# BENDING MOMENT AND SHEAR FORCE COEFFICIENTS FOR TRAPEZOIDAL AND TRIANGULAR LOADS

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## INTRODUCTION

Ethiopian Standard Code of Practice (ESCP 2 – 1983) stipulates rules for the transfer of loads from two-way slabs to the supporting beams that result in generally unsymmetrical trapezoidal and triangular loads on the beams.

Furthermore, in recognition of the fact that it would be too cumbersome to deal with such loads, it gives coefficients to convert the actual loads into equivalent uniform loads.

This technical note gives the derivation of the approximate equations and the maximum percentage errors that may be expected.

## SHEAR FORCE AND POSITIVE MOMENT COEFFICIENTS

Referring to Fig. 1, which shows the shear force and bending moment diagrams due to an unsymmetrical trapezoidal load on a simply supported beam, the exact shear or support reaction and the maximum positive moments may be obtained by elementary theory as follows:

### Exact Coefficients

*Shear Froces*

$$V_1 = R_1 = (1 - \frac{2}{3}\alpha)\frac{\alpha}{2} qL + \frac{2}{3}\beta\frac{\beta}{2} qL$$

$$+ 0.5(1 - \alpha + \beta)(1 - \alpha - \beta)qL$$

$$= \frac{qL}{2}\{1 - \alpha + \frac{(\alpha^2 - \beta^2)}{3}\} \quad (1)$$

$$V_2 = R_2 = \frac{qL}{2}\{1 - \beta + \frac{(\beta^2 - \alpha^2)}{3}\} \quad (2)$$

$$V_3 = \frac{qL}{2}\{1 - 2\alpha + \frac{(\alpha^2 - \beta^2)}{3}\} \quad (3)$$

The terms in brackets, in Eq. (1) and Eq. (2), are the exact end shear force or support reaction coefficients.

*Maximum Positive Bending Moment*

The area of the shear force diagram between the selected points shown in Fig. 1 is calculated first:

$$A_1 = \frac{2}{3}\frac{\alpha qL}{2}\alpha L = \frac{\alpha^2}{3} qL^2 \quad (4)$$

$$A_2 = \alpha L\{1 - 2\alpha + (\frac{\alpha^2 - \beta^2}{3})\}\frac{qL^2}{2}$$

$$= \frac{qL^2}{2}\{\alpha - 2\alpha^2 + \alpha\frac{(\alpha^2 - \beta^2)}{3}\} \quad (5)$$

$$A_3 = \frac{V_3^{2}}{2q}$$

$$= \frac{qL^2}{8}\{1 - 2\alpha + (\frac{\alpha^2 - \beta^2}{3})\}^2 \quad (6)$$

$$M_{max} = A_1 + A_2 + A_3$$

$$= \frac{qL^2}{8}\{\frac{8}{3}\alpha^2 + 4\alpha(1 - 2\alpha + \frac{\alpha^2 - \beta^2}{3})$$

$$+ (1 - 2\alpha + \frac{\alpha^2 - \beta^2}{3})^2\}$$

$$\frac{qL^2}{8}\{1 - \frac{4}{3}\alpha^2 + \frac{2}{3}(\alpha^2 - \beta^2)$$

$$+ \frac{1}{9}(\alpha^2 - \beta^2)^2\} \quad (7)$$

The terms in brackets, in Eq. (7), are the exact positive moment coefficients.

*Approximate Coefficients*

Replacing the unsymmetrical trapezoidal load by a symmetrical one giving the same total load, the maximum load intensity will remain as $q$ and the length of the triangular loads at the ends will be given by $(\frac{\alpha + \beta}{2}) L$

*Shear Coefficients*

Let $\eta = (\frac{\alpha + \beta}{2})$ (8)

The exact end shear or support reaction coefficient for support 1 is obtained from Eq. (1) as:

$$k_v = 1 - \eta + \frac{(\alpha - \beta)}{2}(1 - \frac{4\eta}{3}) \quad (9)$$

Neglecting the last term would give a maximum error of approximately ± 7%. We therefore obtain:

$$k_v \approx 1 - \eta \quad (10)$$

*Maximum Positive Moment Coefficients*

Using only the first two terms of the exact moment coefficient in Eq. (7), but replacing $\alpha$ by $\eta$ as done above for shear,

$$k_m \approx 1 - \frac{4}{3}\eta^2 \quad (11)$$

Simplifying Eq. (7) for direct comparison with Eq. (11),

$$k_m = (1 - \frac{4}{3}\eta^2)\{1 - \frac{(\alpha - \beta)^2}{3}\} \quad (12)$$

It is apparent that the approximate coefficient is very nearly equal to the exact one and always on the conservative side.

The maximum error in use of Eq. (11) is obtained when $(\alpha - \beta) = 0.2$ as + 1.35%.

*Negative moment coefficients*

Using the above approach, it can be shown that the approximate negative moment coefficient may be obtained as:

$$k_m \approx 1 - 2\eta^3 + \eta^3 \quad (13)$$

However, the code recommends the use of Eq. (11) for both positive and negative moment.

This has been recommended in the interest of simplicity and may be justified as follows:

Using Eq. (13) for calculation of support moments in a continuous beam would mean a slight decrease compared to using Eq. (11), but as the simple span moment and, subsequently, the net positive moment should be obtained using Eq. (11), the recommendation of applying the positive moment coefficient simply amounts to an upward redistribution of support moments with an automatic downward correction of span moments.

On the other hand, if the negative moment coefficient were to be applied throughout, the correct supports moments would be obtained but the span moments would be underestimated.

Finally, it may be remarked that, since the values of $k_m$ obtained from Eq. (11) and (13) are quite close, (maximum difference about 7%) the rotational capacity at the support required for the above mentioned redistribution is always available.

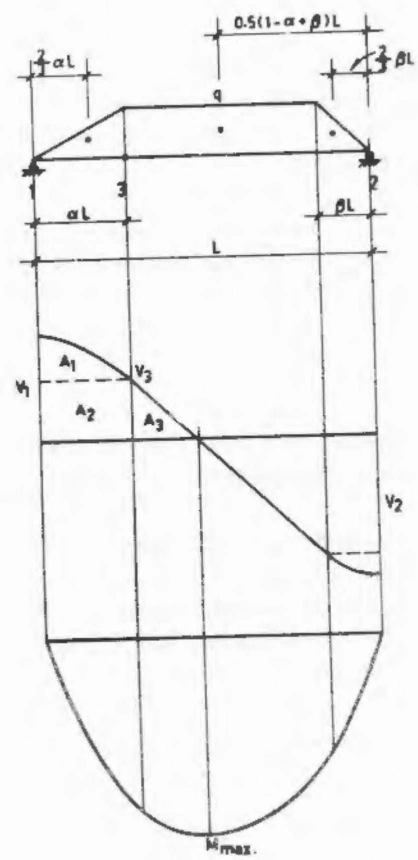


Fig. 1 Shear Force and Bending Moment Diagrams due to Unsymmetrical Trapezoidal Load