PREDICTION OF MAXIMUM DRY DENSITY OF LOCAL GRANULAR FILLS

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ABSTRACT
The paper presents a relation developed to predict maximum dry density (MDD) in terms of the solid density and the gradation coefficients that characterize the grain size distribution of locally employed granular fill materials. For this purpose, two geologically different soils commonly used as selected fill materials are blended. A large number of compaction tests as required by statistical principles are conducted using the modified Proctor compaction procedure. The gradation envelopes, within which the control parameters are varied, are selected on the basis of local specifications for sub-base materials. A strong correlation is found between the MDD and the specific gravity of the solid grains. In contrast, the correlation between the MDD and the gradation coefficients including the coefficient of gradation, $C_g$, and the uniformity coefficient, $C_u$, is weak. The developed predicting formula is compared with existing methods. It is observed that the developed relations predict well the MDD of soils of the nature studied in this work than do the existing methods. A test on a soil of relatively high solid density revealed that the developed relation loses its power of prediction when applied on soils of solid density beyond the range studied.

Keywords: Compaction, gradation coefficients, granular soil, maximum dry density, modified Proctor test, solid unit weight, optimum moisture content

INTRODUCTION
Particle sizes provide quantitative data on the range of sizes of particles and the relative proportions, by weight, of each size range. From this, it is possible to tell whether the soil consists of predominantly gravel, sand, silt or clay sizes and to a limited extent which of these size ranges is likely to control the engineering properties. A subject of active research interest today is the prediction of soil properties based largely on grain size distributions, void ratios, and soil particle characteristics [8]. Studies have been made in the past to establish relationships between the MDD and simple properties like the grain-size distribution of soils. Such undertakings have the objective of enabling the user to predict the MDD for quick applications without the need to perform the compaction test.

The sizes of the soil particles, especially in granular soils, have some effect on the engineering behaviour of a given soil [7]. An indication of the grain size distribution may be numerically obtained from the grain size curve using, parameters like the coefficient of uniformity $C_u = D_{60}/D_{10}$ and the coefficient of gradation $C_g = D_{30}/(D_{10}D_{60})$, where $D_{10}$, $D_{30}$, and $D_{60}$ are the grain sizes corresponding to 10%, 30%, and 60% passing, respectively. These two coefficients are used by different authors to qualitatively describe soil gradations in different ways.

The density of a soil depends mainly on the weight of the individual soil grains, the total number of particles present, and the amount of water present in the voids [2]. Particularly, the maximum dry density of a granular soil can be expressed as:

$$\rho_{d, max} = f(\rho_{solid}, C_u, C_g)$$  \hspace{1cm} (1)

where, $\rho_{d, max}$ is the laboratory maximum dry density and $\rho_{solid}$ is the density of the solid grains.

The shape and texture of individual particles are also important in influencing the density of granular soils. However, there is no refined geotechnical procedure to quantify the same. Thus, they are not included in Eq. (1).

The primary objective of this research work centers on developing an empirical relation to predict the MDD for a quick application of selected local soils employed in road construction in the vicinity of Addis Ababa. In addition, this work compares the predicting power of existing prediction formulas with the one developed here.
In order to meet these objectives, the basic concepts of compaction of soils are briefly presented. Then, previously developed prediction relations are reviewed. Statistical approaches have been followed to determine the number of sampling and to establish the range of the grain size distributions.

Following the common practice of blending soils to get improved products, two soil samples have been taken from the outskirts of Addis Ababa and Addis-Jimma Road Rehabilitation Project site, which were then blended to get a desired mix. The modified Proctor test has been selected for the compaction tests as this has a relatively wider application as compared to the standard Proctor test. Other relevant tests were also conducted, including specific gravity, dry and wet sieve analyses.

The data were analyzed using a commercial statistics software called S-Plus 2000. This analysis essentially focused on finding an appropriate model that best fits the observed data. After testing quite a number of trial polynomial functions, a linear equation has been found to be best fitting. The statistical adequacy of this new relation has also been verified. With a view to assess the influence of the regressor parameter, nine additional compaction tests have been made on a different soil type and the results were analyzed. The prediction capabilities of the existing relations have also been compared against the newly established relation. The results show that the developed relation has a good prediction capacity for the range of parameters considered in the study. Other important inferences have also been made based on the results obtained.

BACKGROUND

The Mechanism of Compaction

At low water content, a soil mass resists the compaction effort imparted to it. As its moisture content is gradually increased, the soil particles are lubricated and they slide over the other thereby becoming closely packed. This results in reduction of air voids. As the free water content increases, the compaction facilitating effect is offset by the increased energy required to move the air out of the soil beyond a certain stage. At this stage the total void volume increases, but not the volume of the air void, which leads to subsequent reduction of the dry density. The limit beyond which the soil shows decrease in dry density is the optimum moisture content, which correspondingly yields the maximum dry density.

Factors Influencing Compaction

Many variables influence the densification of soils. The major ones include the type of soil, moisture content, compaction effort and admixture.

In general, coarse-grained soils can be compacted to higher density than fine-grained soils. With the addition of even a small quantity of fines to a coarse-grained soil, the soil attains a much higher density for a given compaction effort. However, if an excess quantity of a fine fraction is added, the maximum dry density decreases.

Soils are stiff and offer more resistance to compaction at low water content. The increase in water content lubricates the soil particles and the compaction process increases the density the soil. When the optimum water content is achieved, the air voids become approximately constant and further compaction effort cannot force out air from the soil voids.

The compaction effort essentially consists of the mass, size and operating frequency of the compaction equipment. The compactor characteristics influence the stress level and depth of influence of the dynamic force. The frequency of compaction is a function of the compactor-soil system and it changes as the density increases during the process of compaction [4]. In general, an increase in the effort, expressed normally in terms of the energy imparted, tends to decrease the optimum moisture content and increase in the dry density.

Appropriate addition of admixtures gives a better dry density. Blending with soils of wide range of grain size can bring about an increase in dry density.

Laboratory Compaction Tests

Laboratory compaction tests provide the basis for determining the degree of compaction and water content needed to achieve the required engineering properties. They are also important in controlling field compaction to assure that the required density at the specified water content is achieved.

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The commonly used laboratory compaction tests are the standard and the modified proctor compaction tests. The basic difference between the two test procedures lies in the compaction effort and the size of compaction mould. In both tests, the compaction energy is expressed as:

\[ E = \frac{N \times W \times H}{V} \]  

(2)

where \( N \) is number of blows per layer, \( W \) is hammer weight (Newton), \( H \) is the hammer fall height (m), and \( V \) is the mould volume (m\(^3\)).

The modified Proctor test represents a higher compaction effort than that in the standard Proctor test. The test is used to simulate the field condition where heavy compaction equipment is employed as in airport and road constructions. The compaction effort in the modified Proctor test is 4.56 times that of the standard Proctor test. The corresponding increase in maximum dry density is, however, not commensurate with the much larger proportion of expended energy. In actual cases, the improvement in density generally varies depending on the nature of the soil and its gradation, but it seldom exceeds 10% of the standard Proctor tests [4].

Typical values of maximum dry density are around 1600 to 2000 kg/m\(^3\), the maximum range being about 1300 to 2400 kg/m\(^3\). Typical optimum water contents are 10% to 20%, with an outside maximum range of about 5% to 40% [4].

**Existing Prediction Methods**

Different authors have presented prediction formulas for maximum dry density of compacted granular fills. Two of the main approaches are the theoretical and statistical methods.

**Theoretical Approach**

Korfiatis and Manikopoulos [6] approached the problem through theoretical formulations. The approach is based on the observation that any particle-size distribution curve tends to be lognormal in functional form. A straight-line approximation is used to represent the expected lognormal gradation of non-cohesive soils. The straight-line distribution can be completely specified by two parameters: the center point with the coordinates, \((X_0, D_{50})\), and the slope, \(s\), at \(X_0\), where \(X_0\) is the particle size corresponding to 50% passing \(D_{50}\). The value of the slope is computed from the experimental data as

\[ s = \frac{1}{\ln(X_2) - \ln(X_1)} \]

where \(X_1\) and \(X_2\) are abscissas of the end points of the straight line segment.

By defining \(q\) as the packing density of fines alone, and \(t\) as the fraction of fine content (soil grains passing the No 200 sieve), \(\rho_{d_{\text{max}}}\) is obtained as

\[ \rho_{d_{\text{max}}} = \frac{\rho_{\text{solid}}}{\left(\frac{1-t}{(a-b*s)+t/q}\right)}; \]

for \(0.5738 < s < 1.1346\)  

(3a)

\[ \rho_{d_{\text{max}}} = \frac{\rho_{\text{solid}}}{\left(\frac{1-t}{(c-d*s)+t/q}\right)}; \]

for \(0.2 < s < 0.5738\)  

(3b)

in which \(a, b, c,\) and \(d\) are parameters determined by fitting to experimental data.

For practical interest, \(\rho_{\text{solid}}\) was fixed to 2650 kg/m\(^3\). By conducting experiments on several soil samples for \(s > 0.2\) and applying a computer-fitting program, the parameters were determined as \(a = 0.6682 \pm 0.0101, b = 0, c = 0.8565 \pm 0.0238, d = 0.3282 \pm 0.0267\) and \(q = 0.7035 \pm 0.0477\).

According to the researchers [6], the result showed that the average value of deviation between the computed and the measured values of maximum dry density was 3.5%.

**Statistical Approach**

Masih [7] used a statistical approach to predict the maximum dry density of granular soils. The work was based on an assumption of normal distributions of grain sizes with the mean, \(\mu\), and standard deviation, \(\sigma\), ranging between 0.063 mm and 4.75 mm, and 1.0 and 5.0, respectively. The soil mixtures for the tests were prepared using the normal distribution and its probability function [11].

The compaction was done using the standard mold. The results were plotted as MDD versus \(\mu\) for the normal grain sizes. They consistently showed a curve formed of two segments: one segment was similar to the probability curve and the other segment was linear.

The expression for MDD or \(\rho_{d_{\text{max}}}\) was then derived as [7]:

\[ \rho_{d_{\text{max}}} = \frac{\rho_{\text{solid}}}{\left(\frac{1-t}{(c-d*s)+t/q}\right)}; \]

for \(0.2 < s < 0.5738\)
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\[ \rho_{d_{\text{max}}} = \rho_{\text{solid}} \left( 0.61 + \frac{0.11 - 1}{\sqrt{2\pi}} \frac{1}{\sigma} e^{-\sigma^2/2} \right) \]
for \( \sigma \leq \mu \)

\[ \rho_{d_{\text{max}}} = \rho_{\text{solid}} \left( 0.61 + \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma} + 0.0004 \cdot \mu \sigma \right) \]
for \( \sigma > \mu \)

where \( \mu = 0.070 + 0.45\mu^{0.93} \) and \( \sigma = 0.35 + 0.09\mu \).

A term called biasness factor, \( \beta \), was introduced to predict the new density of the soil after mixing it with fine particles (10%-30% by weight to the soil sample). The effect of the biasness factor \( \beta \) in increasing the MDD is given by the following Eq. (7):

\[ \Delta \rho = 0.128 \rho_{\text{solid}} \left( (\mu \beta + 0.1) \sqrt{4/\pi + \beta e} 0.2\mu - \beta \right) \]
for \( \beta \leq 0.8 \) and \( 1.2\mu \leq 1.2\mu \)

The biasness factor is calculated as

\[ \beta = \mu \frac{W_0}{(W_2 + W_0)} \]

where \( W_0 \) is the weight of the added fine material and \( W_0 \) is the weight of the original sample. The \( \mu \) to be used in Eq. (6) is calculated as

\[ \mu = \mu_0 \frac{W_0}{(W_2 + W_0)} \]

where \( \mu_0 \) and \( \mu_0 \) are the means of grain sizes before and after addition of fine materials, respectively.

The MDD after the addition of fines is determined by adding Eq. (6) to Eq. (4) or (5) depending on the given condition.

LABORATORY TEST RESULTS

Two types of soils of different geological origin and commonly employed as sub-base materials in road construction in the vicinity of Addis Ababa were blended for the purpose of this study. The first one was sampled from a widely exploited quarry site in Bole Bulbula area in the south-east corner of the city and is mainly a product of weathered trapolithic rock. The second one is of tuffitic origin and taken from a quarry used by the Addis-Jima road rehabilitation project that is currently underway. About 1000 kg of sample was employed from each soil for the intended purpose. The blended soil has a Los Angeles Abrasion value of 46%. Modified proctor compaction tests were conducted on 43 specimens of different grain size distribution obtained by blending different fractions finer than the 19 mm-sieve size so that they all lie within the envelope selected. The specific gravity of the solid grains in these samples ranges between 2.647 and 2.734, the average being 2.683. For detailed data on the samples including gradation coefficients and specific gravities, the reader is referred to the work in [11].

Selection of Gradation Envelope

Gradation envelope is usually provided as a yardstick to control the quality of the material when the same is used in pavement work. For granular soils that are used as sub-base materials, the locally established gradation requirements are given in Table 1 according to the ERA specifications. These requirements are applied to meet different road quality standards.

To study the effects of gradation coefficients on the maximum dry density, it was found necessary to select a gradation envelope. 'Grading A' is commonly used in gravel roads and it has been selected for this purpose with some modifications. The gradation limits on sieve size 4.75 mm are replaced by the corresponding limits of 'Grading B' in order to widen the gradation envelope. The deficiency of fine materials in the test samples has also urged the authors to modify the gradation limits on 0.075 mm opening sieve. In order to represent the grain size variation at uniform intervals, additional intermediate sieve sizes have been included. The modified gradation envelope is given in Table 2.
Table 1: Local gradation requirements for sub base material [14]

<table>
<thead>
<tr>
<th>OPENING SIZE (MM)</th>
<th>75</th>
<th>37.5</th>
<th>25</th>
<th>4.75</th>
<th>1.18</th>
<th>0.075</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grading A</td>
<td>100</td>
<td>75-85</td>
<td>-</td>
<td>45-65</td>
<td>15-40</td>
<td>0-10</td>
</tr>
<tr>
<td>Grading B</td>
<td>-</td>
<td>100</td>
<td>-</td>
<td>30-70</td>
<td>-</td>
<td>0-15</td>
</tr>
<tr>
<td>Grading C</td>
<td>-</td>
<td>-</td>
<td>100</td>
<td>40-80</td>
<td>-</td>
<td>5-20</td>
</tr>
</tbody>
</table>

Table 2: Modified gradation envelope

<table>
<thead>
<tr>
<th>OPENING SIZE (MM)</th>
<th>38.1</th>
<th>19</th>
<th>9.5</th>
<th>4.75</th>
<th>2.36</th>
<th>1.18</th>
<th>0.6</th>
<th>0.3</th>
<th>0.15</th>
<th>0.075</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Bound</td>
<td>75</td>
<td>63</td>
<td>53</td>
<td>30</td>
<td>26</td>
<td>16</td>
<td>10</td>
<td>6</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Upper Bound</td>
<td>85</td>
<td>79</td>
<td>73</td>
<td>70</td>
<td>56</td>
<td>44</td>
<td>37</td>
<td>25</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

Test Frequency

To establish the required sample size, there are three parameters to be determined. The allowable sampling error, $e$, is one of the parameters and is defined as the ratio of the maximum allowable difference between the estimate made from the sample and the result of testing all the individual specimens. The error, $e$, is taken as 3% for this research work.

The second parameter to be evaluated is the coefficient of variation, COV. It is a parameter useful in comparing the variability of two or more data sets that differ considerably in the magnitude of observation. It is generally available from past records or experience. Since such data are not available for the purpose of this work, COV = 10% is adopted for this work as it is a standard value established for unit weight of soils [3].

Finally, a statistical parameter called variate of the standard normal distribution, $Z_\alpha$, has to be determined. The variable $Z_\alpha$ represents the number of standard deviations from the mean. If $\alpha$ is a dimensionless parameter such that $0 < \alpha < 1$, then $Z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution curve and $a (1- \alpha)$ is the confidence level of the sample. Confidence level is the probability that a specified interval around the value of a parameter estimated from a sample actually includes the value that would be calculated from the whole population.

The required sample set size, $n$, is then determined from [8]:

$$n = \left(\frac{Z_{\alpha/2} \times COV}{e}\right)^2$$  (9)

Finally, for a COV of 10%, an $e$ of 3% and a $Z_{0.05}$ of 1.96 (normal distribution), Eq. (9) yields the required sample size, $n$, of 43 for this study.

Application of Sampling Theories

A simple random sampling is a statistical method that allows every possible combination of sample units to be selected. The possible combinations are limited only by the sample size. Random sampling is accomplished by ensuring that at any stage of sampling the selection of a particular unit is not influenced by other units that have already been selected.

RANDBETWEEN(RB) is an in-built statistical function in the common application software of Microsoft Excel. It is used for selecting a random number between the numbers specified. The underlying principle of this function is that every number has an equal chance to be selected. A new random number is returned every time the Excel worksheet, on which one is working, is manipulated. The syntax of this function is RANDBETWEEN (A, B), where A and B are the
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smallest and largest integer numbers, respectively, which the function will return. The random sampling is applied as shown in Table 3.

By running each function indicated in Table 3, forty three times and dividing the generated numbers by 1000, it was possible to find 43 gradation data. Generally, percentage of a soil passing a certain sieve size must decrease as its particle size increases. However, a close observation the generated numbers reveals that this condition has not been met. A column wise or row wise rearrangement adjustment had to be made to get possible gradation distributions. With the adjustment made, the gradation coefficients were calculated.

Table 3: Application of 'Randbetween' function

<table>
<thead>
<tr>
<th>SIEVE SIZE (MM)</th>
<th>PERCENTAGE PASSING (%)</th>
<th>FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lower Bound</td>
<td>Upper Bound</td>
</tr>
<tr>
<td>38.1</td>
<td>75</td>
<td>85</td>
</tr>
<tr>
<td>19</td>
<td>63</td>
<td>79</td>
</tr>
<tr>
<td>9.5</td>
<td>53</td>
<td>73</td>
</tr>
<tr>
<td>4.75</td>
<td>30</td>
<td>70</td>
</tr>
<tr>
<td>2.36</td>
<td>26</td>
<td>56</td>
</tr>
<tr>
<td>1.18</td>
<td>16</td>
<td>44</td>
</tr>
<tr>
<td>0.6</td>
<td>10</td>
<td>37</td>
</tr>
<tr>
<td>0.3</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>0.15</td>
<td>3</td>
<td>17</td>
</tr>
<tr>
<td>0.075</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

There is another type of sampling method in which the total population is broken into a number of strata or subpopulation and a random sample is taken from each stratum. This sampling method is called stratified sampling, and allows for a statistical analysis of variability within and between strata.

The gradation of any granular soils can be classified under one of the three stratified groups that are established using the ranges of values of gradation coefficients $C_u$ and $C_r$. These three groups are shown in Table 4 as Case A, B, and C. Thus, the required number of test gradations for each stratified sample case is determined by applying the random number function 'RANDBETWEEN (14,15)', as draws of 14 twice and 15 once yielding totally 43 different gradations.

<table>
<thead>
<tr>
<th>Case</th>
<th>$C_u$</th>
<th>$C_r$</th>
<th>Number of Samples</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$1 \leq C_u \leq 3$</td>
<td>$C_r &gt; 4$</td>
<td>13</td>
</tr>
<tr>
<td>B</td>
<td>$C_u &gt; 3$</td>
<td>$C_r &gt; 4$</td>
<td>16</td>
</tr>
<tr>
<td>C</td>
<td>$C_u &lt; 1$</td>
<td>$C_r &gt; 4$</td>
<td>14</td>
</tr>
</tbody>
</table>

Test Results

The laboratory work started by sieving separately the two soil samples. About 1 ton of soil was used from each type. The fractions retained on each sieve size were placed separately and blending was made from each equivalent size of the two granular soils in accordance with the desired gradation to prepare the 43 samples as established by Eq. (9). The Modified compaction test was then conducted. As the test material is expected to undergo a certain degree of disintegration due to the compaction effort, the compaction test was again made at
Maximum Dry Density of Local Granular Fills

exactly the optimum moisture content for each of the forty-three soil mixes. This discrepancy was accounted for by conducting dry and wet sieve analyses on specimens compacted at the OMC. Specific gravity tests were also made for all specimens. All the tests were carried out based on the AASHTO standard testing procedures. The details of the laboratory test results are available in [11].

REGRESSION ANALYSIS

Scatter Plot

Before embarking on any regression analysis, the normality of the dependent variable has to be assessed. As shown in Fig. 2, the MDD is practically normally distributed. As the sample size gets larger, it can be expected that the normal distribution of MDD becomes clearer.

In carrying out the whole statistical analysis, a commercial statistics software named S-Plus 2000 was employed.

The scatter plot of the dependent variable with each of the independent variables helps to see the existence of any correlation and identify the regressors, which influence MDD significantly.

Such plots are presented in Figs. 3 to 5 for MDD against the uniformity coefficient, \( C_u \), the coefficient of gradation, and the solid density, respectively.

![Histogram of the MDD](image)

![Scatter plot of \( C_u \) versus MDD](image)

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Even though a trend of banded distribution can be observed from visual inspection, one has to resort to formal regression analysis in order to study the existence of any possible correlation between the dependent and the independent variables. This involves the testing of different models, which is treated in the following section.

**Relationship Between the Regressor Variables and the Response**

Fitting a regression model requires several assumptions. Estimation of the model parameters requires the assumption that the residuals (actual values less estimated values) are uncorrelated random variables with mean zero and constant variance.

A number of techniques can be used to indicate the adequacy of a multiple regression model, some of which are the standard error, the adjusted coefficient of determination, and the t-test. The standard error of a statistic gives some idea about the precision of an estimate. Estimated standard errors are computed based on sample estimates. During modeling, a variable that shows the least standard error of estimates is chosen.

The adjusted coefficient of determination, $R^2$, shows the percentage of variation of the dependent variable. During regression analysis, a regression model with higher value of the adjusted $R^2$ is usually accepted.

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The t-test is one of the methods used to accept or reject a given hypothesis. The t-value is simply calculated as:

$$t = \frac{t_{\text{value}}}{\text{Standard error of the estimated coefficient}}$$

(10)

The P-value is a parameter that is used together with the t-value. It is simply the smallest level of significance, a, at which a variable is significant. If the P-value is smaller than a, the particular regression is important in explaining the variation of the response variation in the model. If $Z_0$ is the computed value of the test statistic, then the P-value is $2(1-\Phi(Z_0))$ for a two-tailed test. Here, $\Phi(Z_0)$ is the standard normal cumulative distribution at $Z_0$.

Polynomial regression models are widely used when the plotted response has a curvilinear trend [10]. They are generalized as:

$$Y = \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \cdots + \alpha_n x_n + \alpha_0 x_1^2 + \alpha_1 x_1 x_2 + \cdots$$

(11)

where $\alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_n$ are coefficients of the independent variables, $x_1, x_2, x_3, \ldots, x_n$ are the independent variables (repressors) and $Y$ is the response. Polynomial functions of various degrees were fitted to the data with the results obtained as shown in Table 5.

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_4$</td>
<td>-2.635</td>
<td>3.99</td>
<td>-0.66</td>
<td>0.5132</td>
</tr>
<tr>
<td>$C_7$</td>
<td>25.8049</td>
<td>40.4365</td>
<td>0.6382</td>
<td>0.5274</td>
</tr>
<tr>
<td>$C_9$</td>
<td>-84.7582</td>
<td>147.8499</td>
<td>-0.5733</td>
<td>0.5700</td>
</tr>
<tr>
<td>$C_L$</td>
<td>106.8761</td>
<td>237.1424</td>
<td>0.4507</td>
<td>0.6549</td>
</tr>
<tr>
<td>$C_u$</td>
<td>-47.3460</td>
<td>162.2631</td>
<td>-0.2918</td>
<td>0.7721</td>
</tr>
<tr>
<td>$P_{\text{solid}}$</td>
<td>0.6230</td>
<td>0.0139</td>
<td>44.7096</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 5 shows that the most significant variable that describes well the variation in the data is $P_{\text{solid}}$, as depicted by the corresponding least value of the standard error and the P-value. The corresponding predicting equation is given as:

$$\text{MDD} = 0.623 P_{\text{solid}} - 1$$

The value of the adjusted coefficient of determination, $R^2$, is calculated using standard procedures as 0.9998. This indicates that 99.98% of the variation in MDD is explained by the equation. Furthermore, the variable $C_4$ has been found to be insignificant in explaining the variability of the dependent variable. The plot of Eq (12) is shown together with the scatter plot in Fig. 6.

![Figure 6: Plot of fitted data for first trial](Journal of EEA, Vol. 21, 2004)
It is obvious from Table 5 and Eq. (12) that there exists a lack of correlation between MDD and the coefficient of gradation parameters. In order to make sure if this trend holds for other definitions of gradation parameters, a different gradation coefficient has been selected, which is given by [13]:

\[ U = \frac{D_{75}}{D_{25}} \]  

(13)

where \( D_{75} \) and \( D_{25} \) are the grain sizes in millimeters corresponding to the 75% and 25% passing, respectively.

The corresponding formulation then becomes:

\[ \rho_{d,\text{max}} = f(\rho_{\text{void}}, U, C_U) \]  

(14)

For the regression analysis, various exponential values for \( U \) were considered. The best fit that was obtained was for an exponential value of 1/5 as indicated in Table 6. The introduction of this new gradation parameter, \( U \), did not significantly improve the correlation. Nevertheless, the final equation becomes

\[ \text{MDD} = 0.615\rho_{\text{void}} + 0.1784C_U - 1.0 \]  

(15)

<table>
<thead>
<tr>
<th>Term</th>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-value</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^{15} )</td>
<td>0.2930</td>
<td>0.2297</td>
<td>1.2754</td>
<td>0.2095</td>
</tr>
<tr>
<td>( C_U )</td>
<td>0.1784</td>
<td>0.0863</td>
<td>2.0672</td>
<td>0.0452</td>
</tr>
<tr>
<td>( \rho_{\text{void}} )</td>
<td>0.6156</td>
<td>0.0044</td>
<td>140.3228</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Table 6 indicates the significance of \( C_U \) in explaining the variability of the observation data with the introduction of \( U \) in place of \( C_g \). The adjusted coefficient has become 0.9996. Thus, Eq. (15) is taken as the appropriate model for the problem at hand in lieu of Eq. (12). The relation found is presented in Fig. 7, where MDD is plotted against \( C_U \) for the common ranges of specific gravity of granular soils.

**Comparison of Results**

A comparison has been made between the actual MDD results against those predicted by existing prediction methods as well as by the presented method as shown in Table 7. As Maah’s equation is developed for standard Proctor tests, an average theoretical conversion factor of 1.07 has been applied to convert it to modified Proctor test results. It is evident from Table 7 that the formula presented in this work has a much stronger prediction capacity than the existing formulas for the material considered.

![Figure 7: Relationship between MDD and \( C_U \) for selected values of \( G_s \)](image-url)
In order to test the sensitivity of the developed formula for solid densities outside the range of values considered in this study (2.647 to 2.734), nine additional compaction tests were conducted on samples taken from the base-course material used in segments of the Bahir Dar-Merawi asphalt road construction project that is currently underway. This material is of different geological origin derived from crushing of sound basaltic rock that exhibits a Los Angeles (LA) Abrasion value of 16% and a range of specific gravity of solids between 2.93 and 2.98. The results of the comparison show as expected that the developed formula fails to sufficiently predict the MDD. This indicates that the developed formula may be used only for cases with gradations and specific gravities of solid grains falling within the ranges studied.

CONCLUSIONS AND RECOMMENDATIONS

The prediction formula is basically developed for materials similar to those used for the study and with gradation falling within the specified range. From the statistical analyses made in the previous sections, the following conclusions can be drawn.

1. The commonly employed coefficient of gradation $C_g$ is not a significant variable to explain the MDD of the type of soil studied. Instead, introduction of the definition given in Eq. (13) resulted in a better correlation of the MDD with the gradation. However, it can generally be concluded that gradation parameters are much less significant than solid density to characterize MDD.

2. The solid density, $\rho_{\text{solid}}$, and $C_v$ of the given soil are significant in explaining the variability of the observation. Particularly, $\rho_{\text{solid}}$ is found to be the most significant parameter.

3. The direct relationship observed between the specific gravity and MDD of the soil agrees with results of previous works. In fact, the developed relation exhibited the best predicting power for the types of soil studied.

4. From the comparison made with the two existing prediction approaches using the test results, Masih’s statistical method better predicted the MDD of the sample.

5. Further studies are needed to establish correlations for values of the important parameters outside the ranges considered in this paper.

6. It is recommendable to extend the study on the basis of gradation coefficients that could better describe the grain size distribution of granular soils. Such coefficients could possibly be defined by the researcher himself.

REFERENCES


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