UNIFORM LOAD COEFFICIENTS FOR BEAMS IN REINFORCED CONCRETE TWO - WAY SLABS

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ABSTRACT

While designing reinforced concrete two - way slab systems, triangular or trapezoidal loadings are encountered during transferring the slab loading to the supporting beams. When analysing continuous beams, uniform loading conditions are, as much as possible, preferred because of their simplicity. In this paper, respective equivalent uniformly distributed load coefficients are derived based on the Ethiopian Standard Code of Practice (ESCP2) [1] recommendation. Results are tabulated for all the possible cases of slab support conditions. A numerical example has been presented to illustrate the application of the coefficients in actual design problems.

It has also been tried to verify some of the results by comparing the recommended side ratio of the slab loadings with the yield line analysis of slabs, the derived coefficients with elastic analysis of single span beams, the total panel loading with the total load the four supporting beams carry. Under these three aspects investigation has been made on the recommendation of the new Building Code Standards [11] which is to be launched in the near future.

INTRODUCTION

In reinforced concrete buildings, the slab panels cast integrally with the beams behave in a two - way action as long as the side ratio of a panel (longer side / shorter side) is not greater than two. In such cases the panel is said to be supported on all four sides. The proportion of the slab loading to be shared by each of the supporting beam depends on the edge fixity, the side ratio of the panel and the amount and nature of slab reinforcements: isotropic vs. orthotropic and top vs. bottom reinforcement **[5,8,10]**. Furthermore, for the same reasons, the distribution of the portion of the slab loading to be carried by a particular boundary beam may be triangular or trapezoidal.

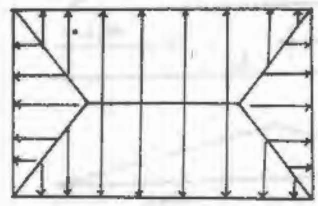


Figure 1 Load dispersion from slab to the four supporting beams

Coefficients have been developed in this paper to convert the resulting triangular or trapezoidal loadings into equivalent uniform loadings. With the help of a short computer program, values for all the possible support conditions as well as for different side ratios have been derived. By using these coefficients, the design engineer can efficiently and more quickly transfer the slab loading to the supporting beams.

Once the equivalent uniform loading from the slab is obtained, the designer needs only to add to this value the beam's own weight and then analyse the beam using his own convenient method.

The coefficients shall be used for moment computation of the beams as recommended in the ESCP2 **[1]**. These coefficients are applicable if the slab panel is bounded by beams on all four sides and the slab loading is uniformly distributed over the panel.

DERIVATION OF EQUATIONS AND COEFFICIENTS

In the Ethiopian Standard code of Practice (ESCP2) **[1]**, it is stated that " Calculation of moments and shears due to trapezoidal and triangular loads may be simplified by using equivalent uniformly distributed loads of intensity equal to the appropriate coefficient k given below times the maximum ordinate of the trapezium or triangle.

The coefficient for moment calculation is given by,

$$k = 1 - 4\alpha^2/3$$

where ,

$$\alpha = (\alpha_1 + \alpha_2)/2 \quad \ldots "$$

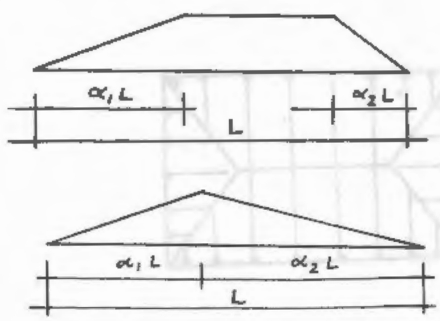


Figure 2 Trapezoidal and triangular loadings

<u>**Case 1:**</u>

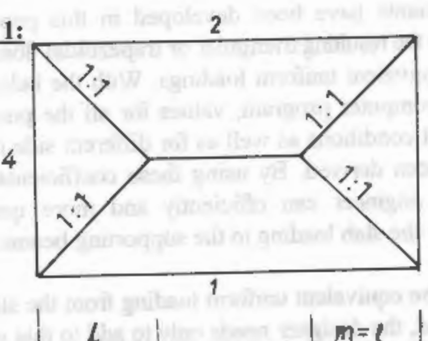


Figure 3 Slab panel simply supported on all four sides

In general

L_y **= the longer side,**
L_x **= the shorter side,**
$r = L_x / L_y$,
ω **= the uniform slab load,**
$l = \alpha_1 L_x$
$m = \alpha_2 L_x$
w_1 , w_2 , w_3 , w_4 **= uniform loads on the four boundary beams supporting the slab.**

Side 1 :

$$l = L_x / 2$$
$$\alpha_1 = \alpha_2 = \alpha = (L_x/2)/L_y = 0.5r$$
$$k = 1 - 4\alpha^2/3 = 1 - r^2/3$$

therefore,

$$w_1 = k(0.5\, L_x\omega) = 0.5(1 - r^2/3)L_x\omega$$
$$k_1 = 0.5\,(1 - r^2/3)$$

Side 2 : this is identical to side 1, and therefore

$$w_2 = w_1$$
$$k_2 = 0.5(1 - r^2/3)$$

Side 3 :

$$\alpha_1 = \alpha_2 = \alpha = (L_x/2)/L_x = 0.5$$
$$k = 1 - 4\alpha^2/3 = 2/3$$

hence,

$$w_3 = k(0.5L_x\omega) = (2/3)(0.5L_x\omega) = (1/3)L_x\omega \ ,$$
$$k_3 = 1/3$$

Side 4 : this is identical to side 3, and therefore

$$w_4 = w_3$$
$$k_4 = 1/3$$

<u>**Case 2:**</u>

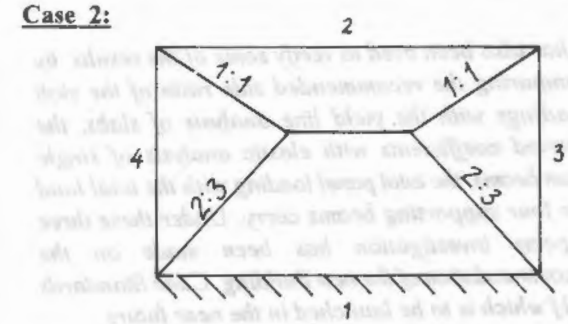


Figure 4 Slab panel continuous along one of the longer sides only.

Side 1 :

$$l = 2L_x/5$$
$$\alpha_1 = \alpha_2 = \alpha = (2L_x/5)/L_y = (2/5)r$$
$$k = 1 - 4\alpha^2/3 = 1 - 16r^2/75$$

therefore,

$$w_1 = k(3L_x\omega/5) = (75 - 16r^2)L_x\omega/125$$
$$k_1 = (75 - 16r^2)/125$$

<u>Side 2 :</u>

$$l = 2L_x/5$$
$$\alpha_1 = \alpha_2 = \alpha = (2L_x/5)/L_y = (2/5)r$$
$$k = 1 - 4\alpha^2/3 = 1 - 16r^2/75$$

therefore,

$$w_2 = k(2L_x\omega/5) = 2(75 - 16r^2)L_x\omega/375$$
$$k_2 = 2(75 - 16r^2)/375$$

Side 3 :

$\alpha_1 = 2/5$, $\alpha_2 = 3/5$, $\alpha = 0.5$

$k = 1 - 4\alpha^2/3 = 2/3$

hence,

$w_3 = k(2/5)L_x\omega = (2/3)(2/5)L_x\omega = (4/15)L_x\omega$

$k_3 = 4/15$

Side 4 : this is identical to side 3, and therefore

$w_4 = w_3$

$k_4 = 1/3$

Case 3:

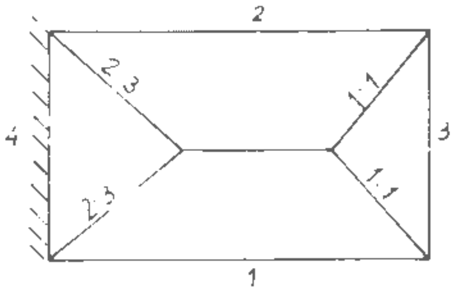


Figure 5 Slab panel continuous along one of the shorter sides only

(a) $L_y/L_x < 1.25$ (b) $L_y/L_x >= 1.25$

For $L_y/L_x < 1.25$,

Side 1 :

$l = 3L_y/5$, $m = 2L_y/5$

$\alpha_1 = (3L_y/5)/L_y = 3/5$

$\alpha_2 = (2L_y/5)/L_y = 2/5$

$\alpha = 1/2$

$k = 1 - 4\alpha^2/3 = 2/3$

therefore,

$w_1 = k(2L_y/5)\omega = (4/(15r))L_x\omega$

$k_1 = 4/(15r)$

Side 2 : this is identical to side 1, and therefore

$w_2 = w_1$

$k_2 = 4/(15r)$

Side 3 :

$\alpha_1 = \alpha_2 = \alpha = (2L_y/5)/L_x = 2/(5r)$,

$k = 1 - 4\alpha^2/3 = 1 - 4(2/(5r))^2/3 = 1 - 16/(75r^2)$

hence,

$w_3 = k(m\ \omega) = ((75r^2 - 16)/(75r^2))(2/(5\,r))L_x\omega$

$= (150r^2 - 32)/(375r^3)L_x\omega$

$k_3 = (150r^2 - 32)/(375r^3)$

Side 4 :

$\alpha_1 = \alpha_2 = \alpha = (2L_y/5)/L_x = 2/(5r)$,

$k = 1 - 4\alpha^2/3 = 1 - 4(2/(5r))^2/3 = 1 - 16/(75r^2$,

hence,

$w_4 = k(\ l\ \omega) = ((75r^2 - 16)/(75r^2))(3/(5r))L_x\omega$

$= (75r^2 - 16)/(125r^3)L_x\omega$

$k_4 = (75r^2 - 16)/(125r^3)$

If $L_y/L_x >= 1.25$, then

Side 1 :

$l = 3L_x/4$, $m = 0.5L_x$

$\alpha_1 = (3L_x/4)/L_y = (3/4)r$

$\alpha_2 = (0.5L_x)/L_y = 0.5r$

$\alpha = 5r/8$

$k = 1 - 4\alpha^2/3 = 1 - 25r^2/48$

therefore,

$w_1 = k(0.5L_x\omega) = (48 - 25r^2)L_x\omega/96$

$k_1 = (48 - 25r^2)/96$

Side 2 : this is identical to side 1, and therefore

$w_2 = w_1$

$k_2 = (48 - 25r^2)/96$

Side 3 :

$\alpha_1 = \alpha_2 = \alpha = 1/2$,

$k = 1 - 4\alpha^2/3 = 2/3$

hence,

$w_3 = k(0.5L_x\omega) = (2/3)(1/2\,L_x\omega) = (1/3)L_x\omega$

$k_3 = 1/3$

Side 4

$\alpha_1 = \alpha_2 = \alpha = 1/2$,

$k = 1 - 4\alpha^2/3 = 2/3$

hence,

$w_4 = k(3/4\,L_x\omega) = (2/3)(3/4)L_x\omega = (1/2)L_x\omega$

$k_4 = 1/2$

For the other cases, similar procedure is employed to arrive at the following results.

Case 4:

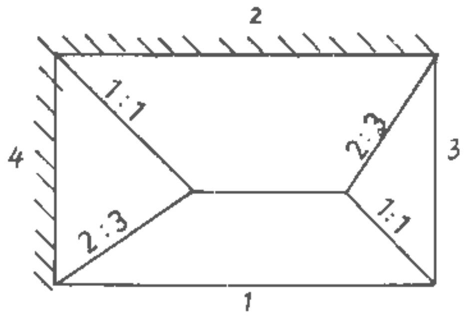


Figure 6 Slab panel continuous along the two adjacent sides only

$k_1 = 2(3 - r^2)/15$
$k_2 = (3 - r^2)/5$
$k_3 = 4/15$
$k_4 = 1/5$

Case 5:

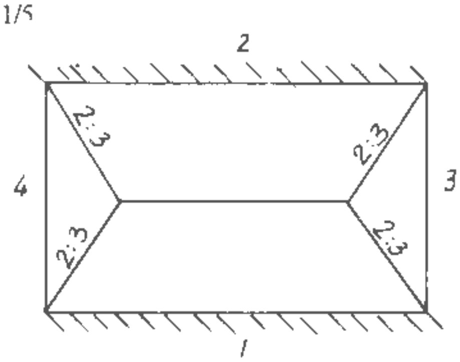


Figure 7 Slab panel continuous along the two longer sides only.

$k_1 = (27 - 4r^2)/54$
$k_2 = (27 - 4r^2)/54$
$k_3 = 2/9$
$k_4 = 2/9$

Case 6:

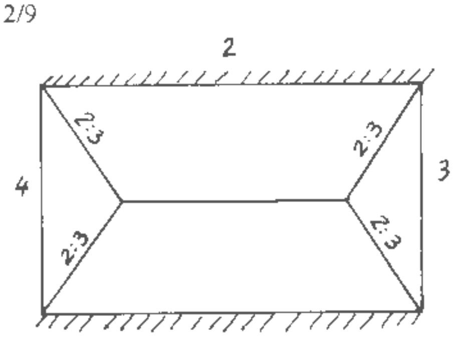


Figure 8 Slab panel continuous along the two shorter sides only

(a) $L_y/L_x < 1.5$ (b) $L_y/L_x >= 1.5$.

If $L_y/L_x <= 1.5$, then

$k_1 = 2/(9r)$
$k_2 = 2/(9r)$
$k_3 = (27r^2 - 4)/(54r^2)$
$k_4 = (27r^2 - 4)/(54r^2)$

If $L_y/L_{xx} >= 1.5$, then

$k_1 = (4 - 3r^2)/8$
$k_2 = (4 - 3r^2)/8$
$k_3 = 1/2$
$k_4 = 1/2$

Case 7:

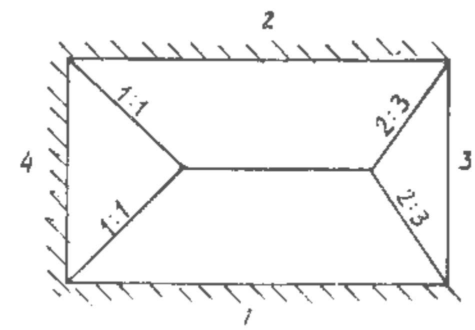


Figure 9 Slab panel simply supported along one of the shorter sides only.

$k_1 = (108 - 25r^2)/216$
$k_2 = (108 - 25r^2)/216$
$k_3 = 2/9$
$k_4 = 1/3$

Case 8:

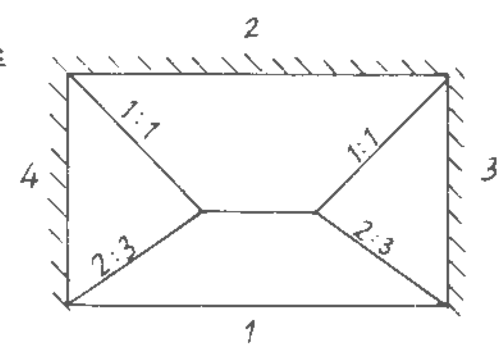


Figure 10 Slab panel simply supported along one of the longer sides only

(a) $L_y/L_x < 1.2$ (b) $L_y/L_x >= 1.2$.

If $L_y/L_x < 1.2$, then

$k_1 = 2/(9r)$
$k_2 = 1/(3r)$
$k_3 = (108r^2 - 25)/(216r^2)$
$k_4 = (108r^2 - 25)/(216r^3)$

If $L_y/L_x >= 1.2$, then

$k_1 = 2(25 - 12r^2)/125$

$k_2 = 3(25 - 12r^2)/125$
$k_2 = 2/5$
$k_4 = 2/5$

Case 9:

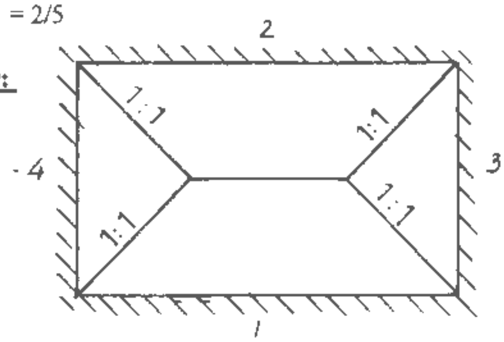


Figure 11 Slab panel continuous along all four sides

$k_1 = 0.5\ (1 - r^2/3)$
$k_2 = 0.5\ (1 - r^2/3)$
$k_2 = 1/3$
$k_4 = 1/3$

Finally, with the help of a short computer program, the various coefficients have been computed and tabulated in Table 1.

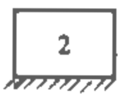
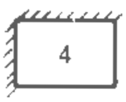
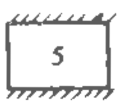
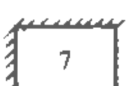
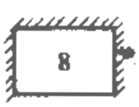
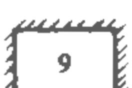
Table 1: Equivalent Uniform Load Coefficients for Moment for Beams Supporting Uniformly Loaded Two - way Slabs.
(For moment based on $k = 1 - 4\ \alpha^2 / 3$)

$$w_i = k_i\ \omega\ L_x$$

In which,

w_i = equivalent uniform load on beam along side i (kN/m) ,
k_i = equivalent uniform load coefficient (Table 1),
ω = uniformly distributed slab loading (kpa),
L_x = short side of the panel (m),
i = the side number of the slab panel (1 to 4).

Table 2: Equivalent Uniform Load Coefficients for Shear for Beams Supporting Uniformly Loaded Two - way Slabs

Support Condition	k_i	L_y / L_x 1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
 1	k_1	0.3333	0.3623	0.3843	0.4014	0.4150	0.4259	0.4349	0.4423	0.4486	0.4538	0.4583
	k_2	0.3333	0.3623	0.3843	0.4014	0.4150	0.4259	0.4349	0.4423	0.4486	0.4538	0.4583
	k_3	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
	k_4	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
 2	k_1	0.4720	0.4942	0.5111	0.5243	0.5347	0.5431	0.5500	0.5557	0.5605	0.5645	0.5680
	k_2	0.3147	0.3295	0.3407	0.3495	0.3565	0.3621	0.3667	0.3705	0.3737	0.3764	0.3787
	k_3	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667
	k_4	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667
 3	k_1	0.2667	0.2933	0.3200	0.3459	0.3671	0.3843	0.3983	0.4099	0.4196	0.4279	0.4349
	k_2	0.2667	0.2933	0.3200	0.3459	0.3671	0.3843	0.3983	0.4099	0.4196	0.4279	0.4349
	k_3	0.3147	0.3264	0.3325	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
	k_4	0.4720	0.4896	0.4988	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
 4	k_1	0.4000	0.4347	0.4611	0.4817	0.4980	0.5111	0.5219	0.5308	0.5383	0.5446	0.5500
	k_2	0.2667	0.2898	0.3074	0.3211	0.3320	0.3407	0.3479	0.3539	0.3588	0.3631	0.3667
	k_3	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
	k_4	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667	0.2667
 5	k_1	0.4259	0.4388	0.4486	0.4562	0.4622	0.4671	0.4711	0.4744	0.4771	0.4795	0.4815
	k_2	0.4259	0.4388	0.4486	0.4562	0.4622	0.4671	0.4711	0.4744	0.4771	0.4795	0.4815
	k_3	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222
	k_4	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222
 6	k_1	0.2222	0.2444	0.2667	0.2889	0.3111	0.3333	0.3535	0.3702	0.3843	0.3961	0.4063
	k_2	0.2222	0.2444	0.2667	0.2889	0.3111	0.3333	0.3535	0.3702	0.3843	0.3961	0.4063
	k_3	0.4259	0.4514	0.4720	0.4873	0.4967	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
	k_4	0.4259	0.4514	0.4720	0.4873	0.4967	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000
 7	k_1	0.3843	0.4043	0.4196	0.4315	0.4409	0.4486	0.4548	0.4600	0.4643	0.4679	0.4711
	k_2	0.3843	0.4043	0.4196	0.4315	0.4409	0.4486	0.4548	0.4600	0.4643	0.4679	0.4711
	k_3	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
	k_4	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222	0.2222
 8	k_1	0.3333	0.3667	0.4000	0.4296	0.4531	0.4720	0.4875	0.5003	0.5111	0.5202	0.5280
	k_2	0.2222	0.2444	0.2667	0.2864	0.3020	0.3147	0.3250	0.3336	0.3407	0.3468	0.3520
	k_3	0.3843	0.3959	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
	k_4	0.3843	0.3959	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000	0.4000
 9	k_1	0.3333	0.3623	0.3843	0.4014	0.4150	0.4259	0.4349	0.4423	0.4486	0.4538	0.4583
	k_2	0.3333	0.3623	0.3843	0.4014	0.4150	0.4259	0.4349	0.4423	0.4486	0.4538	0.4583
	k_3	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333
	k_4	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333	0.3333

(For shear, based on $k = 1 - \alpha$)

$$w_i = k_i \, \omega \, L_x$$

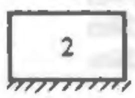
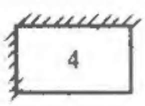
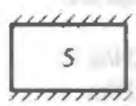
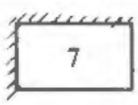
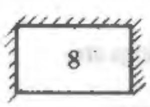
In which,

w_i = equivalent uniform load on beam along side i (kN/m),

k_i = equivalent uniform load coefficient (Table 2),

ω = uniformly distributed slab loading (kpa),

L_x = short side of the panel (m),
i = the side number of the slab panel (1 to 4).

Support Condition		L_y / L_x										
	k_i	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
 1	k_1	0.25	0.27	0.29	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38
	k_2	0.25	0.27	0.29	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38
	k_3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	k_4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
 2	k_1	0.36	0.38	0.40	0.42	0.43	0.44	0.45	0.46	0.47	0.47	0.48
	k_2	0.24	0.25	0.27	0.28	0.29	0.29	0.30	0.31	0.31	0.32	0.32
	k_3	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
	k_4	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27	0.27
 3	k_1	0.20	0.22	0.24	0.26	0.28	0.29	0.30	0.32	0.33	0.34	0.34
	k_2	0.20	0.22	0.24	0.26	0.28	0.29	0.30	0.32	0.33	0.34	0.34
	k_3	0.24	0.23	0.22	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	k_4	0.36	0.35	0.33	0.38	0.38	0.38	0.38	0.38	0.38	0.38	0.38
 4	k_1	0.20	0.22	0.23	0.25	0.26	0.27	0.27	0.28	0.29	0.29	0.30
	k_2	0.30	0.33	0.35	0.37	0.39	0.40	0.41	0.42	0.43	0.44	0.45
	k_3	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20	0.20
	k_4	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
 5	k_1	0.33	0.35	0.36	0.37	0.38	0.39	0.40	0.40	0.41	0.41	0.42
	k_2	0.33	0.35	0.36	0.37	0.38	0.39	0.40	0.40	0.41	0.41	0.42
	k_3	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
	k_4	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
 6	k_1	0.17	0.18	0.20	0.22	0.23	0.25	0.27	0.28	0.29	0.30	0.31
	k_2	0.17	0.18	0.20	0.22	0.23	0.25	0.27	0.28	0.29	0.30	0.31
	k_3	0.33	0.35	0.36	0.37	0.37	0.38	0.38	0.38	0.38	0.38	0.38
	k_4	0.33	0.35	0.36	0.37	0.37	0.38	0.38	0.38	0.38	0.38	0.38
 7	k_1	0.33	0.35	0.36	0.37	0.38	0.39	0.40	0.40	0.41	0.41	0.42
	k_2	0.33	0.35	0.36	0.37	0.38	0.39	0.40	0.40	0.41	0.41	0.42
	k_3	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17
	k_4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
 8	k_1	0.17	0.18	0.20	0.22	0.23	0.24	0.25	0.26	0.27	0.27	0.28
	k_2	0.25	0.27	0.30	0.32	0.34	0.36	0.38	0.39	0.40	0.41	0.42
	k_3	0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
	k_4	0.29	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30	0.30
 9	k_1	0.25	0.27	0.29	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38
	k_2	0.25	0.27	0.29	0.31	0.32	0.33	0.34	0.35	0.36	0.37	0.38
	k_3	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25
	k_4	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0.25

NUMERICAL EXAMPLE

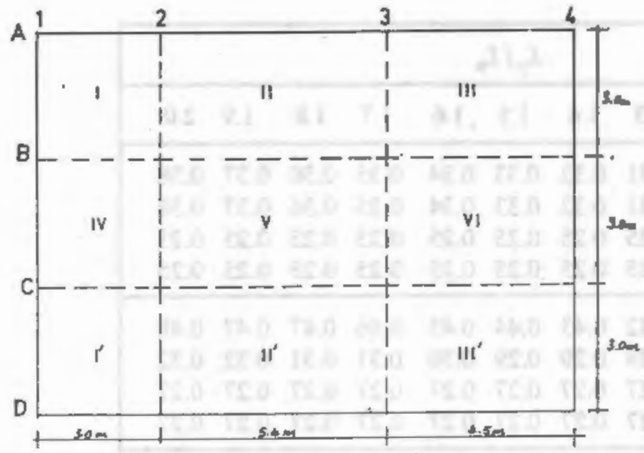


Figure 12 A slab system with three by three panels

Given:

For the slab system shown in Fig 21 above,
Live load = 2.5 kpa,
Slab thickness = 15 cm,
Assume beam own wt. = 2.5 KN/m,

Required: Total equivalent uniform loading on beam along axis *B*.

Solution:

Total uniform slab loading = 2.5 + 0.15(25) = 6.25 kpa

Panel I :

This panel is case 4, for which L_y/L_x = 1, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.2$
$k_2 = 0.3$
$k_3 = 0.2$
$k_4 = 0.3$

and the uniform beam loadings are,

$w_1 = 0.2(6.25)(3) = 3.75$ KN/m
$w_2 = 0.3(6.25)(3) = 5.63$ KN/m
$w_3 = 0.2(6.25)(3) = 3.75$ KN/m
$w_4 = 0.3(6.25)(3) = 5.63$ KN/m

Panel II :

This panel is case 8, for which L_y/L_x = 1.8, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.27$
$k_2 = 0.4$
$k_3 = 0.3$
$k_4 = 0.3$

and the uniform beam loadings are,

$w_1 = 0.27(6.25)(3) = 5.06$ KN/m
$w_2 = 0.4(6.25)(3) = 7.50$ KN/m
$w_3 = 0.3(6.25)(3) = 5.63$ KN/m
$w_4 = 0.3(6.25)(3) = 5.63$ KN/m

Panel III :

This panel is case 4, for which L_y/L_x = 1.5, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.27$
$k_2 = 0.4$
$k_3 = 0.2$
$k_4 = 0.3$

and the uniform beam loadings are,

$w_1 = 0.27(6.25)(3) = 5.06$ KN/m
$w_2 = 0.4(6.25)(3) = 7.50$ KN/m
$w_3 = 0.2(6.25)(3) = 3.75$ KN/m
$w_4 = 0.3(6.25)(3) = 5.63$ KN/m

Panel IV :

This panel is case 7, for which L_y/L_x = 1, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.33$
$k_2 = 0.33$
$k_3 = 0.17$
$k_4 = 0.25$

and the uniform beam loadings are,

$w_1 = 0.33(6.25)(3) = 6.19$ KN/m
$w_2 = 0.33(6.25)(3) = 6.19$ KN/m
$w_3 = 0.17(6.25)(3) = 3.19$ KN/m
$w_4 = 0.25(6.25)(3) = 4.69$ KN/m

Panel V :

This panel is case 9, for which L_y/L'_x = 1.8, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.36$
$k_2 = 0.36$
$k_3 = 0.25$
$k_4 = 0.25$

and the uniform beam loadings are,

$w_1 = 0.36(6.25)(3) = 6.75$ KN/m
$w_2 = 0.36(6.25)(3) = 6.75$ KN/m
$w_3 = 0.25(6.25)(3) = 4.69$ KN/m
$w_4 = 0.25(6.25)(3) = 4.69$ KN/m

Panel VI :

This panel is case 7, for which L_y/L_x = 1.5, the corresponding uniform load coefficients (Table 2) are;

$k_1 = 0.39$
$k_2 = 0.39$
$k_3 = 0.17$
$k_4 = 0.25$

and the uniform beam loadings are,

$w_1 = 0.39(6.25)(3) = 7.31$ KN/m
$w_2 = 0.39(6.25)(3) = 7.31$ KN/m
$w_3 = 0.17(6.25)(3) = 3.19$ KN/m
$w_4 = 0.25(6.25)(3) = 4.69$ KN/m

Accordingly, the loading on beam along axis B is given below and shown in Fig 13.

$F_{1\text{-}2}$ = 2.5(own wt.) + 5.63(from panel I) + 6.19(from panel IV) = 14.32 KN/m

$F_{2\text{-}3}$ = 2.5(own wt.) + 7.50(from panel II) + 6.75(from panel V) = 16.75 KN/m

$F_{3\text{-}4}$ = 2.5(own wt.) + 7.50(from panel III) + 6.75(from panel VI) = 16.75 KN/m

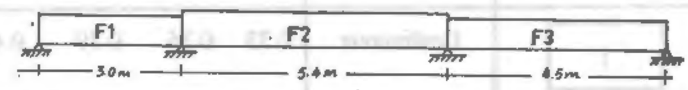


Figure 13 Equivalent uniform loading on beam along axis *B*

THE NEW BUILDING CODE STANDARD

A new Building Code Standard for the structural use of concrete has been prepared and is to be launched within a short time. On the topic of load dispersion from slab to beams, the new code provides a table with coefficients similar to the ones derived in this paper. These coefficients are shown in Table 2 for comparison. According to this new code recommendation,

(1) The design load on beams supporting solid slabs spanning in two directions at right angles supporting uniformly distributed loads may be assessed from the following equation:

$$V_x = \beta_{vx}(g_d + q_d)L_x$$
$$V_y = \beta_{vy}(g_d + q_d)L_x$$

Table 2: Shear Force Coefficients for Uniformly Loaded Rectangular Panels Supported on Four Sides With Provision for Torsion at Corners

Type of panel and location	Edge	β_{vx} for values of L_y/L_x								
		1.0	1.1	1.2	1.3	1.4	1.5	1.75	2.0	β_{vy}
1	Continuous	0.33	0.36	0.39	0.41	0.43	0.45	0. 48	0.50	0.33
2	Continuous	0.36	0.39	0.42	0.44	0.45	0.47	0. 50	0.52	0.36
	Discontinuous	-	-	-	-	-	-	-	-	0.24
3	Continuous	0.36	0.40	0.44	0.47	0.49	0.51	0. 55	0.59	0.36
	Discontinuous	0.24	0.27	0.29	0.31	0.32	0.34	0. 36	0.38	-
4	Continuous	0.40	0.44	0.47	0.50	0.52	0.54	0. 57	0.60	0.40
	Discontinuous	0.26	0.29	0.31	0.33	0.34	0.35	0. 38	0.40	0.26
5	Continuous	0.40	0.43	0.45	0.47	0.48	0.49	0. 52	0.54	-
	Discontinuous	-	-	-	-	-	-	-	-	0.26
6	Continuous	-	-	-	-	-	-	-	-	0.40
	Discontinuous	0.26	0.30	0.33	0.36	0.38	0.40	0.44	0.47	-
7	Continuous	0.45	0.48	0.51	0.53	0.55	0.57	0. 60	0.63	-
	Discontinuous	0.30	0.32	0.34	0.35	0.36	0.37	0. 39	0.41	0.30
8	Continuous	-	-	-	-	-	-	-	-	0.40
	Discontinuous	0.30	0.33	0.36	0.38	0.40	0.42	0. 45	0.48	0.30
9	Discontinuous	0.33	0.36	0.39	0.41	0.43	0.45	0.48	0.50	0.33

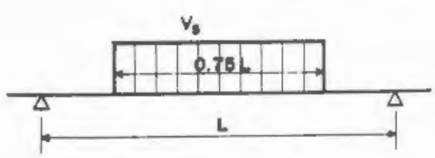


Figure 14 Distribution of Load on a Beam Supporting a Two- Way Spanning Slab

(2) Table 2 gives values of load transfer coefficients. The assumed distribution of the load on a supporting beam is shown in Fig. 14.

Based on this new building code standard recommendation, the loading on Axis B for the example given earlier has been recalculated and the result is shown in Fig.15.

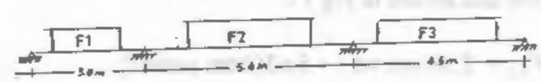


Figure 15 Load on Axis *B* according to the new code EBCS2

RESULT VERIFICATION

The following three points need to be considered in order the results obtained in this paper to be valid. Namely :

1. To what extent is the suggested proportion (i.e. 1:1 and 2:3) of slab loading shared by the supporting beams correct ?

2. **Are the derived equivalent uniform loadings representing the actual situation ? (Or Are the maximum mid span moments and/ or support moments produced by the uniform loadings on the beam similar to the ones produced by the actual triangular or trapezoidal loadings ?)**

3. **Is the total slab loading carried by the four supporting beams of a panel correct ?**

For the first point, the ESCP2 recommendation is based on the yield line pattern as indicated in Reinforced Concrete slab design procedures **[3,6,7,9]**.

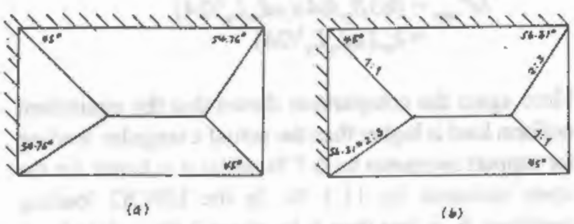


Figure 16 Load dispersion (a) as obtained by the yield line analysis for isotropic slab, (b) as recommended by the ESCP2.

Using the yield line analysis for a rectangular, isotropic slab panel, simply supported along the two adjacent edges and fixed along the other two, Dyaratnam **[3]** has come up with the result shown in Fig.16(a). The suggested slope given in the code, as shown in Fig.16(b), is very close to the yield line result. The reason for the small discrepancy may arise from the fact that the negative moments in slabs being actually higher than the field moments and therefore support reinforcements are normally higher (non-isotropic).

Hence, as recommended, if two adjacent sides have the same fixity, a ratio of 1:1 is to be used, while for different fixity (one side fixed and the adjacent side simply supported), a 2:3 ratio shall be used.

To check whether the equivalent uniform loading gives the same moments at critical locations, the following three panel cases are investigated.

Case 1 Simply supported all round

Sides 1 & 2

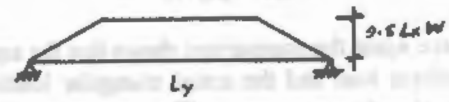


Figure 17 Simply supported beam loaded with trapezoidal loading

Using the actual trapezoidal loading **[2,4]** ,

$$M^{+}_{max} = (0.5 - r^2/6)(\omega L_x L_y^2/8) = k_m(\omega L_x L_y^2/8)$$

Using equivalent uniform loading (Table 1),

$$M^{+}_{max} = k_1(\omega L_x L_y^2/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]** ,

$$M^{+}_{max} = (15\beta_m/16)(\omega L_x L_y^2/8) = k_{n+}(\omega L_x L_y^2/8)$$

k_1 and k_m are equal for all values of r indicating that the equivalent uniform loading does give the same moment at mid span as the actual trapezoidal loading. For this case, the EBCS2 coefficient k_{n+} varies from k_m by about 6%.

Sides 3 & 4,

Using the actual triangular loading **[2,4]** ,

$$M^{+}_{max} = (1/3)(\omega L_x^3/8) = k_m(wL_x^3/8)$$

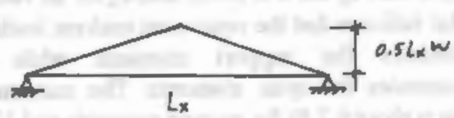


Figure 18 Simply supported beam loaded with triangular loading

Using equivalent uniform loading (Table 1),

$$M^{+}_{max} = k_3(\omega L_x^3/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]** ,

$$M^{+}_{max} = (15\beta_w/16)(\omega L_x L_y^2/8)$$
$$= k_{m+}(\omega L_x L_y^2/8)$$

Here again the comparison shows that the equivalent uniform load and the actual triangular loading have equal mid span moments. The moment coefficient for the EBCS2 loading also has the same variation.

Case 9 Fixed support all round

Sides 1 & 2

Figure 19 Fixed beam loaded with trapezoidal loading

Using the actual trapezoidal loading **[2,4]**,

$$M^{-}_{max} = (0.5 - 0.25r^2 + r^3/16)(\omega L_x L_y^2/12)$$
$$= k_r(\omega L_x L_y^2/12)$$
$$M^{+}_{max} = (0.5 - 0.125r^3)(\omega L_x L_y^2/24)$$
$$= k_m(\omega L_x L_y^2/24)$$

Using equivalent uniform loading (Table 1),

$$M^{-}_{max} = k_1(\omega L_y^2/12)$$
$$M^{+}_{max} = k_1(\omega L_y^2/24)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^{-}_{max} = (117\beta_w/128)(\omega L_x L_y^2/12)$$
$$= k_{r-}(\omega L_x L_y^2/12)$$
$$M^{+}_{max} = (63\beta_w/64)(\omega L_x L_y^2/24)$$
$$= k_{m+}(\omega L_x L_y^2/24)$$

k_1 is higher than k_r but it is lower than k_m for all values of r. This indicates that the equivalent uniform loading overestimates the support moments while it underestimates the span moments. The maximum variation is about 6.7 % for support moments and 11.1 % for span moments, which occurs when r equals 1. In the EBCS2 loading condition, k_{r-} varies from k_r by about 2.5%; while k_{m+} varies from k_m by 12.5% to 1.6% for r equals 1 to 2, respectively.

Sides 3 & 4,

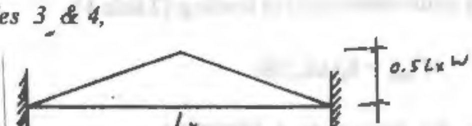


Figure 20 Fixed beam loaded with triangular loading

Using the actual triangular loading **[2,4]**,

$$M^{-}_{max} = (5/16)(\omega L_x^3/12)$$
$$= k_r(\omega L_x^3/12)$$
$$M^{+}_{max} = (3/8)(\omega L_x^3/24)$$
$$= k_m(\omega L_x^3/24)$$

Using equivalent uniform loading (Table 1),

$$M^{-}_{max} = k_3(\omega L_x^3/12)$$
$$M^{+}_{max} = k_3(\omega L_x^3/24)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^{-}_{max} = (117\beta_v/128)(\omega L_x L_y^2/12)$$
$$= k_{r-}(\omega L_x L_y^2/12)$$
$$M^{+}_{max} = (63\beta_v/64)(\omega L_x L_y^2/24)$$
$$= k_{m+}(\omega L_x L_y^2/24)$$

Here again the comparison shows that the equivalent uniform load is higher than the actual triangular loading for support moments by 6.7 % while it is lower for the span moments by 11.1 %. In the EBCS2 loading condition, k_{r-} is less than k_r by about 2.5%; while k_{m+} is less than k_m by 12.5%.

Case 4 Fixed along the two adjacent sides

Side 1,

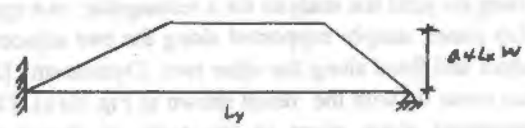


Figure 21 Propped cantilever beam loaded with trapezoidal loading

Using the actual trapezoidal loading **[2,4]**,

$$M^{-}_{max} = (0.4 - 16r^2(220 - 81r + 7.8r^2)/15000)(\omega L_x L_y^2/8)$$
$$= k_r(\omega L_x L_y^2/8)$$

Using equivalent uniform loading (Table 1),

$$M^{-}_{max} = k_1(\omega L_y^2/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^{-}_{max} = (117\beta_w/128)(\omega L_x L_y^2/8)$$
$$= k_{r-}(\omega L_x L_y^2/8)$$

Negative moments are slightly overestimated by the equivalent uniform loading since k_1 is greater than k_t for all values of r, and the maximum variation is 9.6 % (i.e. when $r = 1$). The coefficient K_n for the EBCS2 loading has a 2.4% to 4% variation from k_t.

Side 2,

Using the actual trapezoidal loading **[2,4]**,

$$M^-_{max}=(0.6 - 24r^2(220 - 81r + 7.8r^2)/15000)(\omega L_x L_y^2/8)$$
$$= k_t(\omega L_x L_y^2/8)$$

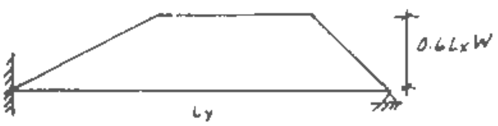


Figure 22 Propped cantilever beam loaded with trapezoidal loading

Using equivalent uniform loading (Table 1),

$$M^-_{max} = k_2(\omega L_x L_y^2/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^-_{max} = (117\beta_{x}/128)(\omega L_x L_y^2/8)$$
$$= k_n(\omega L_x L_y^2/8)$$

Negative moments are slightly overestimated by the equivalent uniform loading since k_2 is greater than k_t for all values of r, and the maximum variation is again 9.6 % (i.e. when $r = 1$). The coefficient K_n for the EBCS2 loading has a 0.1% to 4.6% variation from k_t.

Side 3,

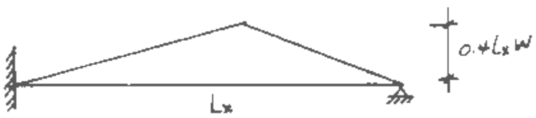


Figure 23 Propped cantilever beam loaded with triangular loading

Using the actual triangular loading **[2,4]**,

$$M^-_{max} = (2282/9375)(\omega L_x^3/8)$$
$$= k_t(\omega L_x^3/8)$$

Using equivalent uniform loading (Table 1),

$$M^-_{max} = k_3(\omega L_x^3/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^-_{max} = (117\beta_y/128)(\omega L_x L_y^2/8)$$
$$= k_n(\omega L_x L_y^2/8)$$

Here again the comparison shows that the equivalent uniform load coefficient k_3 is higher than k_t for the actual triangular loading, overestimating the support moment. The coefficient K_n for the EBCS2 loading has a 2.4% variation from k_t.

Side 4,

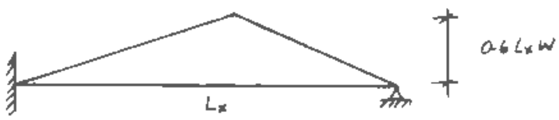


Figure 24 Propped cantilever beam loaded with triangular loading

Using the actual triangular loading **[2,4]**,

$$M^-_{max} = (1141/3125)(\omega L_x^3/8)$$
$$= k_t(\omega L_x^3/8)$$

Using equivalent uniform loading (Table 1),

$$M^-_{max} = k_4(\omega L_x^3/8)$$

Using the new code (EBCS2) recommendation **[2,4,11]**,

$$M^-_{max} = (117\beta_y/128)(\omega L_x L_y^2/8)$$
$$= k_n(\omega L_x L_y^2/8)$$

Comparison, in this case, shows that the equivalent uniform load coefficient k_4 is higher than the k_t value for the actual triangular loading, again overestimating the support moment. The coefficient K_n for the EBCS2 loading has a 0.1% variation from k_t.

Other panel support cases can be similarly investigated. One may conclude that the equivalent uniform loading coefficients derived in this paper overestimate support moments while they underestimates the span moments. Except for very few cases, the new code recommended coefficients produce moments which are closer to the ones produced by the triangular or trapezoidal loadings.

To check whether the total loading carried by the four supporting beams is equal to the total load within a panel, the two values are compared as follows.

Total load within a panel = $\omega L_x L_y$,

According to ESCP2, the four supporting beams carry the following total load (Table 2),

$$\begin{aligned} W &= k_1\,\omega L_x L_y + k_2\,\omega L_x L_y + k_3\,\omega L_x^2 + k_4\,\omega L_x^2 \\ &= (k_1 + k_2 + r\,(k_3 + k_4))\omega L_x L_y \\ &= k_{tot}(\omega) L_x L_y \end{aligned}$$

According to EBCS2, the four supporting beams carry the following total load,

$$\begin{aligned} W &= \beta_1\,\omega L_x L_y + \beta_2\,\omega L_x L_y + \beta_3\,\omega L_x^2 + \beta_4\,\omega L_x^2 \\ &= (\beta_1 + \beta_2 + r\,(\beta_3 + \beta_4))\omega L_x L_y \\ &= \beta_{tot}(\omega) L_x L_y \end{aligned}$$

For very few possible support conditions of slabs and for some values of r, k_{tot} varies between 0.94 and 1.13. However, k_{tot} for most of the cases equals unity. β_{tot} on the other hand is varying between 0.99 and 1.0125.

CONCLUSION

The time and effort required in transferring the slab loading to the four supporting beams can be considerably reduced by using the equivalent uniform load coefficients derived in this paper. These coefficients are based on the Ethiopian standard code of practice ESCP2 recommendation which gives, in the form of a simple equation, the equivalent uniform load coefficients for triangular and trapezoidal loadings on beams. Values are given for all the possible slab support conditions as well as for different side ratios of slab panels.

In trying to verify the results of this study, the following outcomes have been realised:

- The ESCP2 recommended proportion of slab loading (i.e. 1:1 and 2:3 ratios) follows the pattern for the yield line analysis of isotropic slabs.

- For continuous beams, the negative (support) moments by the equivalent uniform load are overestimated by up to 11.1% depending on the slab support condition and panel side ratios, while the positive (span) moments are underestimated by up to 9%. The EBCS2 recommended coefficients result in moments which vary from the actual triangular or trapezoidal loading by less than 5% for several of the cases and up to 12.5% in some cases.

- The total load carried by the four supporting beams of a panel as obtained using the equivalent uniform load coefficients (Table 2) varies from the actual total panel load between -6% to 13% for very few cases only. both the ESCP2 & EBCS2 coefficients provide a reasonablly correct loading.

- The EBCS2 loading, though being uniform, has to be applied on the middle three quarters of the span Therefore, the use of other appropriate equations to determine the fixed-end-actions for the beams so loaded would be essential for the further analysis .

Some of these outcomes suggest that further investigation is still required in order to determine the uniform load coefficients with a better accuracy. For the ESCP2 recommended and currently employed design procedure, however, the coefficients derived in this study are satisfactory and sufficient.

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