

MODELING OF BLENDING PROCESSES

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ABSTRACT

The blending as to essential properties of raw materials tends to growing importance in chemical process industries recently. That is why the modeling of such processes is an urgent need. It can be shown — on the base of the general model for mechanical macroprocesses — that the modeling of blending processes is possible also in the case of stochastic input and output fluctuations. By means of this model, the influence of input fluctuations on the efficiency of separation processes can be precalculated also.

INTRODUCTION

The aim of a blending process in Chemical Process Industries consists in reducing the input fluctuations of mass flow rate and/or essential raw material properties, e.g., particle size distribution, chemical and mineralogical composition, moisture, ash content, calorific value a.o. Input and output fluctuations (time series) can be characterized by mean value for instance and variance or standard deviation, hence a blending process means the variance σ_x^2 (input) shall be reduced to σ_y^2 (output), so that $\sigma_y^2 / \sigma_x^2 < 1$. In the following the blending of properties of material is considered only.

The difference between a mixing and a blending process lies in the manner of consideration of a mixture: the mixing state is evaluated by drawing samples from different places at a fixed time, the spatial coordinates are variable; the blending efficiency can be evaluated by drawing samples at a fixed position (inlet and outlet) as function of time. For both processes, temporal and/or spatial displacement of relatively small volume elements of the material against each other is necessary. That means, both mixing and blending processes can be modeled on the same base, e.g., a partial differential equation in space and time. Reasons for introducing blending processes in chemical process industries are recently:

1. By the mining, raw materials with relatively high variations of the properties interesting can be delivered only (due to the geological situation of the deposits).
2. By means of blending processes, essential improvement of technological characteristics is possible (recovery of valuable components, efficiency of separation processes, etc.).
3. The final products of chemical process plants including blending processes have smaller fluctuations in their quality (e.g., ceramic raw material, coal for combustion in power plants, glass sand, etc.).

4. Important decrease of the expenses for plant measuring and control is possible, because as rule smaller fluctuations are to control.

Principally, blending processes are carried out in special blending equipment, e.g., in stockpiles (stacking: longitudinal direction, reclaiming orthogonally to the direction of stacking) and blending bunkers (cell types and organ pipe bunker). In a stockpile the output fluctuation can be reduced to about 10% of the input fluctuations (rule of thumb), but it is also possible to use it for mixing purposes (different raw meal components for cement production) and buffering.

Not only in special blending equipment blending processes take place, but also in all apparatuses with a certain mixing effect and storage capacity, e.g., flotation cells or machines, mills (ball, rod mills), mechanical classifiers (rake, screw classifiers) and cyclones.

TRANSFER OF STOCHASTIC PROCESS BY LINEAR SYSTEMS

Very often it is possible to describe or to model the input and output fluctuations of essential material properties as stochastic processes, e.g., stochastic input process $X(t)$ and output process $Y(t)$. Then the aim of modeling consists in finding input-output relationships in a mathematical form, e.g., the relationship between the standard deviations σ_x and σ_y . By means of these quantities we can define the following characteristics for the result of a blending process:

$$\text{blending effect} \quad G = \frac{\sigma_y}{\sigma_x} \cdot 100 \text{ in \%} \quad (1a)$$

$$\begin{aligned} \text{efficiency factor} \quad \eta G &= \left(1 - \frac{\sigma_y}{\sigma_x}\right) \cdot 100 \\ &= 100 - G \text{ in \%} \end{aligned} \quad (1b)$$

Input-Output relationships play an important role not only for the modeling of blending processes, a lot of analogous examples exists in other fields of science and technology; e.g., vibrations of mechanical/electrical systems at random excitation, compensation of the quality fluctuations of an entering gas stream by a gas holder, etc. [1].

Stationary stochastic processes possess — because of their relatively simple mathematical treatment — an especial importance for the modeling of blending processes. Stochastic processes can be characterized by special moments (in general time-dependent functions) similar to the moments of a random variable. In general

use are the expectation function, the dispersion or variance function and covariance and correlation functions respectively [2].

By means of these functions, stochastic processes are described in the time domain, another possibility for that (applied especially in electrical engineering) is the description in the frequency domain: A Fourier transformation of the stochastic process results in the spectral density, which gives the composition of the stochastic process as to single frequencies.

The general relationship between the input and the output of a system (both for deterministic and stochastic processes) is given in the time domain by the Duhamel integral (convolution):

$$Y(t) = \int_{t_0}^t h(t, t') X(t') dt' \quad (2)$$

$h(t, t')$ impulse response (Green function, weighting function)

For instance, the calculation of the output expectation function can be done in the following way; under certain conditions an exchange of calculation of expected value and integration is possible:

$$\begin{aligned} m_y(t) &= E \left[\int_{t_0}^t h(t, t') X(t') dt' \right] \\ &= \int_{t_0}^t h(t, t') m_x(t') dt' \end{aligned} \quad (3)$$

In the same manner, the other moment functions of the output process $Y(t)$ can be calculated (dispersion function, correlation functions). A Fourier transformation of the Duhamel integral leads to the following input-output relationship in the frequency domain:

$$S_{xy}(\omega) = F(j\omega) \cdot S_{xx}(\omega) \quad (4a)$$

$$S_{yy}(\omega) = |F(j\omega)|^2 \cdot S_{xx}(\omega) \quad (4b)$$

$S(\omega)$ spectral density

$F(j\omega)$ frequency function of the transfer system

The area under the spectral density is equal to the variance of the stochastic process, that means, in the frequency domain also, there is the desired relationship between the input and output standard deviation. The

procedure described above may be demonstrated at the example of an ideally mixed vessel (Fig. 1). The impulse response is

$$h(t, t') = \frac{1}{T} \exp\left(-\frac{t-t'}{T}\right) \quad (5a)$$

the frequency function and its absolute value respectively

$$F(j\omega) = \frac{1}{1+j\omega T}; |F(j\omega)|^2 = \frac{1}{1+\omega^2 T^2} \quad (5b)$$

The stationary stochastic input process $X(t)$ shall be centered one (zero mean value) with the variance $D^2 X = \sigma_x^2$ and an exponential type autocorrelation function (a.c.f.):

$$R_{xx}(\tau) = \sigma_x^2 \cdot \exp(-a|\tau|) \quad (5c)$$

the spectral density of this a.c.f. is its Fourier transformation:

$$S_{xx}(\omega) = \sigma_x^2 \frac{2a}{a^2 + \omega^2} \quad (5d)$$

In the time domain, the variance function can be calculated herewith (Fig. 2):

$$\begin{aligned} \sigma_{yy}^2 \left(\frac{t-t_0}{T} \right) &= \int_{t_0}^t \int_{t_0}^t h(t, t_1) h(t, t_2) R_{xx}(t_1, t_2) dt_1 dt_2 \\ &= \frac{\sigma_x^2}{(1-aT)(1+aT)} [1 - aT + (1+aT) \exp(-2 \frac{t-t_0}{T}) - 2 \exp(-(1+aT) \frac{t-t_0}{T})] \end{aligned} \quad (6)$$

With Eqs. (4a), (4b), (5b) and (5d) the spectral density of the output process is calculable, an inverse Fourier transformation gives the a.c.f. of that process (the residue method must be used for the integration [8]):

$$R_{yy}(\tau) = \sigma_x^4 \frac{\exp(-a|\tau|) - aT \cdot \exp(-|\tau|/T)}{1 - a^2 T^2} \quad (7)$$

For $\frac{t-t_0}{T} \rightarrow \infty$ (variance function) or $\tau = 0$ (output a.c.f. or cross correlation function also) the blending effect G and the efficiency factor are available:

$$\frac{\sigma_y^2}{\sigma_x^2} = \frac{1}{1+aT}; \quad G = \frac{100}{\sqrt{1+aT}} \text{ in \%};$$

$$\eta G = 100(1 - \frac{1}{\sqrt{1+aT}}) \text{ in \%} \quad (8)$$

It can be concluded that the greater is the volume of the mixer (or the time constant T) the better is the blending effect and the damping of the input fluctuations; the higher the frequency of the fluctuations, the higher is the a.c.f. parameter a , that means, higher frequencies are better damped than lower ones. The procedure of calculating the blending effect for one vessel ideally mixed can be extended to a cascade of n ideally mixed vessels. In this case, the mean residential time of one stage is T/n , that means, the overall mean residential time is independent on the stage number. From the result for the blending effect (Eq. (9), Fig. 3).

$$G^2 = \frac{1}{(1-a^2 T^2/n^2)^n} + \frac{1}{2^n} \sum_{k=0}^{n-1} \binom{n+k-1}{n} \frac{(1-aT/n)^{n-k} - (1+aT)^{n-k}}{2^k (1-a^2 T^2/n^2)^{n-k}} \quad (9)$$

can be drawn the conclusion, that two mixers with $T/2$ have a smaller blending effect than one mixer with a mean residential time T . Other residential time distribution models can be used for the calculation of blending processes also.

GENERAL MODEL OF MECHANICAL MACROPROCESSES

A model of a mechanical macroprocess has to reflect the essential microprocesses, also their dependence on time, space coordinates and interactions in a volume element, concerning mass transfer of particle classes (size classes density classes, etc.):

- mass transfer by convection (directed motion)
- mass transfer by diffusion (random walk)
- mass transfer due to interactions changed (source or sink).

The mathematical formulation gives the following system coupled of partial differential equations for the n particle classes (mass balance or statistical considerations):

$$\frac{\partial [m_v \mu_i]}{\partial t} = \text{div}[m_v \mu_i \vec{v}_i] - \text{div}[D_i \text{grad}(m_v \mu_i)] - G_i$$

temporal alteration of the i th class mass convection term diffusion term interaction term

m_v particle mass in the volume element
 μ_i mass fraction of the i th class in the volume element

This model (not resolvable without simplification) is widely used for comminution, separation and mixing processes. In the case of modeling blending processes the following simplifications are possible:

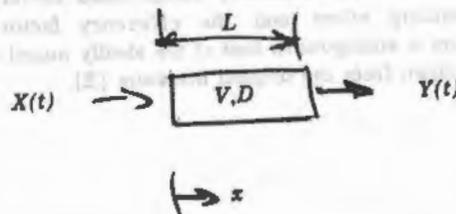
- neither sink nor source ($G_i = 0$)
- the system is regarded as one-dimensional
- v_i and D_i are temporally and spatially constant
- the concentration (mass fraction) of one component is of interest only (tracer substance).

With these simplifications we get the partial differential equation of the so-called dispersion model, analogous to the diffusion or heat conduction equation with convection [4 and 5]:

$$\frac{\partial u}{\partial t} - D \frac{\partial^2 u}{\partial x^2} + v \frac{\partial u}{\partial x} = 0 \quad (11a)$$

$$\frac{\partial u}{\partial(t/T)} - \frac{1}{Bo} \frac{\partial^2 u}{\partial(x/L)^2} + \frac{\partial u}{\partial(x/L)} = 0 \quad (11b)$$

dimensionless, with $Bo = \frac{v \cdot L}{D}$ (Bodenstein number)



The differential equation can be solved (with suited initial and boundary conditions, e.g., according to Danckwerts [1] by means of:

- separation of variables (result: impulse response, time domain)
- Laplace transformation (result: transfer function, complex domain)
- Fourier transformation (frequency function, domain).

The separation of variables results in the impulse response, which corresponds for $x = L$ with the residential time distribution density of the system:

$$h(t/T) = \frac{1}{T} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} \cdot 2 \cdot a_n^2}{a_n^2 + Bo^2/4 + Bo} \exp\left(Bo/2 - \frac{a_n^2 + Bo^2/4}{Bo} t/T\right) \quad (12a)$$

with the eigenvalue equation

$$\text{ctg } a_n = \frac{1}{2} \left(\frac{a_n}{Bo/2} - \frac{Bo/2}{a_n} \right) \quad (12b)$$

In the limit as $Bo \rightarrow 0$ and $Bo \rightarrow \infty$ it can be shown, that the result is the impulse response, transfer function and frequency function of the ideal mixer and the plug flow reactor respectively. Eq. (12a) represents a Fourier series (modified by means of the eigenvalue equation), another representation is possible as series of error functions and their repeated integrals. The first term of this error integral series is the exact solution for a half-enclosed system (used also for modeling mixing processes for relatively short mixing times), the further terms of that series for the closed system originate from reflection at $x = L$ and superposition [4].

In Fig. 4 some results of the calculation of the efficiency factor are represented graphically. Calculations for the instationary state of the system must be done in the time domain, the calculations in the frequency domain are simpler, but possible in the stationary state of the system only. The relatively complicated formulae for the blending effect and the efficiency factor—the procedure is analogous to that of the ideally mixed vessel—can be taken from the original literature [2].

APPLICATION OF THE RESULTS

1. By means of the results mentioned above (curves for the efficiency factor η_G) an approximate determination of the Bodenstein number Bo is possible if the
 - a.c.f. of the input (e.g., the parameter a)
 - time constant T
 - blending effect G (or η_G)

are available by measurements. The advantage of such a method is that no additional tracer is necessary, but a sampling problem has to be solved (sample mass, distance).

2. A further application is possible on separation processes, especially on the model of cross-flow classifier (Fig. 5) [6]. The processes in this type of classifier can be described by Eqs. (10) and (11) respectively, but the convection (that means the settling of particles) is directed downwards. In the stationary state, there is a balance between mixing effects (turbulent diffusion) and segregating ones (settling of particles), resulting in an exponential concentration distribution, the so-called height distribution. The boundary conditions must be formulated so, that the mass flow at $z/H=0$ and $z/H=1$ vanishes, that means reflecting boundaries (Fig. 6). According to the different magnitude of order of the Bodenstein number $Bo_s = v_s \cdot H/D$ we can differ the following states of concentration distribution in the model:

State	Description	Reasons
(a)	sedimentation	1. $D = 0$ (no turbulent diffusion) 2. v_s large (Bo_s, d_p large)
(b)	height distribution (exponential)	both effects in similar order of magnitude
(c)	equidistribution	1. $v_s = 0$ ($\gamma = 0$) 2. $v_s \rightarrow 0$ (very fine particles) 3. $D \rightarrow \infty$ (high intensity turbulence)

On the base of a suited separation model (e.g., pulp division model, Fig. 5), the Tromp distribution curve (or distribution factor curve) can be calculated, in our case:

$$T_i(Bo_s) = \frac{\exp(Bo_s) - \exp(Bo_s(1 - H_{coarse}/H))}{\exp(Bo_s) - 1} = \frac{1 - \exp(-Bo_s \cdot H_{coarse}/H)}{1 - \exp(-Bo_s)} \quad (13)$$

The slope in the middle part of this curve characterizes the sharpness of separation. If the stationary state of the separation process is not yet reached, the sharpness of separation is smaller than for the stationary state (Fig. 7). This conclusion is confirmed by experimental results in hydrocyclones. It can be shown in a similar manner for an one-phase flotation model, that in presence of instationary states (fluctuations of input concentration) the efficiency of separation is decreased.

NOTATION

a	a.c.f. parameter
a_n	eigenvalues
B_o, Bo_s	Bodenstein number (general, for settling)
d_p	particle size
D	diffusion coefficient
$D^2 X$	variance
$E/$	expectation value
$F(j\omega)$	frequency function
G	blending effect
G_i	interaction term (sink or source)
$h(t, t')$	impulse response
H, H_{coarse}	heights in the pulp division model
H_{fine}	
j	imaginary unit
k	summation index
L	length
m_y	mass fraction
$m_x(t)$	input and output expectation function
$m_y(t)$	
n	stage number, summation index
$R_{xx}(\tau)$	input and output a.c.f.
$R_{yy}(\tau)$	
$S_{xx}(\omega)$	spectral densities
$S_{yy}(\omega)$	
$S_{xy}(\omega)$	
T	time constant
t, t_o, t'	time
t_1', t_2'	
u	general concentration
v, \bar{v}, v_s	velocity directed (convection)
$X(t), Y(t)$	stochastic input and output processes
x, y	input and output variables
x	length coordinate
z	height coordinate
γ	particle density
η_G	blending efficiency factor
μ_i	mass fraction of the i^{th} class
	fluid density
σ_x^2, σ_y^2	variances, dispersion
σ_x, σ_y	standard deviations
$\sigma_{yy}^2 \left(\frac{t-t_o}{T} \right)$	dispersion function
τ	correlation time
ω	angular frequency

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