ABSTRACT

While designing reinforced concrete two-way slab systems, triangular or trapezoidal loadings are encountered during transferring the slab loading to the supporting beams. When analysing continuous beams, uniform loading conditions are, as much as possible, preferred because of their simplicity. In this paper, respective equivalent uniformly distributed load coefficients are derived based on the Ethiopian Standard Code of Practice (ESCP2) recommendation. Results are tabulated for all the possible cases of slab support conditions. A numerical example has been presented to illustrate the application of the coefficients in actual design problems.

It has also been tried to verify some of the results by comparing the recommended side ratio of the slab loadings with the yield line analysis of slabs, the derived coefficients with elastic analysis of single span beams, the total panel loading with the total load the four supporting beams carry. Under these three aspects investigation has been made on the recommendation of the new Building Code Standards [11] which is to be launched in the near future.

INTRODUCTION

In reinforced concrete buildings, the slab panels cast integrally with the beams behave in a two-way action as long as the side ratio of a panel (longer side/shorter side) is not greater than two. In such cases the panel is said to be supported on all four sides. The proportion of the slab loading to be shared by each of the supporting beam depends on the edge fixity, the side ratio of the panel and the amount and nature of slab reinforcements: isotropic vs. orthotropic and top vs. bottom reinforcement [5,8,10]. Furthermore, for the same reasons, the distribution of the portion of the slab loading to be carried by a particular boundary beam may be triangular or trapezoidal.
where,
\[ a = \frac{(a_1 + a_2)}{2} \]

\[ \alpha = \frac{(L_1)}{L} \]

**Case 1:**

**Side 1:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 1 - r^2/3 \]

therefore,
\[ w_1 = k(0.5L_1 \omega) = 0.5(1-r^2/3)L_1 \omega \]
\[ k_i = 0.5(1-r^2/3) \]

**Side 2:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 2/3 \]

**Side 3:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 2/3 \]

**Side 4:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 1 - r/3 \]

**Case 2:**

**Side 1:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 1 - 16r^2/75 \]

therefore,
\[ w_1 = k(3L_1 \omega) = (75 - 16r^2)L_1 \omega /125 \]
\[ k_i = (75 - 16r^2)/125 \]

**Side 2:**
\[ a_1 = a_2 = \alpha = (L_2)/L = 0.5 \]
\[ k = 1 - 4 \alpha^2/3 = 1 - 16r^2/75 \]

therefore,
\[ w_1 = k(2L_1 \omega^5) = 2(75 - 16r^2)L_1 \omega^3 /75 \]
\[ k_i = 2(75 - 16r^2)/75 \]
Uniform Load Coefficients for Beams in Two-Way Slabs

3

Side 3:
\[ \alpha_1 = 2/5, \alpha_3 = 3/5, \alpha = 1/5 \]
\[ k = 1 - 4\alpha^2/3 = 2/3 \]

hence,
\[ w_1 = k(2/5)L_1, \omega = (2/3)(2/5)L_1, \omega = (4/15)L_1, \omega \]
\[ k_1 = 4/15 \]

Side 4: this is identical to side 3, and therefore
\[ w_4 = w_3 \]
\[ k_4 = 1/3 \]

Case 3:

Figure 5: Slab panel continuous along one of the shorter sides only

(a) \( L_x/L_y < 1.25 \)  
(b) \( L_x/L_y > 1.25 \)

For \( L_x/L_y < 1.25 \),

Side 1:
\[ l = 3L_x/5, m = 2L_x/5 \]
\[ \alpha_1 = (3L_x/5)/L_y = 3/5 \]
\[ \alpha_2 = (2L_x/5)/L_y = 2/5 \]
\[ \alpha = 1/2 \]
\[ k = 1 - 4\alpha^2/3 = 2/3 \]

therefore,
\[ w_1 = k(2L_x/5), \omega = (4/15)L_x, \omega \]
\[ k_1 = 4/(15r) \]

Side 2: this is identical to side 1, and therefore
\[ w_2 = w_1 \]
\[ k_2 = 4/(15r) \]

Side 3:
\[ \alpha_1 = \alpha_3 = \alpha = (2L_x/5)/L_y = 2/(5r) \]
\[ k = 1 - 4\alpha^2/3 = 1 - 4(2/(5r))^2/3 = 1 - 16/(75r^2) \]

hence,
\[ w_3 = k(m, \omega) = ((75r^2 - 16)/(75r^3))L_x, \omega \]
\[ k_3 = (15r^2 - 32)/(375r^3) \]

Side 4:
\[ \alpha_1 = \alpha_3 = \alpha = (2L_x/5)/L_y = 2/(5r) \]
\[ k = 1 - 4\alpha^2/3 = 1 - 4(2/(5r))^2/3 = 1 - 16/(75r^2) \]

hence,
\[ w_4 = k(l, \omega) = (75r^2 - 16)/(75r^3)L_x, \omega \]
\[ k_4 = (75r^2 - 16)/(125r^3) \]

If \( L_x/L_y > 1.25 \), then

Side 1:
\[ l = 3L_x/4, m = 0.5L_x \]
\[ \alpha_1 = (3L_x/4)/L_y = (3/4)r \]
\[ \alpha_2 = (0.5L_x)/L_y = 0.5r \]
\[ \alpha = 5r/8 \]
\[ k = 1 - 4\alpha^2/3 = 1 - 25r^2/48 \]

therefore,
\[ w_1 = k(0.5L_x, \omega) = (48 - 25r^2)L_x, \omega \]
\[ k_1 = (48 - 25r^2)/96 \]

Side 2: this is identical to side 1, and therefore
\[ w_2 = w_1 \]
\[ k_2 = (48 - 25r^2)/96 \]

Side 3:
\[ \alpha_1 = \alpha_3 = \alpha = 1/2 \]
\[ k = 1 - 4\alpha^2/3 = 2/3 \]

hence,
\[ w_3 = k(1/2), \omega = (2/3)(1/2)L_x, \omega = (1/3)L_x, \omega \]
\[ k_3 = 1/3 \]

Side 4:
\[ \alpha_1 = \alpha_3 = \alpha = 1/2 \]
\[ k = 1 - 4\alpha^2/3 = 2/3 \]

hence,
\[ w_4 = k(1/2), \omega = (2/3)(3/4)L_x, \omega = (1/2)L_x, \omega \]
\[ k_4 = 1/2 \]

For the other cases, similar procedure is employed to arrive at the following results.

Case 4:

Figure 6 Slab panel continuous along the two adjacent sides only

\[ k_1 = \frac{2(3 - r^2)}{15} \]
\[ k_2 = \frac{(3 - r^2)}{5} \]
\[ k_3 = \frac{4}{15} \]
\[ k_4 = \frac{1}{5} \]

Case 5:

Figure 7 Slab panel continuous along the two longer sides only.

\[ k_1 = \frac{(27 - 4r^2)}{54} \]
\[ k_2 = \frac{(27 - 4r^2)}{54} \]
\[ k_3 = \frac{2}{9} \]
\[ k_4 = \frac{2}{9} \]

Case 6:

Figure 8 Slab panel continuous along the two shorter sides only

(a) \( L_{/L_x} < 1.5 \) (b) \( L_{/L_x} ≥ 1.5 \).

If \( L_{/L_x} ≤ 1.5 \), then

\[ k_1 = \frac{2(9r)}{} \]
\[ k_2 = \frac{(27r^2 - 4)/(54r^2)}{} \]
\[ k_3 = \frac{(27r^2 - 4)/(54r^2)}{} \]

If \( L_{/L_x} ≥ 1.5 \), then

\[ k_1 = \frac{4 - 3r^2}{8} \]
\[ k_2 = \frac{4 - 3r^2}{8} \]
\[ k_3 = \frac{1}{2} \]
\[ k_4 = \frac{1}{2} \]

Case 7:

Figure 9 Slab panel simply supported along one of the shorter sides only.

\[ k_1 = \frac{(108 - 25r^2)}{216} \]
\[ k_2 = \frac{(108 - 25r^2)}{216} \]
\[ k_3 = \frac{2}{9} \]
\[ k_4 = \frac{1}{3} \]

Case 8:

Figure 10 Slab panel simply supported along one of the longer sides only

(a) \( L_{/L_x} < 1.2 \) (b) \( L_{/L_x} ≥ 1.2 \).

If \( L_{/L_x} < 1.2 \), then

\[ k_1 = \frac{2(9r)}{} \]
\[ k_2 = \frac{1(3r)}{} \]
\[ k_3 = \frac{(108r^2 - 25)/(216r^2)}{} \]
\[ k_4 = \frac{(108r^2 - 25)/(216r^2)}{} \]

If \( L_{/L_x} ≥ 1.2 \), then

\[ k_1 = \frac{2(25 - 12r^2)}{125} \]

Uniform Load Coefficients for Beams in Two-Way Slabs

\[ k_1 = 3(25 - 12r^2)/125 \]
\[ k_2 = 2/5 \]
\[ k_3 = 2/5 \]

**Case 9:**

Finally, with the help of a short computer program, the various coefficients have been computed and tabulated in Table 1.

Table 1: Equivalent Uniform Load Coefficients for Moment for Beams Supporting Uniformly Loaded Two-Way Slabs.

(For moment based on \( k = 1 - 4 \alpha^2 / 3 \))

\[ w_i = k_i \omega L_s \]

In which,

\( w_i \) = equivalent uniform load on beam along side \( i \) (kN/m),
\( k_i \) = equivalent uniform load coefficient (Table 1),
\( \omega \) = uniformly distributed slab loading (kPa),
\( L_s \) = short side of the panel (m),
\( i \) = the side number of the slab panel (1 to 4).

**Figure 11** Slab panel continuous along all four sides

\[ k_1 = 0.5 (1 - r^2 / 3) \]
\[ k_2 = 0.5 (1 - r^2 / 3) \]
\[ k_3 = 1/3 \]
\[ k_4 = 1/3 \]
Table 2: Equivalent Uniform Load Coefficients for Shear for Beams Supporting Uniformly Loaded Two-way Slabs

<table>
<thead>
<tr>
<th>Support Condition</th>
<th>( L_x / L_z )</th>
<th>1.0</th>
<th>1.1</th>
<th>1.2</th>
<th>1.3</th>
<th>1.4</th>
<th>1.5</th>
<th>1.6</th>
<th>1.7</th>
<th>1.8</th>
<th>1.9</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k_1 )</td>
<td></td>
<td>0.3333</td>
<td>0.3623</td>
<td>0.3843</td>
<td>0.4014</td>
<td>0.4150</td>
<td>0.4259</td>
<td>0.4349</td>
<td>0.4423</td>
<td>0.4486</td>
<td>0.4538</td>
<td>0.4583</td>
</tr>
<tr>
<td>( k_2 )</td>
<td></td>
<td>0.3333</td>
<td>0.3623</td>
<td>0.3843</td>
<td>0.4014</td>
<td>0.4150</td>
<td>0.4259</td>
<td>0.4349</td>
<td>0.4423</td>
<td>0.4486</td>
<td>0.4538</td>
<td>0.4583</td>
</tr>
<tr>
<td>( k_3 )</td>
<td></td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
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<td>0.3333</td>
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<td>0.3333</td>
</tr>
<tr>
<td>( k_4 )</td>
<td></td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
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<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
<td>0.3333</td>
</tr>
</tbody>
</table>

For shear, based on \( k = 1 - \alpha \)

\[ w_x = k \omega L_z \]

In which,

- \( w_x \) = equivalent uniform load on beam along side \( L_z \) (kN/m)
- \( k \) = equivalent uniform load coefficient (Table 2)
- \( \omega \) = uniformly distributed slab loading (kPa)

Uniform Load Coefficients for Beams in Two-Way Slabs

\( L_x \) = short side of the panel (m),
\( i \) = the side number of the slab panel (1 to 4).

<table>
<thead>
<tr>
<th>Support Condition</th>
<th>( k_i )</th>
<th>( L_x/L_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.3</td>
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<tr>
<td></td>
<td></td>
<td>1.4</td>
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<td></td>
<td></td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6</td>
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<td></td>
<td></td>
<td>1.7</td>
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<tr>
<td></td>
<td></td>
<td>1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2.0</td>
</tr>
</tbody>
</table>

| 2                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 3                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 4                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 5                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 6                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 7                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 8                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

| 9                 | 1.0      | 1.1         |
|                   |          | 1.2         |
|                   |          | 1.3         |
|                   |          | 1.4         |
|                   |          | 1.5         |
|                   |          | 1.6         |
|                   |          | 1.7         |
|                   |          | 1.8         |
|                   |          | 1.9         |
|                   |          | 2.0         |

*Journal of EAEA, Vol. 13, 1996*
NUMERICAL EXAMPLE

Figure 12 A slab system with three by three panels

Given:
For the slab system shown in Fig 21 above,
Live load = 2.5 kpa,
Slab thickness = 15 cm,
Assume beam own wt. = 2.5 KN/m,

Required: Total equivalent uniform loading on beam along axis B.

Solution:
Total uniform slab loading = 2.5 + 0.15(25) = 6.25 kpa

Panel I:
This panel is case 4, for which \( L/L_d = 1 \), the corresponding uniform load coefficients (Table 2) are;
\[
\begin{align*}
    k_1 &= 0.2 \\
    k_2 &= 0.3 \\
    k_3 &= 0.2 \\
    k_4 &= 0.3 \\
\end{align*}
\]
and the uniform beam loadings are,
\[
\begin{align*}
    w_1 &= 0.2(6.25)(3) = 3.75 KN/m \\
    w_2 &= 0.3(6.25)(3) = 5.63 KN/m \\
    w_3 &= 0.2(6.25)(3) = 3.75 KN/m \\
    w_4 &= 0.3(6.25)(3) = 5.63 KN/m \\
\end{align*}
\]

Panel II:
This panel is case 8, for which \( L/L_d = 1.8 \), the corresponding uniform load coefficients (Table 2) are;
\[
\begin{align*}
    k_1 &= 0.27 \\
    k_2 &= 0.4 \\
    k_3 &= 0.3 \\
    k_4 &= 0.3 \\
\end{align*}
\]
and the uniform beam loadings are,
\[
\begin{align*}
    w_1 &= 0.27(6.25)(3) = 5.06 KN/m \\
    w_2 &= 0.4(6.25)(3) = 7.50 KN/m \\
    w_3 &= 0.3(6.25)(3) = 5.63 KN/m \\
    w_4 &= 0.3(6.25)(3) = 5.63 KN/m \\
\end{align*}
\]

Panel III:
This panel is case 4, for which \( L/L_d = 1.5 \), the corresponding uniform load coefficients (Table 2) are;
\[
\begin{align*}
    k_1 &= 0.27 \\
    k_2 &= 0.4 \\
    k_3 &= 0.2 \\
    k_4 &= 0.3 \\
\end{align*}
\]
and the uniform beam loadings are,
\[
\begin{align*}
    w_1 &= 0.27(6.25)(3) = 5.06 KN/m \\
    w_2 &= 0.4(6.25)(3) = 7.50 KN/m \\
    w_3 &= 0.2(6.25)(3) = 3.75 KN/m \\
    w_4 &= 0.3(6.25)(3) = 5.63 KN/m \\
\end{align*}
\]

Panel IV:
This panel is case 7, for which \( L/L_d = 1 \), the corresponding uniform load coefficients (Table 2) are;
\[
\begin{align*}
    k_1 &= 0.33 \\
    k_2 &= 0.33 \\
    k_3 &= 0.17 \\
    k_4 &= 0.25 \\
\end{align*}
\]
and the uniform beam loadings are,
\[
\begin{align*}
    w_1 &= 0.33(6.25)(3) = 6.19 KN/m \\
    w_2 &= 0.33(6.25)(3) = 6.19 KN/m \\
    w_3 &= 0.17(6.25)(3) = 3.19 KN/m \\
    w_4 &= 0.25(6.25)(3) = 4.69 KN/m \\
\end{align*}
\]
Uniform Load Coefficients for Beams in Two-Way Slabs

Panel V:
This panel is case 9, for which $L/L' = 1.8$, the corresponding uniform load coefficients (Table 2) are:

- $k_1 = 0.36$
- $k_2 = 0.36$
- $k_3 = 0.25$
- $k_4 = 0.25$

and the uniform beam loadings are,

- $w_1 = 0.36(6.25)(3) = 6.75$ KN/m
- $w_2 = 0.36(6.25)(3) = 6.75$ KN/m
- $w_3 = 0.25(6.25)(3) = 4.69$ KN/m
- $w_4 = 0.25(6.25)(3) = 4.69$ KN/m

Panel VI:
This panel is case 7, for which $L/L' = 1.5$, the corresponding uniform load coefficients (Table 2) are:

- $k_1 = 0.39$
- $k_2 = 0.39$
- $k_3 = 0.17$
- $k_4 = 0.25$

and the uniform beam loadings are,

- $w_1 = 0.39(6.25)(3) = 7.31$ KN/m
- $w_2 = 0.39(6.25)(3) = 7.31$ KN/m
- $w_3 = 0.17(6.25)(3) = 3.19$ KN/m
- $w_4 = 0.25(6.25)(3) = 4.69$ KN/m

Accordingly, the loading on beam along axis B is given below and shown in Fig 13.

$$ F_{1x} = 2.5(\text{own wt.}) + 5.63(\text{from panel II}) + 6.19(\text{from panel IV}) = 14.32 \text{ KN/m} $$

$$ F_{2x} = 2.5(\text{own wt.}) + 7.50(\text{from panel II}) + 6.75(\text{from panel V}) = 16.75 \text{ KN/m} $$

Figure 13 Equivalent uniform loading on beam along axis B

THE NEW BUILDING CODE STANDARD

A new Building Code Standard for the structural use of concrete has been prepared and is to be launched within a short time. On the topic of load dispersion from slab to beams, the new code provides a table with coefficients similar to the ones derived in this paper. These coefficients are shown in Table 2 for comparison. According to this new code recommendation,

1. The design load on beams supporting solid slabs spanning in two directions at right angles supporting uniformly distributed loads may be assessed from the following equation:

$$ V_x = \beta_x (g_d + q_d) L_x $$

$$ V_y = \beta_y (g_d + q_d) L_y $$

Table 2: Shear Force Coefficients for Uniformly Loaded Rectangular Panels Supported on Four Sides With Provision for Torsion at Corners

<table>
<thead>
<tr>
<th>Type of panel and location</th>
<th>Edge</th>
<th>$\beta_{cn}$ for values of $L/L_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1.0</td>
</tr>
<tr>
<td>1 Continuous</td>
<td>0.33 0.36 0.39 0.41</td>
<td></td>
</tr>
<tr>
<td>2 Continuous</td>
<td>0.36 0.39 0.42 0.44</td>
<td></td>
</tr>
<tr>
<td>2 Discontinuous</td>
<td>- - - -</td>
<td></td>
</tr>
<tr>
<td>3 Continuous</td>
<td>0.24 0.27 0.29 0.31</td>
<td></td>
</tr>
<tr>
<td>3 Discontinuous</td>
<td>0.40 0.44 0.47 0.50</td>
<td></td>
</tr>
<tr>
<td>4 Continuous</td>
<td>0.26 0.29 0.31 0.33</td>
<td></td>
</tr>
<tr>
<td>4 Discontinuous</td>
<td>0.40 0.43 0.45 0.47</td>
<td></td>
</tr>
<tr>
<td>5 Continuous</td>
<td>- - - -</td>
<td></td>
</tr>
<tr>
<td>5 Discontinuous</td>
<td>0.26 0.30 0.33 0.36</td>
<td></td>
</tr>
<tr>
<td>6 Continuous</td>
<td>0.45 0.48 0.51 0.53</td>
<td></td>
</tr>
<tr>
<td>6 Discontinuous</td>
<td>0.30 0.32 0.34 0.35</td>
<td></td>
</tr>
<tr>
<td>7 Continuous</td>
<td>0.30 0.33 0.36 0.38</td>
<td></td>
</tr>
<tr>
<td>7 Discontinuous</td>
<td>0.30 0.33 0.36 0.38</td>
<td></td>
</tr>
<tr>
<td>8 Continuous</td>
<td>0.33 0.36 0.39 0.41</td>
<td></td>
</tr>
<tr>
<td>8 Discontinuous</td>
<td>0.33 0.36 0.39 0.41</td>
<td></td>
</tr>
</tbody>
</table>

Figure 14 Distribution of Load on a Beam Supporting a Two-Way Spanning Slab

Figure 15 Load on Axis B according to the new code EBCS2

RESULT VERIFICATION

The following three points need to be considered in order the results obtained in this paper to be valid. Namely:

1. To what extent is the suggested proportion (i.e. 1:1 and 2:3) of slab loading shared by the supporting beams correct?
Uniform Load Coefficients for Beams in Two-Way Slabs

2. Are the derived equivalent uniform loadings representing the actual situation? (Or are the maximum mid span moments and/or support moments produced by the uniform loadings on the beam similar to the ones produced by the actual triangular or trapezoidal loadings?)

3. Is the total slab loading carried by the four supporting beams of a panel correct?

For the first point, the ESCP2 recommendation is based on the yield line pattern as indicated in Reinforced Concrete slab design procedures [3,6,7,9].

![Figure 16 Load dispersion (a) as obtained by the yield line analysis for isotropic slab, (b) as recommended by the ESCP2.](image)

Using the yield line analysis for a rectangular, isotropic slab panel, simply supported along the two adjacent edges and fixed along the other two, Dyaratnam [3] has come up with the result shown in Fig. 16(a). The suggested slope given in the code, as shown in Fig. 16(b), is very close to the yield line result. The reason for the small discrepancy may arise from the fact that the negative moments in slabs being actually higher than the field moments and therefore support reinforcements are normally higher (non-isotropic).

Hence, as recommended, if two adjacent sides have the same fixity, a ratio of 1:1 is to be used, while for different fixity (one side fixed and the adjacent side simply supported), a 2:3 ratio shall be used.

To check whether the equivalent uniform loading gives the same moments at critical locations, the following three panel cases are investigated.

**Case 1. Simply supported all round**

*Sides 1 & 2*

![Figure 17 Simply supported beam loaded with trapezoidal loading](image)

Using the actual trapezoidal loading [2,4],

\[ M_{max} = \frac{1}{2} \left( \frac{1}{2} \rho \right) \left( \frac{1}{8} \right) = k_a \left( \frac{1}{8} \right) \]

Using equivalent uniform loading (Table 1),

\[ M_{max} = k_a \left( \frac{1}{8} \right) \]

Using the new code (EBCS2) recommendation [2,4,11],

\[ M_{max} = \left( \frac{15}{16} \right) \left( \frac{1}{8} \right) = k_a \left( \frac{1}{8} \right) \]

\( k_a \) and \( k_w \) are equal for all values of \( r \) indicating that the equivalent uniform loading does give the same moment at mid span as the actual trapezoidal loading. For this case, the EBCS2 coefficient \( k_a \) varies from \( k_w \) by about 6%.

*Sides 3 & 4*

Using the actual triangular loading [2,4],

\[ M_{max} = \left( \frac{1}{3} \right) \left( \frac{1}{8} \right) = k_a \left( \frac{1}{8} \right) \]

Using equivalent uniform loading (Table 1),

\[ M_{max} = k_a \left( \frac{1}{8} \right) \]

Using the new code (EBCS2) recommendation [2,4,11],

\[ M_{max} = \left( \frac{15}{16} \right) \left( \frac{1}{8} \right) = k_a \left( \frac{1}{8} \right) \]

Here again the comparison shows that the equivalent uniform load and the actual triangular loading have equal mid-span moments. The moment coefficient for the EBCS2 loading also has the same variation.

Case 9. Fixed support all round

**Sides 1 & 2**

![Figure 19](image)

**Figure 19** Fixed beam loaded with trapezoidal loading.

Using the actual trapezoidal loading [2,4],

\[
M_{\text{max}} = (0.5 - 0.25r^2 + r^2/16)(awL_x L_y^2/12) = k_s(awL_x L_y^2/12)
\]

\[
M_{\text{max}} = (0.5 - 0.125r^2)(awL_x L_y^2/24) = k_s(awL_x L_y^2/24)
\]

Using equivalent uniform loading (Table 1),

\[
M_{\text{max}} = k_s(awL_x L_y^2/12)
\]

\[
M_{\text{max}} = k_s(awL_x L_y^2/24)
\]

Using the new code (EBCS2) recommendation [2,4,11],

\[
M_{\text{max}} = (117\beta_s/128)(awL_x L_y^2/12)
\]

\[
M_{\text{max}} = (63\beta_s/64)(awL_x L_y^2/24)
\]

\[k_s\] is higher than \[k_r\] but it is lower than \[k_u\] for all values of \(r\). This indicates that the equivalent uniform loading overestimates the support moments while it underestimates the span moments. The maximum variation is about 6.7% for support moments and 11.1% for span moments, which occurs when \(r\) equals 1. In the EBCS2 loading condition, \(k_u\) varies from \(k_r\) by about 2.5%; while \(k_s\) varies from \(k_u\) by 12.5% to 1.6% for \(r\) equals 1 to 2, respectively.

**Sides 3 & 4**

![Figure 20](image)

**Figure 20** Fixed beam loaded with triangular loading.

Using the actual trapezoidal loading [2,4],

\[
M_{\text{max}} = (15\beta_s/16)(awL_x L_y^2/8)
\]

\[
M_{\text{max}} = k_u(awL_x L_y^2/8)
\]

Using the actual triangular loading [2,4],

\[
M_{\text{max}} = (5/16)(awL_y^2/12)
\]

\[
M_{\text{max}} = k_s(awL_y^2/12)
\]

\[
M_{\text{max}} = (3/8)(awL_y^2/24)
\]

\[
M_{\text{max}} = k_s(awL_y^2/24)
\]

Using equivalent uniform loading (Table 1),

\[
M_{\text{max}} = k_s(awL_y^2/12)
\]

\[
M_{\text{max}} = k_s(awL_y^2/24)
\]

Using the new code (EBCS2) recommendation [2,4,11],

\[
M_{\text{max}} = (117\beta_s/128)(awL_y^2/12)
\]

\[
M_{\text{max}} = (63\beta_s/64)(awL_y^2/24)
\]

Here again the comparison shows that the equivalent uniform load is higher than the actual triangular loading for support moments by 6.7% while it is lower for the span moments by 11.1%. In the EBCS2 loading condition, \(k_s\) is less than \(k_r\) by about 2.5%; while \(k_s\) is less than \(k_u\) by 12.5%.

Case 4. Fixed along the two adjacent sides

**Side 1**

![Figure 21](image)

**Figure 21** Propped cantilever beam loaded with trapezoidal loading.

Using the actual trapezoidal loading [2,4],

\[
M_{\text{max}} = (0.4 - 16r^2(220 - 81r + 7.8r^2)/15000)(awL_x L_y^2/8)
\]

\[
M_{\text{max}} = k_s(awL_x L_y^2/8)
\]

Using equivalent uniform loading (Table 1),

\[
M_{\text{max}} = k_s(awL_x L_y^2/8)
\]

Using the new code (EBCS2) recommendation [2,4,11],

\[
M_{\text{max}} = (117\beta_s/128)(awL_x L_y^2/8)
\]

\[
M_{\text{max}} = k_s(awL_x L_y^2/8)
\]

\[
M_{\text{max}} = (63\beta_s/64)(awL_x L_y^2/24)
\]

\[
M_{\text{max}} = k_s(awL_x L_y^2/24)
\]
Negative moments are slightly overestimated by the equivalent uniform loading since \( k_1 \) is greater than \( k \) for all values of \( r \), and the maximum variation is 9.6% (i.e. when \( r = 1 \)). The coefficient \( K_{\text{max}} \) for the EBCS2 loading has a 2.4% to 4% variation from \( k \).

**Side 2.**

Using the actual trapezoidal loading [2,4],

\[
M_{\text{max}} = (0.6 - 24r^2)(220 - 81r + 7.8r^2)/15000)(\omega L/L_2)^3/8) = k(\omega L/L_2)^3/8)
\]

![Figure 22: Propped cantilever beam loaded with trapezoidal loading](image)

Using equivalent uniform loading (Table 1),

\[
M_{\text{max}} = k(\omega L/L_2)^3/8)
\]

Using the new code (EBCS2) recommendation [2,4,11],

\[
M_{\text{max}} = (117\beta_{\text{r}}/128)(\omega L/L_2)^3/8) = k(\omega L/L_2)^3/8)
\]

Here again the comparison shows that the equivalent uniform load coefficient \( k \) is higher than \( k_1 \) for the actual triangular loading, overestimating the support moment. The coefficient \( K_{\text{max}} \) for the EBCS2 loading has a 2.4% variation from \( k \).

**Side 3.**

Using the actual triangular loading [2,4],

\[
M_{\text{max}} = (2282/9375)(\omega L/L_2)^3/8) = k(\omega L/L_2)^3/8)
\]

Using equivalent uniform loading (Table 1),

\[
M_{\text{max}} = k(\omega L/L_2)^3/8)
\]

Using the new code (EBCS2) recommendation [2,4,11],

\[
M_{\text{max}} = (117\beta_{\text{r}}/128)(\omega L/L_2)^3/8) = k(\omega L/L_2)^3/8)
\]

Comparison, in this case, shows that the equivalent uniform load coefficient \( k \) is higher than the \( k_1 \) value for the actual triangular loading, again overestimating the support moment. The coefficient \( K_{\text{max}} \) for the EBCS2 loading has a 0.1% variation from \( k \).

Other panel support cases can be similarly investigated. One may conclude that the equivalent uniform load coefficients derived in this paper overestimate support moments while they underestimates the span moments. Except for very few cases, the new code recommended coefficients produce moments which are closer to the ones produced by the triangular or trapezoidal loadings.

*Journal of EAEA, Vol. 13, 1996*
To check whether the total loading carried by the four supporting beams is equal to the total load within a panel, the two values are compared as follows.

Total load within a panel \( = \omega L_x L_y \).

According to ESCP2, the four supporting beams carry the following total load (Table 2).

\[
W = k_1 \omega L_x L_y + k_2 \omega L_x L_y + k_3 \omega L_x L_y + k_4 \omega L_x L_y = (k_1 + k_2 + r (k_3 + k_4)) \omega L_x L_y
\]

According to EBSC2, the four supporting beams carry the following total load,

\[
W = \beta_1 \omega L_x L_y + \beta_2 \omega L_x L_y + \beta_3 \omega L_x L_y + \beta_4 \omega L_x L_y = (\beta_1 + \beta_2 + r (\beta_3 + \beta_4)) \omega L_x L_y
\]

For very few possible support conditions of slabs and for some values of \( r, k_{su} \) varies between 0.94 and 1.13. However, \( k_{su} \) for most of the cases equals unity. \( \beta_{su} \) on the other hand is varying between 0.99 and 1.0125.

**CONCLUSION**

The time and effort required in transferring the slab loading to the four supporting beams can be considerably reduced by using the equivalent uniform load coefficients derived in this paper. These coefficients are based on the Ethiopian standard code of practice ESCP2 recommendation which gives, in the form of a simple equation, the equivalent uniform load coefficients for triangular and trapezoidal loadings on beams. Values are given for all the possible slab support conditions as well as for different side ratios of slab panels.

In trying to verify the results of this study, the following outcomes have been realised:

- The ESCP2 recommended proportion of slab loading (i.e., 1:1 and 2:3 ratios) follows the pattern for the yield line analysis of isotropic slabs.

- For continuous beams, the negative (support) moments by the equivalent uniform load are overestimated by up to 11.1% depending on the slab support condition and panel side ratios, while the positive (span) moments are underestimated by up to 9%. The EBSC2 recommended coefficients result in moments which vary from the actual triangular or trapezoidal loading by less than 5% for several of the cases and up to 12.5% in some cases.

- The total load carried by the four supporting beams of a panel as obtained using the equivalent uniform load coefficients (Table 2) varies from the actual total panel load between -6% to 13% for very few cases only. Both the ESCP2 & EBSC2 coefficients provide a reasonably correct loading.

- The EBSC2 loading, though being uniform, has to be applied on the middle three quarters of the span. Therefore, the use of other appropriate equations to determine the fixed-end actions for the beams so loaded would be essential for the further analysis.

Some of these outcomes suggest that further investigation is still required in order to determine the uniform load coefficients with a better accuracy. For the ESCP2 recommended and currently employed design procedure, however, the coefficients derived in this study are satisfactory and sufficient.

**ACKNOWLEDGEMENT**

I am grateful to Prof. Negussie Teghese and Ato Bekele Mekonen for the encouragement, advise and support they provided me and especially for introducing me to the revised Ethiopian Building Code Standard (EBSC2). Thanks is also due to Dr. Asnake Adamu for his encouragement and advice which helped in making this work become successful.

**REFERENCES**


Uniform Load Coefficients for Beams in Two-Way Slabs


