VARIABLE MESH STIFFNESS OF SPUR GEAR TEETH USING FINITE ELEMENT METHOD

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ABSTRACT

The objective of this paper is to determine the variable mesh stiffness of spur gear teeth using the finite element method. There are many factors for the variation of stiffness. In this paper only the numbers of contact tooth pairs and applied load are taken into considerations. For accomplishing the objective, a computer program named VMS (Variable Mesh Stiffness) has been developed, which needs only a little effort from the user. The program is capable of determining the plot of variable mesh stiffness and has also the ability of discretization of a single gear tooth, single pair teeth, and double pair teeth. The results of the developed program are found to agree with the analytical results.

INTRODUCTION

Gears transmission plays an important role in modern technology. It transfers both power and motion, and is employed in various kinds of machines and control systems. Systematic studies in gear dynamics first started in the 1920s by Ross and Buckingham. The basic concern in those studies was the prediction of tooth dynamic loads for designing gears at higher speeds. In later studies of gear dynamics the concern ranged widely from calculating dynamic transmission errors to predict gear noise. The predicting of gear tooth dynamic loads and gear noise has always been a major concern in gear design. Nevzat *et al.* [19] have presented a review of literature relating to the gear dynamics. It is widely accepted that the noise generated by a pair of gears is strongly related to the gear pairs' transmission error [21], which is defined by Silichai as the difference between the actual position of the driven gear and the position it would assume if the gear drive were "perfect". As the teeth are loaded, the mesh stiffness of each gear changes throughout the mesh cycle causing variations in displacements along the line of action. In other words variations in gear mesh stiffness are primary source of parametric vibration excitation in geared systems, by virtue of large magnitude change in stiffness that occurs during gear engagement. A gear mesh kinematic simulation will be developed to simulate the meshing action of two spur gears.

Recently, greater emphasis has been played on the creation of analytical tools that may be employed to predict gear noise and to improve the dynamic performance of gears. To do this the characteristics of the mesh stiffness should be known beforehand Methods for determining mesh stiffness variations have been considered by many researchers. The methods could be analytical method [1, 2, 3, 5, 6, 11, 12, 17, and 18], the finite element method [10, 16, and 25], transfer matrix method [4, 23], experimental method [7], or both analytical and experimental methods [3, 9].

The objective of this study is to see the effect of the number of teeth in contact and applied load on mesh stiffness of spur gear teeth using the finite element method. The theory and concepts of the finite element method used in this paper are taken from literatures published recently [8, 14, and 20]. It is based on the linear theory of elasticity, and the type of element used is triangular element.

MATHEMATICAL MODELING OF VARIABLE MESH STIFFNESS

The solution of the unknown displacements will be manipulated to find one of the secondary variables, i.e. Variable Mesh Stiffness.

Numerous mathematical models have been developed for gear analysis [11, 19 and 22]. According to Nevzat, *et al.* [19] tooth compliance model is appropriate for VMS of a spur gear tooth. The assumptions for the model are the following:

1. The gear tooth is assumed to be a non uniform cantilever beam
2. Only the stiffness due to gear tooth is considered. All other elements (gear body, shaft flexibility, bearings, etc.) are assumed to be perfectly rigid.
3. Friction between mating gear teeth is neglected.
4. Contact assumed to occur only along the line of action.

5. Manufacturing errors are neglected.
6. Constant input torque is assumed.

The VMS of spur gear tooth in mesh at particular positions through the mesh cycle can be obtained by rotating both gears (pinion and gear) then creating finite element model in that particular position. The deflections are obtained from FEM analysis. Then these have been projected along the line of action (Fig. 1).

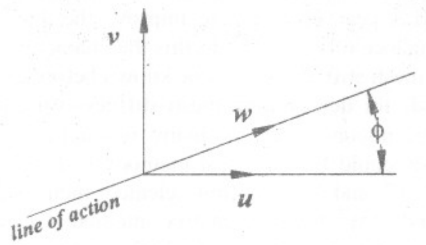


Figure 1 FEM displacement at a particular point on the line of action.

The displacement of the tooth at any point along the line of action (w) can be defined as:

$$w = u\cos\phi + v\sin\phi \qquad (1)$$

where:

u is the displacement in the u direction;
v is the displacement in the v direction;

Since it is a linear elasticity problem, the stiffness at the particular position will be as a function of the deflection and force along the line of action:

$$k = F_n / w \qquad (2)$$

where:

k is stiffness along the line of action, N/mm;
F_n is force along the line of action, N;
w is deflection along the line of action, mm;

In order to develop representative results, a large number of finite element models at different meshing positions of the gear tooth are considered. This process has been continued until one complete tooth meshing cycle is completed.

Meshing Cycle of Tooth Gear

The numbers of pair of teeth in contact vary due to the geometry of the gears. The gears are designed in such a way that a pair of teeth begins contact before the previous gear pairs has ended its contact. This action is characterized by contact ratio, which is equal to the line of contact g_a divided by the base pitch P_b. The contact ratio value is greater than unity (See Figs. 2and 3).

The numbers of pair of teeth in contact vary during the meshing cycle due to the fact that the contact ratio value is greater than unity. As explained in Figs. 2 and 3.

In one complete tooth mesh cycle, the contact starts at point B_1, as shown in Fig. 2, where the addendum circle of the gear intersect the line of actions. The mesh cycle ends at point B_2, where the addendum circle of the pinion intersects the line of action.

The important intersection points shown in Figs. 2 and 3 are defined as:

K1 and K2: Intersection points between the line of action and base circle diameters of the pinion and gear.
B1 and B2: Intersection points between the line of action and outer diameters of the pinion and gear.
A1 and A2: Contact points of the teeth along the line of action (see Fig. 3).

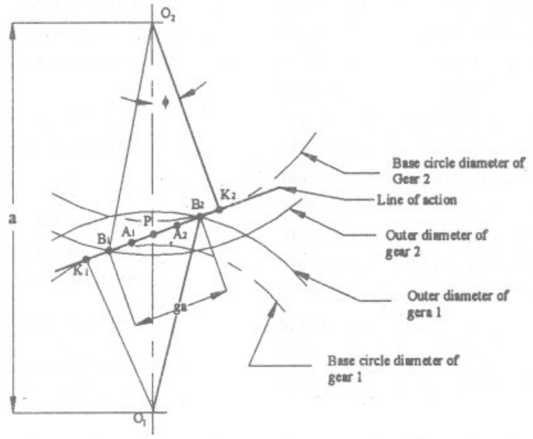


Figure 2 Meshing cycle along the line of action

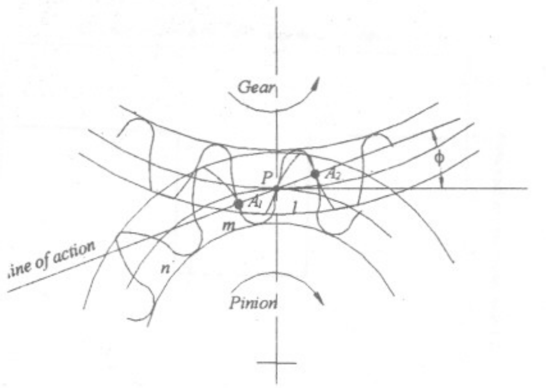


Figure 3 Position of mesh pair teeth

The entire period of meshing cycle can be divided into four regions, as follows:

Region I- Between points B_1 and A_1 when tooth m and tooth l are simultaneously engaged. tooth m being in approach while tooth l is in recess.

Region II- Between points A_1 and P when tooth m alone transmits the entire load and is in approach, while tooth l is disengaged.

Region III- Between points P and A_2 when tooth m is still the only one engaged, but it is in recess.

Region IV- Between points A_2 and B_2 when tooth m and n are engaged, teeth m being in recess while tooth n is in approach.

This is one complete meshing cycle. In general, Region I and Region IV are double teeth pair regions; Region II and Region III are single teeth pair regions. It means that the load will be shared in Regions I and IV, but in Regions II and III the load will be carried only by single teeth pair.

Now it is possible to model the stiffness by considering the two regions, i.e., the single tooth pair and double tooth pair regions.

Normal Mesh Stiffness Modeling for Single Tooth Pair

Figure 4 shows the single tooth pair mesh schematically.

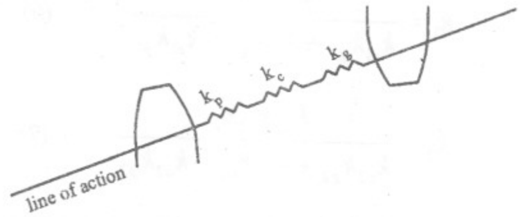


Figure 4 Mesh stiffness along line of action.

As shown in Fig. 4, k_p and k_g are normal mesh stiffness at a particular position of the pinion and gear tooth respectively; and k_c is normal mesh due to Hertizian contact.

k_p and k_g are obtained from Eq. (2); where as k_c can be determined according to Yang *et al.* [26], from:

$$k_c = \frac{\pi E b}{4(1-\nu^2)} \quad (3)$$

where:

E is Modulus of elasticity, N/mm^2
b is Face width of gear tooth, mm
ν is Poisson ratio for steel; 0.3

Hence, the total normal mesh stiffness at a particular position along the line of action will be:

$$\frac{1}{k} = \frac{1}{k_p} + \frac{1}{k_c} + \frac{1}{k_g} \quad (4)$$

$$k = \frac{k_p k_c k_g}{k_c k_g + k_p k_g + k_c k_p} \quad (5)$$

Normal Mesh Stiffness Modeling for Double Tooth Pair

Figure 5 represents schematically double tooth-pair normal mesh.

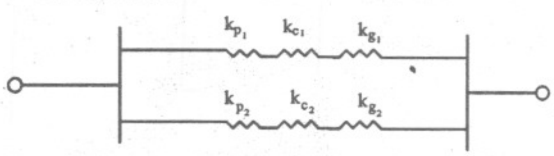


Figure 5 Springs connected in series and parallel.

For the double tooth pair in contact, the normal mesh stiffness is obtained by combining the single pair tooth normal mesh stiffness as springs connected in series as shown in Fig. 5.

$$k_1 = \frac{k_{p1}k_{c1}k_{g1}}{k_{c1}k_{g1} + k_{p1}k_{g1} + k_{c1}k_{p1}} \tag{6}$$

$$k_2 = \frac{k_{p2}k_{c2}k_{g2}}{k_{c2}k_{g2} + k_{p2}k_{g2} + k_{c2}k_{p2}} \tag{7}$$

Combining Eqs. (6) and (7), the total normal mesh stiffness at a particular position along the line of action will be

$$k = k_1 + k_2 \tag{8}$$

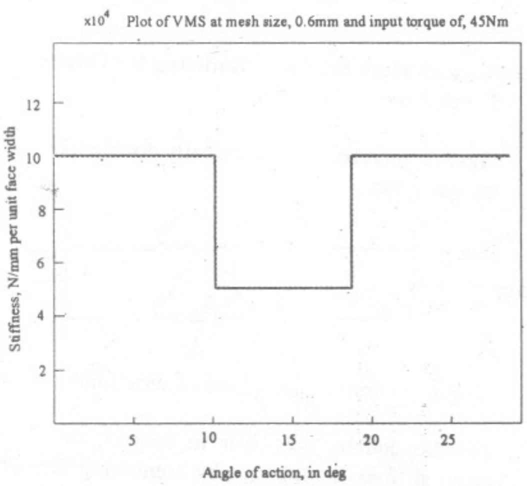
RESULTS AND DISCUSSIONS

This study has considered the variation of stiffness due to the applied (input torque) and the number of contacting pairs.

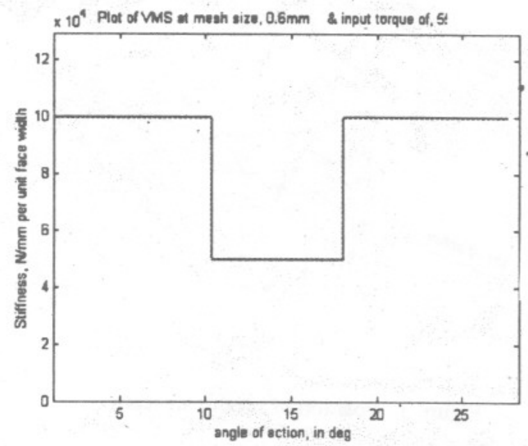
The stiffness of meshed gears depends on the value of contact ratio. At contact ratios greater than 1, there is a possibility of load sharing among teeth. In case of spur gears, there will be period during which only one pair of teeth will be taking the entire load.

Variation due to Input Torque

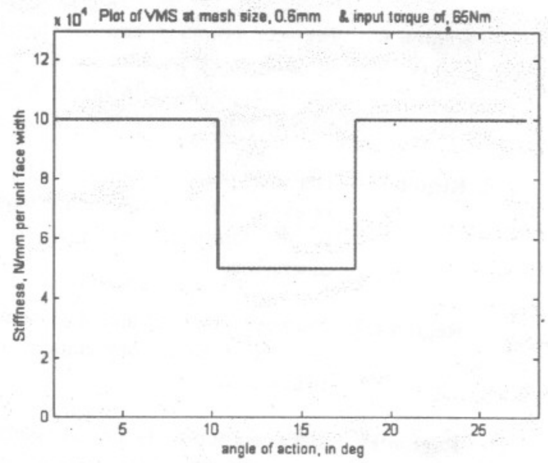
It is assumed that the input torque is constant. So the effect of constant torque values will be considered here so as to see the variation of stiffness over one meshing cycle. 45 Nm, 55 Nm, 65Nm, and 75Nm are taken. The results are shown in Fig. 6.



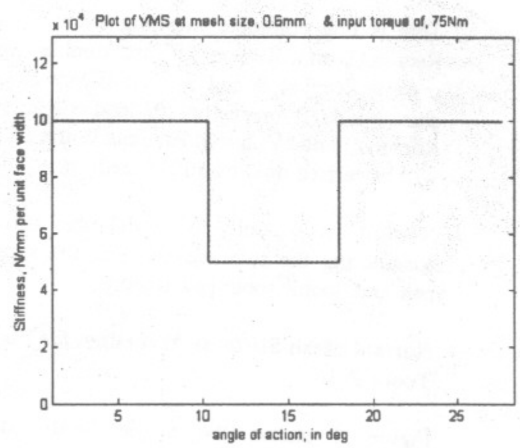
(a)



(b)



(c)



(d)

Figure 6 Simulation results of variable stiffness at input torques

a) 45 Nm, b) 55 Nm, c) 65 Nm and d) 75 Nm

It is clearly observed in Fig. 6 that input torque has no effect on the variation of stiffness. This leads to the conclusion that the relationship between the load and displacement can be taken as linear.

Variation due to Number of Contacting Pairs

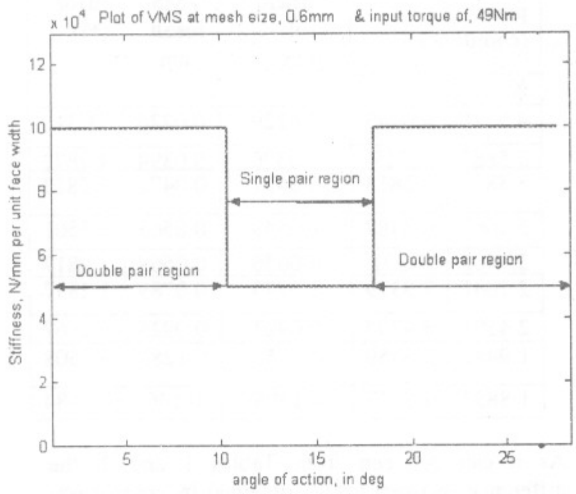


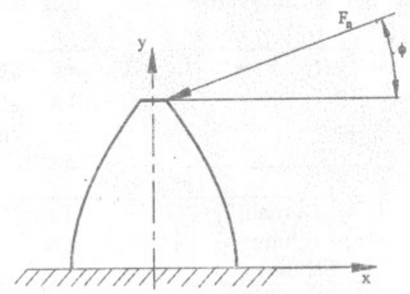
Figure 7 Simulation results of variable stiffness due to variation of contacting pairs

As shown in Fig. 7, the variable stiffness is maximum in the region of double contact pairs and minimum in the region of single contacting pairs. In the region of a double tooth mesh, the stiffness of two teeth arranged in parallel is considered and the stiffness of each tooth is added to the stiffness of the meshing tooth. Hence, the stiffness in the region of a double pair tooth contact is twice bigger than of the region of a single pair tooth contact (see Figs. 4 and 5).

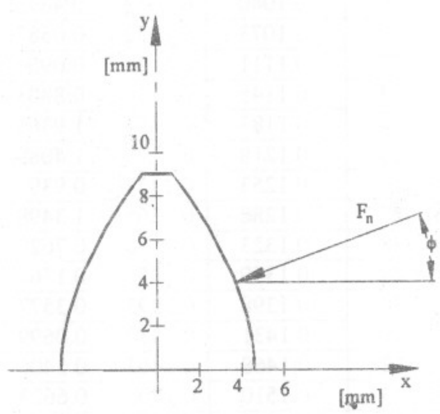
The characteristic of the variable mesh stiffness observed in Fig. 6 and Fig. 7 is similar to that of Leishman *et al.* [15] and Wang [24].

VERIFICATION OF THE COMPUTER PROGRAM

In this section the validity of the program developed is checked by considering two cases. In the first case, the load is applied at the tip of the tooth, and results are obtained for different number of teeth. In the second case, the load is applied at a number of points along the tooth profile, and results are obtained for a fixed number of teeth. Figure 8 shows the line of action of the applied load for both cases.



a) First case



b) Second case

Figure 8 Normal force applied on the gear tooth

ϕ is the pressure angle and F_n is the normal force along the line of action. The program is verified by comparing the results obtained from VMS with those obtained by Cornell [7] for both cases.

The comparisons for both cases are shown in Tables 1and 2, respectively.

Table 1 Deflection results for the first case

$E = 1.9x10^{11} N/m^2$ $\nu = 0.3$ $\phi = 20^0$	$b = 25.4mm$ $F_n = 3x10^4 N$		
No. of teeth	Deflection		
	Cornell[7] result (mm)	VMS result (mm)	Error (%)
17	0.0933	0.0941	0.4695
18	0.0969	0.0981	0.7042
19	0.1002	0.0991	0.6455
20	0.1040	0.1032	0.4695
21	0.1073	0.1072	0.0587
22	0.1111	0.1110	0.0900
23	0.1145	0.1130	0.8803
24	0.1183	0.1150	1.9366
25	0.1218	0.1194	1.4085
26	0.1253	0.1237	0.939
27	0.1288	0.1265	1.3498
28	0.1323	0.1310	0.7629
29	0.1359	0.1356	0.1761
30	0.1398	0.1393	0.3577
31	0.1431	0.1430	0.0699
32	0.1468	0.1463	0.3406
33	0.1510	0.1500	0.6623
34	0.1547	0.1537	0.6464
35	0.1578	0.1566	0.7605
36	0.1616	0.1616	0.0000
37	0.1650	0.1639	0.6455
38	0.1692	0.1675	0.9977
39	0.1704	0.1696	0.4695

Table 2 Deflection results for the second case

$E = 1.9x10^{11} N/m^2$ $\nu = 0.3$ $\phi = 20^0$ $m = 5\,mm$		$b = 25.4mm$ $F_n = 3x10^4 N$ $N = 30$		
Normal force (F_n) application point (coordinate)		Deflection		
		Cornell[7] result (mm)	VMS result (mm)	Error (%)
X	y			
4.1326	4.0043	0.0229	0.0226	1.31
3.5887	5.719	0.0376	0.0394	4.7872
3.3831	6.2813	0.0457	0.0472	3.2823
3.1667	6.8387	0.0549	0.0563	2.5501
2.9402	7.391	0.0656	0.0669	1.9817
2.7041	7.9379	0.0779	0.0789	1.2837
2.4591	8.4794	0.0923	0.0925	0.2167
1.9442	9.5459	0.1359	0.1289	5.1508
1.8852	9.6627	0.1395	0.140	0.3584

As it can be seen from Tables 1 and 2, the difference between results obtained by comparison and VMS results are very small. Due to linear relationship between force and deflection, the stiffness also will have the same margin of error. Therefore, VMS has the potential to model the variation of stiffness of spur gear teeth.

CONCLUSION

From the study carried out, it can be concluded that, the variation in gear mesh stiffness is mainly due to variation in the number of contacting gear tooth pairs. The period of maximum stiffness corresponds to double tooth pair contact, and that of minimum stiffness to single tooth pair contact.

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