A LOW-PHASE NOISE FREQUENCY MULTIPLIER CHAIN

Mohammed Abdo
Electrical Engineering Department
Addis Ababa University

ABSTRACT

In this paper a low phase-noise frequency multiplier chain from 10MHz to 480MHz, built in two stages and intended for use as a base for the synthesis of frequencies up to the far infrared (FIR) region, is presented. Different methods of frequency multiplication were used and the associated circuit diagrams given. To investigate the performance of the chain, its spectral purity was measured. Measurements were made both in frequency and time domain and a set of graphs have been given. It has been verified that this chain is very stable and that it can be used for frequency synthesis up to 360GHz.

INTRODUCTION

The techniques of frequency multiplication are extensively used in many branches of Radio Engineering and applied physics ([3]. In fact the synthesis of extremely accurate frequency up to the far infrared (FIR) region has put new vistas on the metrology of time and frequency. The measurements of time and frequency are basic operations which are influenced by the development of frequency synthesis techniques ([10]. This development has impact not only on the measurement of time and frequency but also on different disciplines such as length standards and metrology, spectroscopy, chronometry, communication and relativistic experiments. The realization of microwave frequency standards (cesium, rubidium) and the subsequent development of new devices in the sub-millimeter (Mg beam, HCN laser) and the infrared region have imposed stringent specifications on frequency synthesis. Consequently, the driving crystal oscillators and the first multiplier stages must exhibit very reliable performance so as to be used in this field.

The realization of the desired accuracy and precision becomes easy and less expensive if there is careful optimization of the design regarding noise. The long term goal of frequency synthesis is to create a simple, reliable and less expensive model for synthesizing frequencies up to FIR and the visible radiation (VR) with good accuracy, stability and reproducibility as that of the driving source. In this way the accuracy and stability of excellent signal sources in the microwave region can be transferred into the FIR regions. Conversely, if in the future, it is discovered that most stable signal sources are in the FIR, then it will be simpler to transfer their stability and accuracy into other frequency spectrum region.

THEORETICAL DESCRIPTION OF FREQUENCY MULTIPLIERS

A. An Oscillator Model

For an ideal oscillator the output signal can be expressed by:

\[ v(t) = V_o \sin (2\pi f_o t + \varphi_o) \]  

where \( V_o \) and \( f_o \) are the amplitudes and the nominal frequency respectively, \( \varphi_o \) is the initial phase angle. For such a signal the power spectral density is:

\[ S_v(f) = P_o \delta (f - f_o) \]  

where \( \delta \) is Dirac's delta

\( P_o \) is the emitted power

The total power is:

\[ P_T = \int_{-\infty}^{\infty} S_v(f) df = \int_{-\infty}^{\infty} P_0 \delta(f - f_o) df \]

\[ = P_0 \]

For real oscillators, perturbations introduced by noise has to be considered. In such a case:

\[ u(t) = (V_0 + \epsilon(t)) \sin (2\pi f_0 t + \phi(t)) \]

where \( \epsilon \) and \( \phi \) are the amplitude and phase fluctuations, respectively

The angular frequency is:

\[ \nu(t) = \nu_0 + \frac{1}{2\pi} \left( \frac{d\varphi}{dt} \right) \]

and if it exists, \( \nu_0 \) represents the angular frequency fluctuation.

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Rewriting (5), \[ \nu(t) = \nu_0 + \Delta \nu(t) \] [5a]

where \[ \Delta \nu(t) = \frac{1}{2\pi} \frac{d\phi}{dt} \] [5b]

is a random process characterizing the frequency noise. For high quality oscillators

\[ |\Delta \nu(t)| << \nu_0 \] and \[ \nu_0 \] and \[ e(t) \] << 1

An important parameter is the instantaneous frequency deviation \( y(t) \)

\[ y(t) = \frac{\Delta \nu(t)}{\nu_0} \]

which remains unaltered in the process of frequency multiplication or division.

The amplitude noise, \( e(t) \), is generally negligible with respect to the phase noise, \( \phi(t) \), or the frequency noise \( \Delta \nu \), so that equation (4) can be written as

\[ u(t) = V_0 \sin(2\pi \nu_0 t + \phi(t)) \] [6]

The frequency stability of the signal in eqn. (6) is usually characterized by the spectral density of the phase fluctuation, \( S_\phi(f) \), in the frequency domain and by the Allan variance, \( \sigma^2_T(\tau) \) in the time domain.

**Definition 1**

The spectral density of the phase noise, \( \phi \)

\[ S_\phi(\psi_0, f) = \int_{-\infty}^{\infty} R(\tau) \exp(-j2\pi f \tau) \, d\tau \]

\[ = 2 \int_{0}^{\infty} R(\tau) \cos(2\pi f \tau) \, d\tau \, (\text{rad}^2/\text{Hz}) \] [7]

where \( R(\tau) = <\phi(t), \phi(t+\tau)> \)

\[ \sigma^2_T(\tau) = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \phi(t) \phi(t+\tau) \, dt \]

It can be shown that:

\[ y_k = \frac{1}{\tau} \int_{t_k}^{t_k+\tau} \phi(\theta) \, d\theta \] [8]

\[ y_k = \frac{\theta(t_k + \tau) - \phi(t_k)}{2 \pi \nu_0 \tau} \] [9]

where the numerator indicates the phase error accumulated over the interval \( t_k \) and \( t_k + \tau \). A measurement of duration \( \tau \) provides one sample \( y_k \). Assuming \( y(t) \) and hence \( y_k \) has a zero mean, the variance is equal to the mean square value of \( y_k \):

\[ \sigma^2_T(\tau) = \langle y_k^2 \rangle \] [10]

In the literature \( \sigma^2_T(\tau) \) is usually indicated by \( \nu_T^2(\tau) \) and it is called the true variance.

The sample variance is defined in several ways; one of which is:

\[ \sigma^2(N, T, \tau) = \frac{1}{N-1} \sum_{j=1}^{N} \left( y_j - \frac{1}{N} \sum_{j=1}^{N} y_j \right)^2 \] [11]

For white noise and for \( T = \tau \), one has

\[ \langle \sigma^2(N, T, \tau) \rangle = \nu^2(\tau) \] [12]

with \( N = 2, T = \tau \) we have the so-called Allan Variance, defined as:

\[ \langle \sigma^2(y, 2, \tau) \rangle = \frac{2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \left( y_i - \frac{1}{N} \sum_{j=1}^{N} y_j \right)^2 \] [13]
B. Characterization of a frequency Multiplier Chain

B.1 The ideal frequency multiplier

Frequency multiplication is generally obtained by utilizing a non linear device which is capable of producing harmonics of the input signal. If such a device is ideal then the input/output characteristics is of the form:

\[ v_o(t) = f(v_i(t)) \]  

(14)

For a sinusoidal input signal, \( V_i = A_0 \sin(2\pi f_0 t + \phi) \) the output is of the form (using power series):

\[ v_o = \sum_{n=-\infty}^{\infty} a_n V_n^2(t) = \sum_{n=0}^{\infty} a_n A_0^n \sin(2\pi f_0 t + \phi) \]  

(15)

The coefficients, \( a_n \), depend on the non-linear device used and the frequency of the input signal. For a signal perturbed with noise:

\[ V_i(t) = A_0 \sin(2\pi f_0 t + \phi(t)) \]  

(16)

The \( N^{th} \) harmonic can be written as:

\[ V_n(t) = A'_n \sin(2\pi f_0 t + N\phi(t)) \]  

(17)

where \( A'_n \) depend on \( A_0 \) and \( a_n \).

Thus the autocorrelation function of the phase noise of the \( N^{th} \) harmonic is

\[ R_{\phi N}(\tau) = \langle N\phi(t), N\phi(t + \tau) \rangle = N^2 K_{\phi N}(\tau) \]  

(18)

and the spectral density of the phase noise is

\[ S_{\phi N}(v_0, f) = N^2 S_{\phi}(v_0, f) \]  

(19)

From equation (19) we observe that the spectral purity of a signal, even under the assumption of ideal devices, is being deteriorated by a factor of \( N^2 \), where \( N \) is the frequency multiplication factor. This assumption is valid as long as \( N^2 \phi^2 << 1 \text{rad}^2 \).

B.2. The practical frequency multiplier

In the real case the frequency multipliers are affected by their own internal noise and like any other non-linear device, have the tendency of AM to PM conversion of the input signal.

The characterization of the noise of a frequency multiplier is normally conducted on the spectral density of the phase-noise of the multiplier chain referred to the input (2). Hence

\[ S_{\phi}(v_0, f) = S_{\phi}^{\text{sig}}(v_0, f) G^2(f) \]

\[ + S_{\phi}^{\text{ch}}(v_0, f) K(f) \]  

(20)

where

\[ v_0 \] the input signal frequency

\[ f \] the spectral frequency referred to \( v_0 \)

\( S_{\phi}^{\text{sig}}(v_0, f) \) Power spectral density of the phase of the input signal

\( S_{\phi}^{\text{ch}}(v_0, f) \) Power spectral density of the amplitude of the input signal

\( G^2(f) \) transfer function of the chain at the Fourier frequency, \( f \)

\( K(f) \) the AM to PM conversion coefficient of the chain at the frequency \( f \)
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The power spectral density of the phase-noise of the chain referred to the frequency \( v_0 \) is given by

\[
S_{\phi}^{ch}(v, f) = \left( \frac{P}{K_T e B_W} \right) \frac{S}{N_\phi}
\]

where \( P \) is the signal power at the desired frequency, \( K_T e B_W \) is the noise power in the circuit, \( T_e \) is the effective noise temperature.

The signal to noise ratio (S/N) deteriorates along the chain as the square of the frequency multiplication factor, \( n \), and for \( n \) sufficiently large, the phase-noise in the side bands reduces the carrier power further such that at a certain frequency \( \nu_{\text{coll}} \), the S/N ratio collapses. The collapse frequency, \( \nu_{\text{coll}} \), can be expressed as

\[
\nu_{\text{coll}} = \nu_0 \left( \frac{2 P}{K_T e B_W} \right) \frac{1}{\alpha} \left( \frac{\phi}{\nu} \right) \frac{\nu}{\nu_{\text{coll}}} \frac{1}{e^{-\phi}}
\]

(21)

where \( \phi = \int_0^\infty S_{\phi}^\omega (v) df = \left( \frac{v}{\nu_{\text{coll}}} \right)^{-1} \)

\( B_w \) is the bandwidth of the filter of the spectrum analyzer.

The spectral purity is guaranteed at every point of the chain if and only if

\[
\Delta \nu_p = 2B \frac{\frac{\phi}{\nu}}{1-e^{-\phi}} \frac{1}{\alpha}
\]

(23)

THE FREQUENCY MULTIPLIER CHAIN FROM 10MHz to 480MHz

The frequency multiplier chain from 10MHz to 480MHz was built in two stages (Fig. 1). The first stage is from 10MHz to 60MHz and the second one is from 60MHz to 480MHz. The chain is driven with a 10MHz quartz oscillator. The collapse frequency of this oscillator is calculated to be 6.7 THz. Before it can be used to drive the chain, the quartz oscillator has to be isolated and also amplified and therefore we need the isolation amplifier.

A. The 10MHz to 60MHz Multiplier Chain

The symmetrical transistor configuration was used to realize this chain. Two chains were built which will later be used to make phase-noise measurements by the method of comparison. The first one is in two stages (3x2) i.e., a tripler in the push-pull configuration followed by a doubler in the push-push configuration (Fig. 2). The second one is built in only one stage (2x1) using a push-push configuration (Fig. 3). The output is tuned on the 6th harmonic and then fed into a 60MHz quartz filter. The quartz filter is not only to filter the undesired harmonic but mainly to reduce the side band of the noise (it has \( \alpha = 40 \text{dB/dec} \) and \( 2B = 20 \text{KHz} \)).

B. THE 60MHz to 480MHz Multiplier Chain

To realize this stage a step-recovery (SRD) circuit is used (Fig. 4). The SRD has the advantage of generating strong harmonics and therefore has a high efficiency of multiplication in only one step. However, one has to keep in mind that the SRD must be used in the last and not in the first stage because it doesn't satisfy the necessary spectral purity required in the 1st stage.
Fig. 1: Block diagram of the frequency multiplier chain from 10MHz to 480MHz

Fig. 2: The frequency multiplier chain from 10MHz to 60MHz in two steps.

- $L_1$: 3x48 turns
- $L_2$: prim = 2x15 turns; sec = 2x8 turns
- $L_3$: prim = 12 turns; sec = 5 turns
- $L_4$: prim = 14 turns; sec = 4 turns
Fig. 3: The frequency multiplier chain from 10MHz to 60MHz in one step.

Toroidal Core Coils in Flexiglass

- \( L_1 \): 3x45 turns
- \( L_2 \): Primary = 18 turns; secondary = 4 turns

Fig. 4: The frequency multiplier chain from 60MHz to 480MHz
To eliminate spurious harmonics an interdigital filter tuned at 480MHz is used. This filter guarantees an output signal with spurious components not more than -70 dBc in order to avoid intermodulation in the following stage. Figures 5 (a) and (b) show respectively the 480 MHz signal before and after the interdigital filter.

THE PHASE NOISE MEASUREMENTS

A. Measurements in the Frequency Domain

What is being done here is to measure $S_{\phi}(\nu_0, f)$. The experimental set up is shown in figure 6 and the measurement is done on the two 60 MHz multiplier chains. The vector voltmeter measures the signal level at the inputs of the mixer (which has to be equal), i.e. a phase displacing apparatus and it is varied until the two signals are in quadrature. The FFT $512/8$ is a low frequency spectrum analyzer and it measures the spectral density $S_{\nu}(f)$ of the noise at various Fourier frequencies.

The sensitivity of the Measurement set up

By sensitivity of the measurement set up we mean the noise in the system i.e. $S_{\phi}(\nu_0, f) = \frac{1}{2K_D} S_{\nu}(f)$

$$S_{\phi}(\nu_0, f) = \frac{1}{2} S_{\nu}(f)$$

where

$S_{\nu}(f)$ the noise spectrum at the output of the mixer (V$^2$/Hz$^2$)

$K_D$ the sensitivity of the mixer (V/rad)

This measurement is done in the absence of the two multiplier chains. The signal at the output of the isolation amplifier is fed into both inputs of the mixer.

On the screen of the spectrum analyzer we read the value of $S_{\nu}(f) = \gamma^2 S_{\phi}(f) / G^2(f)$

$$S_{\phi}(\nu_0, f) = \frac{1}{2} \frac{S_{\nu}(f)}{K_D \gamma^2 G^2(f)}$$

where $S_{\phi}(\nu_0, f)$ is read on the spectrum analyzer

$K_D = \frac{v_1 - v_2}{2} = 0.2$ volt/rad (measured)

$\gamma = 2$ dB (from Table 1)

$G(f) = 80$ dB (fixed)

The value of $S_{\phi}(\nu_0, f)$ evaluated in this way for sufficient number of frequencies is plotted [Fig. 7 (a)]

The phase noise of the isolation amplifier is also measured by applying the signal at the output of the two isolation amplifiers into the two input of the mixer. The spectrum analyzer reads $S_{\phi}(f)$ and $S_{\phi}(\nu_0, f)$ is calculated as indicated in eqn. 27 and then plotted (Fig. 7 b). Finally the phase noise of the two multiplier chains at 60 MHz are measured and the result plotted [Fig. 7 (c)]. Since we are interested in the phase noise referred to the input, the measurement done at 60MHz has to be referred to the input (i.e. to 10MHz). Thus we have to subtract the contribution of the noise due to the multiplication factor $N$, which is $10 \log N^2 = 10 \log 36 = 15.6$ dB. The result is plotted in Fig. 7, (d).

### Table 1

<table>
<thead>
<tr>
<th>Type of noise</th>
<th>$\gamma^2$</th>
</tr>
</thead>
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<tr>
<td>$f^0$ white phase noise</td>
<td>1.6 dB</td>
</tr>
<tr>
<td>$f^{-1}$ flicker phase noise</td>
<td>2 (3dB)</td>
</tr>
<tr>
<td>$f^{-2}$ white frequency noise</td>
<td>3.2 (5dB)</td>
</tr>
<tr>
<td>$f^{-3}$ flicker frequency noise</td>
<td>5 (7dB)</td>
</tr>
</tbody>
</table>

Expressing $S_{\phi}(\nu_0, f)$ in terms of $S_{\nu}(f)$ we have

$$S_{\phi}(\nu_0, f) = \frac{1}{2} \frac{S_{\nu}(f)}{K_D \gamma^2 G^2(f)}$$

where $S_{\nu}(f)$ is read on the spectrum analyzer

$S_{\phi}(\nu_0, f)$ evaluated in this way for sufficient number of frequencies is plotted [Fig. 7 (a)]

$K_D = \frac{v_1 - v_2}{2} = 0.2$ volt/rad (measured)

$\gamma = 2$ dB (from Table 1)

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Fig. 5(a): The 480MHz signal spectrum before the interdigital filter.

Vertical axis: 10dB/div.
Horizontal axis: 60MHz/div.
The 480MHz signal level: 3.6mW/50Ω

Fig. 5(b): The 480MHz signal spectrum after the interdigital filter.

Vertical axis: 10dB/div.
Horizontal axis: 60MHz/div.
The 480MHz signal level: 1.3mW/50Ω

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Fig. 6

Fig. 7

a) Noise of the measurement set up.
b) Noise of the 10MHz isolation amplifiers.
c) Noise of the multipliers at 60MHz.
d) Noise of the 60MHz multipliers referred to 10 MHz.

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Fig. 8: Block diagram for the measurement of the Allan variance of the Quartz Oscillator.

Table 2

<table>
<thead>
<tr>
<th>(Gate Time $T$) (Sec)</th>
<th>1</th>
<th>10</th>
<th>100</th>
<th>1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of samples ($N$)</td>
<td>100</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\sigma_y$ (measured)</td>
<td>$4.73 \times 10^{-11}$</td>
<td>$1.33 \times 10^{-11}$</td>
<td>$0.42 \times 10^{-11}$</td>
<td>$1.33 \times 10^{-11}$</td>
</tr>
<tr>
<td>$\sigma_y$ (from manual)</td>
<td>$0.5 \times 10^{-11}$</td>
<td>$0.5 \times 10^{-11}$</td>
<td>$1 \times 10^{-11}$</td>
<td></td>
</tr>
</tbody>
</table>

Fig. 9: Allan Variance of the Quartz oscillator.
B. Measurement in the Time Domain

In the frequency domain measurement, the accuracy of the spectrum analyzer in the low Fourier frequency (very near the carrier) is not reliable. For this reason time domain measurement is necessary. The stability measurement in the time domain essentially boils down to measuring the Allan Variance, \( \sigma_y^2(T) \).

Since the flicker noise of the frequency multiplier is \( \alpha (1/f) \) while that of the quartz oscillator is \( 1/f \), it means that near the center frequency, the dominant noise is that of the quartz. Consequently, in the time domain it is sufficient to measure the Allan Variance of the quartz oscillator. The measurement set up is given in Fig. 8.

The measurement of the Allan Variance is done with the help of a computer (HP9815) using a program intended for this purpose. The data required by the computer are: the nominal frequency (f MHz), the order of multiplication of the "difference" multiplier \( M = 100 \) and the number of samples \( N \) desired for each gate time \( r \). By taking the average for a certain number of samples, the following results are obtained (Table 2) which are then plotted (Fig. 9).

CONCLUSION

In the circuits built transistors with high gain and low noise are used. Moreover, the currents in the multipliers are made as low as possible to reduce noise. Inspite of these precautions, the phase noise of the multipliers at 50MHz has been 110dB (in the white noise region) above the sensitivity of the measurement set up utilized. Part of this noise is due to the power amplifiers used externally to raise the power level in order to drive the mixer.

It has been verified that this chain is very stable and can be used for frequency synthesis up to 300GHz. To go higher in frequency it is necessary to reduce further the noise in the first stage. One way of doing it may be to realize the 1st stage with a mixer using a Schottky diode followed by a normal doubler.

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