THE CONVERGENCE CHARACTERISTIC OF THE FAST DECOUPLED LOAD FLOW TECHNIQUE

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ABSTRACT

A Fortran program is written using matrix sparsity for the inversion of the coefficient matrix and two aspects of the convergence characteristic of the fast decoupled load flow technique are investigated.

These are, effects of the starting values and effects of R/X ratio of transmission lines.

The analysis is based on computational results and not on theoretical study. A threshold level of R/X ratio above which convergence is not reliable is determined.

INTRODUCTION

A load flow problem consists of the calculation of power flows and voltages of a network for specified terminal or busbar conditions. A solution of the system equations for a load flow problem can be carried out by any of the classical numerical methods, such as Newton-Raphson, Direct Inversion, Relaxation and Gauss-Seidel. Difficulties arise from the fact that no one method possesses the desirable features of the others. The main desirable features of a particular method of load flow analysis are usually identified on the basis of:

1. Storage requirements
2. Speed of Convergence
3. Complexity of programming.

In general direct methods [1] converge in few iterations; however, since the methods involve the inversion of the system admittance matrix they make large demands on computer storage space and calculating time when the inversion is carried out. A system with N busbars results in an $N \times N$ admittance matrix. Thus the memory requirements and computing time of direct methods increase as the same power of problem size, thereby limiting their effectiveness to small problems. The iterative methods converge slowly but their memory requirements are minimal and directly proportional to problem size, and the number of iterations for solution increases in proportion with problem size. For large problems only iterative methods have proved practical. This paper looks into the convergence characteristic of one of the iterative methods namely the fast decoupled load flow method. This method [2] has been described as a simple and extremely fast method with possible practical application in on-line and off-line computation. However, in the derivation of the method certain assumptions were involved. The authors of the original paper presented results to justify some of the assumptions but not all of them. This paper looks into two more aspects of the method, namely

(a) the effects of the starting values on the speed of convergence, and

(b) the effect of R/X ratio of transmission lines on the convergence characteristic.

A Fortran program is written using matrix sparsity for the inversion of the coefficient matrix. The iteration scheme used is the $(1\theta, 1V)$ type of approach which is used in most applications; i.e., no acceleration factor is used. The fundamental aim here is to determine a range of starting values for fast convergence and a threshold level of R/X ratio above which convergence becomes unreliable.

THEORY

The current flowing between two nodes, say from node k to node j, I_{kj} is

$$I_{kj} = (\bar{V}_k - \bar{V}_j)\, Y_{kj} \quad (1)$$

where $\bar{V}_j$, $\bar{V}_k$ are the node voltages and Y_{kj} is the admittance between the nodes.

With all nodes connected to each other, the injected current at node k minus the current in the shunt elements of the node is

$$\sum_{j=1}^{n} (\bar{V}_k - \bar{V}_j)\, Y_{kj},\ j \neq k \quad (2)$$

which can be written as

$$\sum_{j=1}^{n} V_k Y_{kj} - \sum_{j=1}^{n} Y_{kj} V_j,\ j \neq k \quad (3)$$

The current in the shunt elements at node k is

$$I_{ksh} = Y_{ksh} \cdot \bar{V}_k$$

The injected current at node k is therefore

$$I_k = \sum_{j=1}^{n} Y_{kj} \bar{V}_k - \sum_{j=1}^{n} Y_{kj} \bar{V}_j + Y_{ksh} \bar{V}_k,\ j \neq k$$

$$= \sum_{j=1}^{n} Y_{kj} \bar{V}_j$$

where $Y_{kk} = \sum_{j=1}^{n} Y_{kj} + Y_{ksh},\ j \neq k \quad (4)$

The complex power at a node is

$$S = P + jQ = VI^* \quad (5)$$

where * denotes complex conjugation.

In polar co-ordinates if $V_k \angle \theta_k$ and $V_m \angle \theta_m$ are the voltages at node k and node m respectively and if the admittance between node k and node m is $Y_{km} = G_{km} + jB_{km}$, then the complex power at node k is

$$\begin{aligned} S_k &= P_k + jQ_k = \bar{V}_k \bar{I}_k^* \\ &= V_k \angle \theta_k \sum_{m \varepsilon k} [V_m \angle \theta_m (G_{km} + jB_{km})]^* \\ &= V_k \sum_{m \varepsilon k} V_m (G_{km} \cos\theta_{km} + B_{km} \cos\theta_{km} + jG_{km} \sin\theta_{km} - jB_{km} \cos\theta_{km}) \end{aligned} \quad (6)$$

Note that $m \varepsilon k$ signifies that busbar m is connected to busbar k including the case $m = k$. Also $\theta_{km} = \theta_k - \theta_m$.

Taking the real part of the complex power to be the active power and the imaginary part to be the reactive power, the real power P_k and the reactive power Q_k at node k are given by the following two expressions:

$$P_k = V_k \sum_{m \varepsilon k} V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \quad (7)$$

$$Q_k = V_k \sum_{m \varepsilon k} V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \quad (8)$$

At each busbar four variables, namely the real power, the reactive power, the voltage magnitude and the phase angle of the nodal voltage can be defined. In load flow calculation three types of busbars are represented.

1. A slack busbar.
2. Load busbars (PQ busbars).
3. Voltage controlled busbars (PV busbars).

At any busbar only two of the above four quantities are specified. The slack busbar provides the additional real and reactive power to supply the transmission losses, since they are unknown until the final solution is obtained.

NEWTON-RAPHSON METHOD

Solving a load flow problem means obtaining the solution to a set of non-linear equations. The Newton-Raphson technique [3], [4], which is one of the main methods available at present for solving load flow problems, uses the first two terms of a Taylor's series expansion. This approach transforms the non-linear load flow equations into a linear form and thus simplifies the problem. The Newton-Raphson method solves iteratively an equation of the type $F(x) = 0$ which is first expanded about the point x, as follows:

$$F(x) = F(x_o) + \frac{1}{1!}\frac{dF(x_o)}{dx}(x - x_o) + \frac{1}{2!}\frac{d^2F(x_o)}{dx^2}(x - x_o)^2 + \ldots\ldots + \text{higher order terms} \ldots\ldots \tag{9}$$

If x_o, the initial starting point, is near the solution point x, then $x - x_o$ will be small and hence second and higher order terms of $x - x_o$ may be negligibe so that $F(x)$ can be approximated as follows:

$$F(x) = 0 \triangleq F(x_o) + \frac{dF(x_o)}{dx}(x - x_o) \tag{10}$$

solving for x, $$x = x_o - \frac{F(x_o)}{F'(x_o)} \tag{11}$$

After writing a Taylor's series expansion for an n variable function and neglecting second and higher order terms, the approximate solution at the $(n + 1)^{th}$ iteration becomes

$$x_i^{n+1} = x_i^n - J^{n^{-1}} F(x_i) \tag{12}$$

where $|J|$ is a matrix of partial derivatives and is called the Jacobian matrix. Eq. 12 enables us to find new values of the variables x_i iteratively until two successive values for each x_i differ only hy a specified tolerance. The elements of the Jacobian matrix can be calculated at the beginning of each iteration or can be calculated after a certain number of iterations. It can also be calculated once and kept constant. When it is calculated at the beginning of each iteration it is equivalent to finding the tangent at each point and as result convergence is quadratic. But when it is calculated once and kept constant it is equivalent to using constant slope.

Application to Load Flow

The most popular and successful formulation of the load flow problem is that in which F is the set of busbar active and reactive power mismatches (the difference between the specified and computed values) and the solution variables are the unknown angle and magnitude of the busbar voltage. The real and reactive power mismatches at busbar k are:

$$\Delta P_k = P_k^{sp} - V_k \sum_{m \in k} V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \tag{13}$$

$$\Delta Q_k = Q_k^{sp} - V_k \sum_{m \in k} V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \tag{14}$$

When Eqs. 13 and 14 are partially differentiated with respect to θ and V, a set of linear equations can be formed giving the relationship between small changes in the voltage magnitude and angle and small changes in real and reactive power.

$$O = \Delta P + \frac{\partial \Delta P}{\partial \theta}\Delta\theta + \frac{\partial \Delta P}{\partial V}\Delta V \tag{15}$$

$$O = \Delta Q + \frac{\partial \Delta Q}{\partial \theta} + \frac{\partial \Delta Q}{\partial V} V \tag{16}$$

where $\Delta P = P_k^{sp} - P_k(\theta_k, V_k)$,

$$\Delta Q = Q_k^{sp} - Q_k(P_k, V_k)$$

re-arranging, it can be written in matrix form

$$\begin{Bmatrix} \Delta P \\ \Delta Q \end{Bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{Bmatrix} \Delta\theta \\ \dfrac{\Delta V}{V} \end{Bmatrix} \quad (17)$$

where H, N, J and L are submatrices of the Jacobian matrix and are given by

$$\left[H\right] = \left[-\frac{\partial \Delta P}{\partial \theta}\right], \left[N\right] = \left[-V\frac{\partial \Delta P}{\partial V}\right]$$

$$\left[J\right] = \left[-\frac{\partial \Delta Q}{\partial \theta}\right] \text{ and } \left[L\right] = \left[-V\frac{\partial \Delta Q}{\partial V}\right]$$

In the Newton-Raphson method Eq. 17 is solved for $\Delta\theta$ and $\frac{\Delta V}{V}$, and hence ΔV to update θ and V at every iteration till the difference between θ_{i+1} and θ_i and V_{i+1} and V_i are within an acceptable tolerance.

It can be shown (refer to Appendix I) that for a small change in the magnitude of the busbar voltage the real power at the busbar does not change appreciably. Similarly for a small change in the phase angle of the busbar voltage the reactive power does not change significantly. Since their elements are very small the coupling submatrices $[N]$ and $[J]$ can be neglected giving two decoupled equations.

$$[\Delta P] = [H]\,[\Delta Q] \quad (18)$$

$$[\Delta Q] = [L]\,\left[\frac{\Delta V}{V}\right] \quad (19)$$

Thus Eqs. 18 and 19 result in the method of decoupling between $MW - \theta$ and $MVAr - V$. Since the actual difference in angle between two nodes in practical power systems is very small, the following assumptions can be made.

$$\cos\theta_{km} \cong 1,\ G_{km}\sin\theta_{km} \ll B_{km} \text{ and}$$

$$\theta_k \ll B_{kk}\ V_k^2\,.$$

As a result of these assumptions the decoupled equations reduce to

$$[\Delta P] = [V \cdot B' \cdot V]\ [\Delta\theta] \quad (20)$$

$$[\Delta Q] = [V \cdot B'' \cdot V]\ \left[\frac{\Delta V}{V}\right] \quad (21)$$

1. Omitting from $[B']$ the representation of those network elements that predominantly affect $MVAr$ flows, i.e., shunt reactances and off-nominal in phase-transformer taps.
2. Omitting from $[B'']$ the shifting effects of phase shifters.
3. Taking the left-hand V terms in Eqs. 20 and 21 on to the left-hand sides of the equations and in Eq. 20 removing the influence of $MVAr$ flows on the calculation of $[\Delta\theta]$ by setting all the right-hand V terms to 1p.u.

Thus the final form for the fast-decoupled load flow equations becomes

$$\left[\frac{\Delta P}{V}\right] = [B']\ [\Delta\theta] \quad (22)$$

$$\left[\frac{\Delta Q}{V}\right] = [B'']\ [\Delta V] \quad (23)$$

COMPUTATION

Consider the power system shown in Fig. 1, whose data are tabulated below:

Bus Code $P - q$	Impedance $R + jx$	Line Charging B
1 − 2	0.04 + j0.12	j0.03
1 − 4	0.02 + j0.06	j0.05
1 − 5	0.08 + j0.24	j0.06
2 − 4	0.02 + j0.06	j0.05
2 − 5	0.04 + j0.12	j0.03
3 − 4	0.06 + j0.18	j0.04
3 − 5	0.02 + j0.06	j0.05
3 − 3	− −	j0.05

Busbar Code	Voltage Magnitude	Generation MW	Generation MVAR	Load MW	Load MVAR
1	−	0	0	25	10
2	−	0	0	40	15
3	−	0	0	50	20
4	1.04	60	35	40	20
5	1.05	0	0	0	0

The problem now is to calculate the magnitude and the angle of busbar voltages and the amount of active and reactive power which have to be generated by the slack busbar to meet the load demand and the power losses on the transmission lines.

First the admittances of the lines are calculated.

Bus Code	Line Admittance	Line Charging B/2
1 − 2	2.5 − j7.5	j0.015
1 − 4	5 − j15	j0.025
1 − 5	1.25 − j3.75	j0.03
2 − 4	5 − j15	j0.025
2 − 5	2.5 − j7.5	j0.015
3 − 4	1.67 − j5.0	j0.02
3 − 5	5 − j15	j0.025

The admittance matrix is as follows:

$$Y_{BUS} = \begin{bmatrix} 8.75-j26.18 & -2.5+j7.5 & 0 & -5+j5 & -1.25+j3.75 \\ -2.5+j7.5 & 10-j29.945 & 0 & -5+j15 & -2.5+j7.5 \\ 0 & 0 & 6.67-j16.995 & -1.67+j5 & -5+j15 \\ -5+j15 & -5+j15 & -1.67+j5 & 11.67-j34.93 & 0 \\ -1.25+j3.75 & -2.5+j7.5 & -5+j15 & 0 & 8.75-j26.18 \end{bmatrix}$$

In the power system shown in Fig. 1, nodes 1,2 and 3 are *PQ* nodes and nodes 4 and 5 are *PV* nodes. Node 5 is taken as the slack busbar.

With a flat voltage start, i.e. with 1p.u. at an angle 0° for all the *PQ* nodes, P_1, P_2, P_3 and P_4 are calculated using Eq. 7. Similarly Q_1, Q_2 and Q_3 are calculated using Eq. 8. Then the active and reactive power mismatches at each node are determined. If the mismatches are within the acceptable range, it means that the magnitudes and angles of the busbar voltages are obtained and the total power loss and total generation of the slack busbar can be calculated. If the mismatches are not within the acceptable range, the magnitudes and angles of the voltages at the nodes are updated and the process is repeated till the mismatches are within the allowable tolerance.

Starting Values

A theoretical study of the convergence of the fast decoupled load flow has been published [5]. This theoretical study gave little numerical result. It is also impractical to perform the convergence test stated in the paper before each load flow study. The intent here is not to analyse theoretically the convergence characteristic but to look into the practical aspect, i.e., to investigate the convergence of the method for a defined range of starting values. Normally starting values for a load flow study are taken as (1 + j0) p.u. for all the *PQ* nodes (load nodes). In this paper the range investigated is from (0.85 + j0) p.u. to (1.5 + j0) p.u. with a step interval of (0.05 + j0)p.u. All the *PQ* nodes are assumed to have the same initial starting values. Individual starting values for individual *PQ* nodes are not investigated because this will create numerous combinations and also it is doubtful whether a meaningful conclusion can be drawn. Finally, a starting value equal to the slack busbar coltage is also investigated. The three systems used for testing the starting values are the I.E.E.E. 14, 30 and 118 busbars with 21, 43 and 200 branches respectively. The results are tabulated in Table 1 and convergence is assumed if mismatch is of the order of 0.1 MW/MAVr.

Table 1. Numerical Results

Starting voltage in per unit	Number of iterations required for convergence		
	14 Busbars	30 Busbars	118 Busbars
0.85	15	12	7
0.90	15	12	6
0.95	14	12	6
1.00	14	12	6
1.05	13	12	6
1.10	13	12	7
1.15	13	12	7
1.20	12	11	7
1.25	12	11	7
1.30	11	11	7
1.35	11	12	7
1.40	10	12	7
1.45	10	12	7
1.50	9	12	7
V (slack)*	13	12	6

*V (slack) is 1.06 for the 14 and 30 busbars and 0.955 for the 118 busbars system.

Table 1 shows that the fast decoupled load flow is relatively insensitive to the starting values, particularly for larger systems. The normal method of a flat voltage start, i.e., $(1 + j0)$ p.u. appears to be the best. In the case of the 14 busbars, 9 iterations are recorded for a starting value of $(1.5 + j0)$ p.u. However this appears to be exceptional rather than normal. For the 118 busbar system the number of iterations for convergence differs little over the considered range of starting value.

R/X Ratio

The convergence of the fast decoupled method has been criticised for being very sensitive to the R/X ratio of transmission lines. For large R/X ratio the method becomes unreliable. For large R/X ratio the method becomes unreliable. However, for system control it is necessary to determine a threshold value of the R/X ratio above which convergence may not be achieved. It is the intention here to determine this threshold value.

One of the proposals [6] to overcome transmission lines with large R/X ratio is to create a fictitious node in the middle of the branch and thus divide the branch into two sections. One branch will have a resistance equal to the resistance of the original branch and a reactance equal to the amount which gives the desired R/X ratio. The other branch will be with zero resistance and a reactance value which when added to the reactance of the other branch gives the reactance of the original branch. The result is such that one branch has zero R/X ratio and the other has a R/X ratio of the desired value.

Table 2 shows the results of computation carried out on the three I.E.E.E. systems already indicated. For each system the heavily loaded (H) mediumly loaded (M) and lightly loaded (L) lines are considered. F represents failure to converge after 100 iterations and D represents divergence.

The results of Table 2 shows that the convergence characteristic is very sensitive to R/X ratio, particularly for larger systems. A further test is performed on the three I.E.E.E. systems with step increment of R/X ratio of 0.1 for the mediumly loaded line and the result is given in Table 3.

Table 3. Convergence Characteristics

$\frac{R}{X}$ ratio	Number of iterations required for convergence		
	14 Busbars	30 Busbars	118 Busbars
0.5	14	12	6
0.6	14	12	7
0.7	14	12	8
0.8	14	12	10
0.9	14	12	13
1.0	14	17	19
1.1	18	28	32
1.2	31	79	F
1.3	F	F	—

A careful study of Table 2 and Table 3 indicates that the threshold level for the R/X ratio is unity. This is in fact smaller than what was originally expected. A random check of the convergence characteristic for R/X ratio less than 0.5 appears to produce no improvement as far as the number of iterations is concerned. It now appears that for the method to produce a reliable and fast solution the R/X ratio should be approximately 0.6. Solution can still be obtained with R/X ratio upto 0.9 but at the expense of computing time. For R/X ratio of unity or greater convergence can no longer be guaranteed.

CONCLUSION

The convergence characteristic of the fast decoupled load flow is unaffected by the starting voltage. Thus the flat voltage start, that is, 1.0 p.u. volt for the load nodes and zero degree for all the angles is a good starting point in the fast decoupled load flow solution. However, the method is very sensitive to the R/X ratio of transmission lines. Irrespective of the size of the system an R/X ratio of unity may lead to a large number of iterations before convergency can be achieved, or sometimes results even in divergence. This is particularly pronounced in large systems. It is recommended that input data to the algorithm should be screened so as to avoid R/X ratio of 0.9 or greater. The creation of a fictitious node as described in the previous section can be employed so as to restore the reliability of the method.

Appendix I

A simple transmission link and its phasor diagram are shown in Fig. 2 and Fig. 3 [7]

From the phasor diagram

$$E^2 = (V + \Delta V)^2 + (\delta V)^2$$

$$= (V + IR\cos\phi + IX\sin\phi)$$

$$+ (IX\cos\phi - IR\sin\phi)^2$$

$$= (V + \frac{RP}{V} + \frac{XQ}{V})^2 + (\frac{XP}{V} - \frac{RQ}{V})^2$$

$$\therefore \Delta V = \frac{RP + XQ}{V}$$

and $\delta V = \dfrac{XP - RQ}{V}$

For most transmission lines $X \gg R$ and hence

$$\delta V = \frac{XP - RQ}{V} \alpha P$$

and

$$\Delta V = \frac{RP + XQ}{V} \alpha Q$$

but for small angles $\delta V = \theta \alpha P$

i.e. the flow of power between two nodes is determined largely by the transmission angle and the flow of reactive power is determined by the scalar voltage difference between two nodes.

In transmission lines in-phase-transformer taps do not change the transmission angle and hence do not affect the flow of active power, and phase shifters do not change the magnitude of the voltage and as a result do not affect the flow of reactive power.

Table 2. Convergence Characteristics

R/X Ratio	Number of iterations required for convergence								
	14 Busbars			30 Busbars			118 Busbars		
	L	M	H	L	M	H	L	M	H
0.5	14	14	14	12	12	12	6	6	7
1.0	14	14	14	26	17	13	27	25	D
1.5	F	F	F	D	F	F	D	D	–

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