achieved by simply using an electrically thick substrate [3].

In the method presented, the cavity model is developed first and then based on it the full wave modal expansion of the fields is derived. Once the full wave modal expansion is found, the radiation pattern, directivity, resonance frequency, quality factor, input impedance, bandwidth and radiation efficiency of the antenna are computed. It is experimentally proved that dielectric thickness will not have significant effect on the radiation pattern of the antenna [3]. Thus the radiation pattern is computed only by using the dominant mode. Dielectric thickness affects the resonant frequency since it increases fringing effect. Hence a more accurate expression is used taking the fringing effect into account. Input impedance is another parameter that is significantly affected by substrate thickness. Therefore, input impedance is computed by taking
Microstrip Lines for the Realization of Patch Antennas

The effect of substrate thickness into consideration. Expressions for bandwidth and radiation efficiency for the thick rectangular probe-fed patch antenna are also presented using the analysis method developed. As it is modeled in the cavity model, microstrip antennas resemble dielectric loaded cavities, and they exhibit higher order resonance. The normalized fields within the dielectric substrate can be found more accurately by treating that region as a cavity bounded by electric conductors (above and below it) and by magnetic walls (to simulate open circuit) along the perimeter of the patch [6], [9], [11].

The dominant rectangular patch resonance occurs at the frequency for which

$$k = k_{1n}.$$  

Assuming the probe current $I_0$ is one ampere, Eq. (3) predicts that the dominant $TM_{10}$ cavity mode is excited at resonance with maximum amplitude of

$$A_{10} = -\frac{\mu_0}{k_L} \frac{1}{k_{10} L_{10}^{1/2}} \frac{\cos (\pi y/L_{10})}{W_{10}^{1/2}}.$$  

This equation is used later to derive the formula for input resistance at resonance.

Radiation Pattern

Once the field is known, the field at the aperture between the rectangular plate and the ground plane can be represented by an equivalent magnetic current from which the radiated fields will be obtained. Assuming that the dominant mode in the cavity is $TM_{10}$ mode, the electric and the magnetic field components reduce to Eq. (7) as given in [2].

$$E_x = A_{10} \cos \left( \frac{\pi x}{L_{10}} \right), \quad H_y = \frac{\alpha_0 L_{10} \cdot E_x}{\pi x},$$

where $A_{10}$ is given in Equation (6).

Applying Huygen’s principle to the outer surface of the cavity, one obtains on the magnetic wall of the cavity $M_0 = -2n x z F_z$. $M_0$ is found to be

$$M_0 = -2(\alpha_0 x \alpha_1) A_{10} = \alpha_2 A_{10} \quad \text{at } x' = 0.$$  

Now applying this theory to the rectangular slots (apertures) of the rectangular patch antenna, the radiated electric field for the two slots of rectangular patch is given in [2]

$$E_x = \frac{\alpha_0 E_{10} \exp^{-jr}}{2\pi} \left[ \cos \theta \cos \phi \sin \frac{\pi y}{Y} \sin \frac{\pi Z}{Z} \right] (AF),$$

$$E_y = \frac{\alpha_0 E_{10} \exp^{-jr}}{2\pi} \left[ \sin \theta \sin \frac{\pi y}{Y} \sin \frac{\pi Z}{Z} \right] (AF),$$

where $(AF)_y = 2 \cos \left( \frac{k w}{2} \sin \theta \sin \phi \right)$ and

$$Z = \frac{k h}{2} \cos \theta.$$  

with

$$k_{1n} = \left( \frac{m \pi}{L_{10}} \right)^2 + \left( \frac{n \pi}{W_{10}} \right)^2.$$  

The ideal magnetic wall allows for a simple modal expansion of the fields in terms of an eigenfunction expansion. The electric field $E_z$ inside the cavity, as well as the eigenfunctions, are independent of $z$, provided the assumption is made that the probe current $I_0$ is constant. The eigenfunctions $\varphi_{mn}(x,y)$ satisfy the eigenvalue Eq. (9)

$$\nabla^2 \varphi(x,y) + k_{mn}^2 \varphi_{mn}(x,y) = 0$$  

(1)

The eigenfunctions are cavity modes that can exist inside the magnetic-wall cavity, and the eigenvalues $k_{mn}^2$ are the corresponding wave numbers of the resonant cavity modes. Because of the ideal cavity approximation, the eigenfunctions are complete and orthogonal, and the total field excited by the feed may be expanded in terms of these functions. The eigenfunction expansion of the electric field inside the cavity is given in [9] by

$$E_x = \sum_{m=n=0}^{\infty} A_{mn} \varphi_{mn}(x,y)$$  

(2)

where, $A_{mn} = j \omega \mu_0 \left( \frac{I_0}{\varphi_{mn}, \varphi_{mn}} \right)^{-1/2}$

(3)

The effective complex wave number $k_e$ is defined by the relation $k_e^2 = k_e^2(1 - j_{le})$, where $k_e^2 = k_0^2 \mu_0$, and $j_{le} = 1/Q$, is the effective loss tangent of the substrate. This effective loss tangent accounts for all loss mechanisms, including radiation into space, surface waves, dielectric loss, and conductor loss. For the rectangular patch, the eigenfunctions are

$$\varphi_{mn}(x,y) = \cos \left( \frac{m \pi x}{L_{10}} \right) \cos \left( \frac{n \pi y}{W_{10}} \right)$$  

(4)

with eigenvalues

$$k_{mn}^2 = \left( \frac{m \pi}{L_{10}} \right)^2 + \left( \frac{n \pi}{W_{10}} \right)^2.$$  

(5)

Directivity

It is found from the radiation intensity (U) and radiated field \( P_{\text{rad}} \) and it is given in [2]

\[
D_0 = \frac{4\pi f(0,0)}{P_{\text{rad}}} = 4\pi r^2 \frac{W_{\text{rad}}(0,0)}{P_{\text{rad}}} \quad (12a)
\]

\[
W_{\text{rad}}(0,0) = \frac{1}{2\pi} |F_d(0,0)|^2 \quad (12b)
\]

\[
D_0 = \frac{8}{\eta \pi} \left( \frac{\omega_0^2 \sin(k_j \lambda)}{2\pi} \right)^2 \quad (12c)
\]

where, \( P_{\text{rad}} = P_w \) given in Eq. (28).

Resonance frequency

Taking fringing effect into consideration, the resonance frequency is given in [2] as:

\[
(f_r)_h = \frac{1}{2L_{\text{eff}}} \left( \frac{1}{\sqrt{\mu_r \varepsilon_r}} \right) \quad (13a)
\]

\[
(f_r)_0 = \frac{1}{2L_{\text{eff}}} \left( \frac{1}{\sqrt{\mu_r \varepsilon_r}} \right) \quad (13b)
\]

where, \( L_{\text{eff}} = L + \Delta L \) and

\[
\varepsilon_{\text{eff}} = \varepsilon_r + \frac{1}{2} \left( \frac{1 + \frac{12}{\Delta L}}{2} \right) \quad (14)
\]

\[
\Delta L = 0.412 \frac{h}{w} \left( \frac{\varepsilon_{\text{eff}} - 0.35}{\varepsilon_{\text{eff}} - 0.258} \right) \quad (15)
\]

Quality factor

The effective loss tangent is related to the \( Q \) of the patch by the relation

\[
\tan \delta = \frac{1}{Q} \quad (16)
\]

The total quality factor \( Q \) may be expressed in terms of the \( Q \) factors associated with radiation into space \((Q_w)\), radiation into surface waves \((Q_{\text{sw}})\), dielectric loss \((Q_d)\), and conductor loss \((Q_c)\), through the relation

\[
\frac{1}{Q} = \frac{1}{Q_w} + \frac{1}{Q_{\text{sw}}} + \frac{1}{Q_d} + \frac{1}{Q_c} \quad (17)
\]

Formulas for \( Q_w \) and \( Q_{\text{sw}} \) are well known. Assuming the magnetic-wall approximation, a relatively straightforward analysis yields [2]

\[
Q_w = \frac{1}{I_d} \quad \text{and} \quad Q_{\text{sw}} = \frac{1}{2} \frac{\eta \mu_r \sin(k_j h)}{R_s} \quad (18)
\]

where \( I_d \) is the actual loss tangent of the substrate, and \( R_s \) is the surface resistance of the patch and ground plane metal. The surface resistance is calculated from the usual formula

\[
R_s = \frac{\omega_0^2 \sin(k_j \lambda)}{2\pi} \quad (19)
\]

The remaining two factors, \( Q_w \) and \( Q_{\text{sw}} \), determine the amount of power radiated into space and surface waves. To relate these two \( Q \) factors, a radiation efficiency \( e_r \) is defined, which is the radiation efficiency assuming no dielectric loss \((\sigma = 0)\) and no conductor loss \((\sigma = \infty)\). This efficiency accounts only for power loss due to the excitation of surface waves. In terms of the \( Q \) factors,

\[
e_r = \frac{Q_{\text{sw}}}{Q_w} \quad (20)
\]

where the radiation quality factor \( Q_{\text{sw}} \) is defined as

\[
\frac{1}{Q_r} = \frac{1}{Q_{\text{sp}}} + \frac{1}{Q_{\text{sw}}} \quad (21)
\]

To a good approximation, the efficiency \( e_r \) may be approximated by the radiation efficiency \( e_{\text{rad}} \) of a horizontal electric dipole on top of the lossless substrate (the loss tangent \( \tan \delta \) of the substrate is set to zero). A simple algebraic manipulation of Eqs. (20) and (21) yields

\[
Q_{\text{sw}} = Q_{\text{sp}} \left( \frac{e_{\text{rad}}}{1 - e_{\text{rad}}} \right) \quad (22)
\]

Hence, to a good approximation,

\[
Q_{\text{sw}} = Q_{\text{sp}} \left( \frac{e_{\text{rad}}}{1 - e_{\text{rad}}} \right) \quad (23)
\]

Where the radiation efficiency for a horizontal electric dipole may be written as

\[
e_{\text{rad}} = \frac{P_{\text{rad}}}{P_{\text{rad}} + P_{\text{sw}}} \quad (24)
\]

where \( P_{\text{rad}} \) is the power radiated into space by a unit-strength horizontal electric dipole on the lossless substrate, and \( P_{\text{sw}} \) is the power radiated into surface waves by the dipole.

The space wave \( Q \) factor is then determined from the standard formula for \( Q \)

\[
Q_{\text{sw}} = \omega_0 \frac{P_{\text{rad}}}{P_{\text{sw}}} \quad (25)
\]
Microstrip Lines for the Realization of Patch Antennas

Where \( U_s \) is the energy stored inside the cavity and \( P_{sp} \) is the power radiated into space by the patch current. \( U_s \), for the dominant mode is found to be

\[
U_s = \frac{h}{4\mu} w_{ef} L_{ef}
\]  

(26)

To calculate the power radiated into space, the patch is first replaced by an equivalent dipole that has the same dipole moment \( m_{eq} \), defined from

\[
m_{eq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} J_{\omega}(x',y') dx' dy' = \frac{2}{\pi} (w_{ef} L_{ef})
\]

(27)

The space-wave power radiated by the patch to be is given as

\[
P_{sp} = \rho m_{eq}^2 \rho_{sp}^{bed}
\]

(28)

After a lot of computations, \( \rho \) is derived to be

\[
p = 1 + \frac{qa}{\alpha^2} + 2\alpha_2 \left( \frac{3}{360} \right) (k_{c0} w_{ef})^2
\]

\[
+ \frac{c_2}{\alpha_2} \left( \frac{k_{c0} L_{ef}}{k_{c0} w_{ef}} \right)^2 + \alpha_2 c_2 \left( \frac{2}{30} \right) (k_{c0} w_{ef})^2
\]

\[
\times (k_{c0} L_{ef})^2
\]

(29)

where \( a_2 = -0.16605, a_3 = 0.00761, c_2 = -0.0914153 \)

After substitution and some simplification, the final result for the space wave quality factor is

\[
Q_w = \frac{3}{16} \left( \frac{e_x}{p c^2} \right) \left( \frac{b_{ef}}{W_{ef}} \right) \left( \frac{1}{h \sigma} \right)
\]

(30)

**Input Impedance**

The input impedance is calculated from an average voltage, obtained by integrating over the probe, as [6]

\[
Z_m = V_{avg} = \frac{\hbar E_{max}(x_c, y_c)}{I_0} = \frac{\hbar(J_x, E_0)}{I_0}
\]

(31)

which yields

\[
Z_m = -j\omega \mu \hbar \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \left( \frac{J_x}{\sigma_m} \right) \left( \frac{1}{k_x^2 - k_m^2} \right)
\]

(32)

For the rectangular patch, Eq. (32) then becomes (setting \( L_{ef} = \text{lamp} \))

\[
Z_m = -j\omega \mu \hbar \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\cos \left( \frac{m\pi x}{L_{ef}} \right) \cos \left( \frac{n\pi y}{w_{ef}} \right) \sin \left( \frac{m\pi x}{L_{ef}} \right)}{4 w_{ef} (k_x^2 - k_m^2)}
\]

(33)

where the notation \( \delta_{m0} \) denotes the Kronecker delta (1 if \( m=0 \) and 0 otherwise).

If a narrow frequency region around \( \omega_{10} \), the dominant rectangular patch mode resonance, is considered, the impedances for all other RLC circuits can be lumped into a single inductance, which models the stored energy in these other modes. The simple network model shown in Fig. 2 is then obtained.

A simple circuit analysis allows the input impedance of the simple model in Fig. 2 to be written near resonance as

\[
Z_m = jX_f + \frac{R}{1 + j2Q (f_f - 1)}
\]

(34)

where \( X_f = \omega L_{ef} \) and the frequency ratio \( f_f \) is defined as

\[
f_f = \frac{f}{f_0}
\]

(35)

**Figure 2** Simplified Network model valid when \( f_m \) is well separated from all other resonant frequencies.

The input resistance \( R \) is then determined from Eq. (31) as

\[
R_m = \frac{\omega \mu \hbar}{2 \left( \frac{L_{ef}}{k_x^2 l_c} \right)^2} \left( \frac{1}{w_{ef} L_{ef}} \right) \cos \left( \frac{\pi x_{ef}}{L_{ef}} \right)
\]

(36)

Using the relation \( k_{c0} L_{ef} = \pi \), valid at resonance, this simplifies to

\[
R_m = \frac{4 \pi}{\omega \mu \hbar} \left( \frac{l_c}{w_{ef}} \right)^2 \left( \frac{h}{\sigma_0} \right)^2 \cos \left( \frac{\pi x_{ef}}{L_{ef}} \right)
\]

(37)

**Bandwidth**

The bandwidth (BW) of the patch may be calculated by assuming the patch is matched to an incoming
transmission line with characteristic impedance $z_0 = R$ at the resonance frequency (neglecting the feed inductance), and calculating the lower and upper frequency limits $f_1$ and $f_2$ at which the standing wave ratio (SWR) is some specified value. The bandwidth is thus defined as

$$BW = \frac{f_2 - f_1}{f_r} \times 100\%$$  \hspace{1cm} (38)

A simple calculation yields the result

$$BW = \frac{SWR - 1}{Q \sqrt{SWR}} \times 100\%$$  \hspace{1cm} (39)

In this paper the specific choice of $SWR = 2.0$ will be adopted, so that

$$BW = \frac{1}{\sqrt{2Q}} \times 100\%$$  \hspace{1cm} (40)

Substituting for $Q$ and simplifying results in:

$$BW = \frac{1}{\sqrt{2Q} \left[ \frac{L_{ef}}{4} \right]^2 \left[ \frac{R_{ref} L_{ef}}{4} \right]^2 \left[ \frac{R_{ref} W_{ef}}{4} \right]^2}$$  \hspace{1cm} (41)

Radiation Efficiency

The radiation efficiency, $\varepsilon_r$, of the patch is defined as the power radiated into space divided by the total power. In terms of the $Q$ factors [9],

$$\varepsilon_r = \frac{Q}{Q_w}$$  \hspace{1cm} (42)

Substituting $Q$ and $Q_{ef}$, and simplifying it results

$$\varepsilon_r = \frac{1 + e_{\text{had}}}{1 + e_{\text{had}} \left[ \frac{R_{ref} L_{ef}}{4} \right]^2 \left[ \frac{R_{ref} W_{ef}}{4} \right]^2}$$  \hspace{1cm} (43)

Design Procedures

Based on the simplified formulation that has been described, a design procedure is outlined which leads to practical designs of thick rectangular microstrip patch antennas. The procedure is as follows:

1. For an efficient radiator, a practical width that leads to good radiation efficiencies is

$$w = \frac{1}{2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \sqrt{\frac{2}{\varepsilon_r + 1}}$$  \hspace{1cm} (44)

2. Determine the effective dielectric constant of the microstrip antenna using Eq. (15).

3. Once $W$ is found using (44), determine the extension of the length $\Delta L$ using Eq. (14).

4. The actual length of the patch is determined as

$$L = \frac{1}{2} \sqrt{\frac{\mu_o}{\varepsilon_o}} \frac{\varepsilon_r}{2} \Delta L$$  \hspace{1cm} (45)

5. The feed position $X_o$ can now be found solved from:

$$X_o = \frac{L_{ef} \cos \left( \frac{R_{ref} L_{ef}}{4} \right)}{4 \pi L_{ef}} \left( \frac{\varepsilon_r}{\varepsilon_o} \right)$$  \hspace{1cm} (46)

RESULTS

In this section of the paper, efficient Matlab programs are developed for the analysis of thick rectangular probe fed patch antenna. All the Matlab programs are developed using the method presented in the previous section. The results computed by the Matlab programs are compared with measured and numerical results taken from different published materials [3]. Because of this, the comparison is made only at few points.

Figure 3a Radiation pattern plotted by the Matlab program for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_o=2.33$. 
Figure 3b Radiation pattern for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_r=2.33$ measured in [3].

Figure 4 Directivity (dB) versus substrate thickness plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, and $\varepsilon_r=2.2$.

Figure 5 Normalized frequency plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, and $\varepsilon_r=2.2$.

Figure 6 Input resistance Versus Substrate thickness plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$ and $\varepsilon_r=2.2$.

Figure 7 Input resistance versus frequency plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_r=2.33$.

Figure 8 Input resistance and reactance versus frequency for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_r=2.33$ measured in [3].
Mihret Tesfaye and Mohammed Abdo

Figure 9 Probe reactance versus substrate thickness plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, and $\varepsilon_r=2.33$

Figure 10 Probe reactance versus frequency plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_r=2.33$

Figure 11 Bandwidth versus substrate thickness plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, and $\varepsilon_r=2.33$

Figure 12 Radiation efficiency versus substrate thickness plot for $L=1.1\text{cm}$, $W=1.7\text{cm}$, $h=0.3175$ and $\varepsilon_r=2.33$

Radiation Pattern: As can be seen from Fig. 3a and Fig. 3b, the radiation pattern plotted in this paper is in good agreement with that of the measured radiation pattern given in [3].

Directivity: Here the directivity can be computed for different values but there is no measured data available, in literature, to compare with. However, the result is in good agreement with values which can be computed for thin microstrip antennas.

Resonance Frequency: The resonance frequency computed by the Matlab program and measured results taken from [3] are compared in Tables 1 and 2 below. In the computation of Table 1, the formula in Eq. (13b) uses $\omega_{ef}$ in calculating $L_{ef}$ in Eq. (15). As seen from the table, the formula is more accurate for $h < 0.1\lambda_0$.

Table 1: Comparison of resonance frequency with measured value in [3].

<table>
<thead>
<tr>
<th>$W$</th>
<th>$L$</th>
<th>$f_{d,\text{measured}}$ (GHz)</th>
<th>$f_{d,\text{computed}}$ (GHz)</th>
<th>$h/\lambda_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7</td>
<td>3.8</td>
<td>2.31</td>
<td>2.3566</td>
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</tr>
<tr>
<td>4.55</td>
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<td>2.89</td>
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</tr>
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<td>4.24</td>
<td>4.1813</td>
<td>0.0533</td>
</tr>
<tr>
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<td>1.3</td>
<td>5.84</td>
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</tr>
<tr>
<td>1.7</td>
<td>1.1</td>
<td>6.8</td>
<td>6.3822</td>
<td>0.0945</td>
</tr>
</tbody>
</table>

In Table 2, the formula in Eq. (13a) makes use of $\varepsilon_r$ in calculating $L_{ef}$ in Eq. (15). This formula is more accurate for $h > 0.1\lambda_0$. 

Microstrip Lines for the Realization of Patch Antennas

Table 2: Comparison of resonant frequency with measured results in [3].

<table>
<thead>
<tr>
<th>W</th>
<th>L</th>
<th>Measured f_c (GHz)</th>
<th>Computed f_c (GHz)</th>
<th>h/\lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7</td>
<td>1.1</td>
<td>6.8</td>
<td>6.8974</td>
<td>0.0945</td>
</tr>
<tr>
<td>1.4</td>
<td>0.9</td>
<td>7.7</td>
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<td>1.2</td>
<td>0.8</td>
<td>8.27</td>
<td>8.4800</td>
<td>0.1300</td>
</tr>
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<td>1.05</td>
<td>0.7</td>
<td>9.14</td>
<td>9.1528</td>
<td>0.1486</td>
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<tr>
<td>0.9</td>
<td>0.6</td>
<td>10.25</td>
<td>9.9133</td>
<td>0.1733</td>
</tr>
</tbody>
</table>

In general, as can be observed from the table, accurate result is achieved in both ways. However, it would be more accurate if we use the first formula (Eq. (13a)) for thin patches (h<0.1\lambda) and the second one (Eq. (13b)) for thicker patches (h>0.1\lambda). In our case, the second formula is used for electrically thick rectangular probe fed patch antennas. The normalized plot is shown in Fig. 5, which is in a very good agreement with the measured data presented in [3] which are plotted together in the same figure as astrix (*).

Input impedance: Here the input resistance is remarkably in good agreement with the numerical result in [8]. As can be seen in Table 3 below, the result of the program is better than that of the Carver's cavity method and it is closer to the Moment Method (MMm).

Table 3: Comparison of input resistance with Carver and Moment Method in [8]

<table>
<thead>
<tr>
<th>W</th>
<th>Carver</th>
<th>MMm</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f_r(GHz)</td>
<td>R_in</td>
<td>f_r(GHz)</td>
</tr>
<tr>
<td>4.14</td>
<td>2.392</td>
<td>2.27</td>
<td>397</td>
</tr>
<tr>
<td>6.858</td>
<td>2.204</td>
<td>108</td>
<td>2.238</td>
</tr>
<tr>
<td>10.8</td>
<td>2.18</td>
<td>53</td>
<td>2.21</td>
</tr>
</tbody>
</table>

As can be seen in Figs. 6 and 7, the resistance versus substrate thickness plot is in good agreement with the measured plot in Fig. 8 given in [3]. This implies that the method developed really works for thick patch antenna. The reactance versus substrate thickness plot is also in good agreement with the measured plot presented in [3]. This can be observed in Fig. 8 and 9. The agreement implies that the expression developed for the reactance works for thick patch antenna. Fig. 10 shows probe reactance versus frequency plot.

Bandwidth: The bandwidth is given in Table 4 with the corresponding measured values taken from [3]. As may be observed, the result is in a good agreement with that of the measured values given in [3]. It also indicates that a bandwidth up to 20% is achieved by increasing the substrate thickness. Fig. 11 shows the bandwidths versus substrate thickness plot.

Table 4: Comparison of Bandwidth with measured values in [3].

<table>
<thead>
<tr>
<th>W in cm</th>
<th>L in cm</th>
<th>Measured BW in %</th>
<th>Calculated (BW) in %</th>
<th>h/\lambda</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.7</td>
<td>3.8</td>
<td>3.117</td>
<td>3.6900</td>
<td>0.0274</td>
</tr>
<tr>
<td>4.55</td>
<td>3.05</td>
<td>4.083</td>
<td>4.5567</td>
<td>0.0341</td>
</tr>
<tr>
<td>2.95</td>
<td>1.95</td>
<td>6.007</td>
<td>7.0852</td>
<td>0.0533</td>
</tr>
<tr>
<td>1.95</td>
<td>1.3</td>
<td>6.509</td>
<td>10.1668</td>
<td>0.0800</td>
</tr>
<tr>
<td>1.7</td>
<td>1.1</td>
<td>8.699</td>
<td>12.0123</td>
<td>0.0945</td>
</tr>
<tr>
<td>1.4</td>
<td>0.9</td>
<td>11.441</td>
<td>14.1614</td>
<td>0.1156</td>
</tr>
<tr>
<td>1.2</td>
<td>0.8</td>
<td>14.221</td>
<td>14.9213</td>
<td>0.1300</td>
</tr>
<tr>
<td>1.05</td>
<td>0.7</td>
<td>23.85</td>
<td>16.3279</td>
<td>0.1486</td>
</tr>
<tr>
<td>0.9</td>
<td>0.6</td>
<td>2.666</td>
<td>17.9393</td>
<td>0.1733</td>
</tr>
</tbody>
</table>

Radiation Efficiency: Here data couldn't be found in [3] for comparison of the simulation results obtained for the radiation efficiency. However, the result is verifying the theory and it is in a good agreement with the results given in different literature for thin patch antenna [1]. Figure 12 shows the radiation efficiency versus substrate thickness plot.

CONCLUSIONS

The main disadvantage of thin microstrip antennas is their narrow bandwidth. However, recently it is demonstrated that different broadbanding techniques are discovered. The simplest technique to achieve a broadband is using thick substrate patches. Electrically thin microstrip antenna can be conveniently analyzed using the two simple models: transmission line model and ideal cavity model [1]. However, these simple models are not accurate for the thick microstrip antennas. This necessitates developing a simpler analytical analysis method to the electrically thick microstrip antennas.

Here in this paper, a simpler analytical method is used to analyze a thick rectangular probe-fed patch antenna. The theory is presented and based on it.
expressions for radiation pattern, directivity, resonance frequency, quality factor, input impedance, bandwidth and radiation efficiency have been developed and presented. In general, the results are in good agreement with measured and numerical results taken from different literature. The radiation pattern of electrically thick antennas was seen to be very similar to the measured pattern. It is also demonstrated that the resonant frequency and bandwidth have a very good agreement with the measured values. Expressions for resonant frequency, bandwidth and radiation efficiency are the most accurate ones, and it can work well for thickness up to 0.15\(\lambda\) and fairly up to 0.25\(\lambda\). Additional point here is that the theory can also be applied for other patch shapes such as circular with slight modifications.

RECOMMENDATIONS FOR FURTHER STUDY

- As can be observed from the result, the bandwidth for thick substrate is still less than 20\%. Yet a bandwidth up to 35\% can be achieved by using an electrically thick L-shaped probe-fed patch antenna [4], [5]. Therefore, developing simpler method of analysis for an L-shaped probe fed patch antenna can be the next attractive research topic for any one interested.

- Additionally, as mentioned before, the method used here can work well for substrate thickness of up to 0.15\(\lambda\). Hence, a further study can be made to develop a simple and more accurate analysis method that can work for substrate thickness of up to 0.25\(\lambda\).

In summary, the results obtained show that it is possible to achieve a bandwidth up to 20\% only by increasing substrate thickness. It is also demonstrated that the analysis method presented works well for electrically thick rectangular probe fed microstrip patch antenna with substrate thickness of up to 0.15\(\lambda\).

REFERENCES


