AN EFFICIENT STRUCTURAL REANALYSIS MODEL FOR GENERAL DESIGN CHANGES AND BASED ON THE FLEXIBILITY METHOD

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ABSTRACT

Approximate structural reanalysis methods have long been pursued in quest for efficiency so as to enhance their practical application in the assessment and verification of designs following design modifications. These have been of significant practical importance with particular emphasis on design optimization of large-scale structural systems in which thousands of design modifications need to be carried out during the search operation to identify optimal systems. However, the consequential problems of accuracy and reliability call for the development of better-quality mathematical models that enhance computational quality while keeping the computational effort at a fraction of what might be required if complete and exact analysis would be carried out. This paper presents an efficient structural reanalysis mathematical model that is based on the flexibility method and exhibits high approximation qualities in evaluating structural responses of a newly proposed design from those of an initial trial design.

The mathematical model to be presented here is a flexibility-method based binomial-series approximation of structural responses for a general and unconditional change in design coordinates. It makes use of one set of results from a single exact analysis usually carried out for the initial trial design. Novel concepts of scaling and norm minimization have been introduced to gain accuracy, efficiency, and reliability. The proposed model has been compared with existing ones and also with exact analysis outputs and it has shown excellent approximating qualities even under significantly large design modifications.

A numerical example has been presented to show potential capabilities of the proposed model.

INTRODUCTION

Structural response quantities, such as generalized forces, displacements and vibration properties, are necessary in the process of assessing the adequacy of a proposed design with respect to some established criteria. There are a number of conditions that call for reanalysis of a structural system in such a process. Among these are included changes in structural properties such as the cross-sectional, geometrical, topological and material parameters. In this respect, therefore, rapid reanalysis of structures following changes in design coordinates is a problem of considerable practical importance.

Structural response quantities are usually implicit functions of the various structural properties mentioned above. In all changes that affect these and similar parameters, there is a need to carry out complete reanalysis to establish the magnitude and nature of structural responses under the new condition. When such changes take place repeatedly, such as, for example, in an iterative structural design optimization process or when the changes are significantly large as compared to the initially known design coordinate, the reanalysis operation calls for extensive computational effort, sometimes for thousands of such repeated operations.

An exact analysis procedure that is repeated several times to produce such quantities becomes prohibitive from view point of computational cost, its inefficiency and numerical instability. Apart from this, many available analysis theories and procedures are not tailored towards making use of already available structural response quantities to predict corresponding response parameters at different design coordinates. This calls for complete response determination leaving behind previous response values that otherwise could be effectively used in predicting and determining new response quantities that correspond to new design coordinates. In this regard, therefore, efficient method for reanalysis of structures that makes use of already available response data is a desirable phenomenon of significant practical importance. The need to use approximate reanalysis methods is further justified in the classical trial-and-change design operations as it is often unnecessary to analyze intermediate designs accurately. In order to alleviate these shortcomings of carrying out complete exact reanalysis of the design at every stage of the change, several methods have been proposed for approximate reanalysis of structures (see, for example [1,2]).

The primary objective of any approximate structural reanalysis model is to alleviate the need to carry out complete analysis of the structure while generating estimates of critical and potentially critical response quantities at the new design coordinate at a required level of accuracy and efficiency, and to compute the design sensitivity information [3,4]. These methods make use of as much information as possible that is generated at the preceding design coordinates. Accordingly, for any structural reanalysis technique to be worthy of consideration, its computational cost has to be less than that required by a fresh exact analysis and the procedure must possess features that help its integration into the repetitive design operation. In view of these underlying factors, the need to develop efficient and reliable approximate reanalysis algorithms with high level of accuracy becomes obvious. To this effect, therefore, this paper presents an efficient, reliable and high quality approximate reanalysis model with special emphasis on the force method of analysis.

In most cases, reliability and computational efficiency have been secured together with improved level of accuracy by limiting the amount of design changes in search algorithms. This approach, however, entails more computational effort both in the synthesis and the analysis phases as reanalysis is to be carried out at significantly large number of design coordinates in the design space. A more efficient procedure would then be to devise some means of enlarging the quality zones of the approximation models so that a search in a particular iteration stage can be effected in as few steps as possible. Such a procedure will reduce the total computational effort while it keeps the level of accuracy as high as possible.

In recent years, interest in the force method of analysis as a means of design-oriented reanalysis tool is growing. Among the promising features of force method of analysis for this purpose are comparatively lower degree of static indeterminacy compared to displacement degrees of freedom; relatively low sensitivity of forces to changes in design coordinates as compared to those of displacements; comparatively less influence of approximation errors on forces than on displacements; and, finally, option of selecting the analysis unknowns in the force method.

In the finite element force method of structural analysis, a structure is discretized into a number of finite elements connected at nodal points and nodal forces are taken as the unknowns. This method of analysis [5] is based on the overall enforcement of equilibrium conditions in the structural system and the subsequent satisfaction of displacement compatibility. The equations of equilibrium relate externally applied forces **P** to internal reaction forces **T** through the equilibrium matrix **E**, and for the entire structure, these equations are assembled to a form (see Annex):

$$\mathbf{E\,T} = \mathbf{P} \tag{1}$$

In statically indeterminate structures, the number of unknown internal forces **T** are larger than the number of available equilibrium equations. Consequently, the set of equations Eq. (1) is not sufficient for an explicit determination of **T**. The deficiency in solving for the internal force distribution is supplied for by a second set of equations from consideration of displacement compatibility. The compatibility equation system is:

$$\mathbf{F\,N} = \mathbf{D} \tag{2}$$

in which **F** is the flexibility matrix, **N** is the vector of redundant forces, and **D** the displacement compatibility vector corresponding to the generalized redundant forces.

Once the unknown quantities **N** are determined, it is a straightforward matter to find all other member forces from which stresses, displacements and other behavioral quantities of interest can be computed.

The approximate reanalysis methods to be presented here employ one set of exact analysis results in predicting similar quantities at new design points. To facilitate certain computational aspects of the resulting expressions, use is made of intervening variables in the form of cross sectional properties **X** (specifically, reciprocal values of these variables) in place of the corresponding original properties. As these variables usually represent cross-sectional areas or inertia moments, elements of both the flexibility matrix and the displacement compatibility vector in the compatibility equations, Eq. (2), become linear functions of these inverse variables. Hence, formulations in terms of reciprocal variables, **Y**, that is,

$$Y_i = \frac{1}{X_i} \qquad (3)$$

will give expressions that can easily be processed in the compatibility matrices.

In all subsequent formulations, availability of the decomposed initial flexibility matrix $\mathbf{F}_0$ into an upper and lower triangular matrices, $\mathbf{U}$ and $\mathbf{U}^T$, respectively, is assumed such that:

$$\mathbf{F}_0 = \mathbf{U}^T \mathbf{U} \qquad (4)$$

The triangularization of the flexibility matrix $\mathbf{F}_0$ in the form of Eq. (4) facilitates the determination of the forces $\mathbf{N}$ in Eq. (2) by forward and back substitutions rather than through inversion.

In the trial-and-change structural design approach, design changes can be introduced in various ways. These include general change in each design variable such as changes in cross-sectional parameters or property, material property, changes in geometry and topology of the system. Changes may also be introduced in a controlled manner such as those introduced along an arbitrary line in a design space or those along a design scaling line. In this study, reanalysis models are developed for the first type of change. The other forms will be presented in subsequent studies.

APPROXIMATE REANALYSIS MODELS FOR GENERAL DESIGN CHANGES

Binomial Series Approximation Model

This reanalysis model is obtained when an approximate solution to the compatibility equations is expressed in a binomial series expansion [6]. For a general design change $\Delta\overline{\mathbf{Y}}$, the compatibility equation Eq. (2) may be written as:

$$\left(\mathbf{F}_0 + \Delta\overline{\mathbf{F}}\right)\mathbf{N} = \mathbf{D}_0 + \Delta\overline{\mathbf{D}} \qquad (5)$$

where $\mathbf{N}$ is the behavior response to be approximated. Pre-multiplying both sides of Eq. (5) by $\mathbf{F}_0^{-1}$, and using $\overline{\mathbf{C}} = \mathbf{F}_0^{-1}\Delta\overline{\mathbf{F}}$, $\overline{\mathbf{N}}_D = \mathbf{F}_0^{-1}\Delta\overline{\mathbf{D}}$ and $\overline{\mathrm{N}} = \mathrm{N}_0 + \overline{\mathrm{N}}_D$ for brevity, solving for $\mathbf{N}$:

$$\mathrm{N} = \left(\mathrm{I} + \overline{\mathrm{C}}\right)^{-1} \overline{\mathrm{N}} \qquad (6)$$

Expanding $\left(\mathrm{I} + \overline{\mathrm{C}}\right)^{-1}$ in binomial series under the assumption that $\overline{\mathrm{C}}$ is a convergent matrix [7]):

$$\left(\mathrm{I} + \overline{\mathrm{C}}\right)^{-1} = \mathrm{I} - \overline{\mathrm{C}} + \overline{\mathrm{C}}^2 - \overline{\mathrm{C}}^3 + \ldots \qquad (7)$$

Equation (7) provides the binomial series approximation model for $\mathbf{N}$:

$$\mathrm{N} = \left(\mathrm{I} - \overline{\mathrm{C}} + \overline{\mathrm{C}}^2 - \overline{\mathrm{C}}^3 + \ldots\right)\overline{\mathrm{N}} \qquad (8)$$

The validity of a binomial series approximation model depends on the convergence properties of the matrix $\overline{\mathbf{C}}$. The accuracy of this model improves when more terms of the series are employed. Convergence of the series is guaranteed if the Euclidean norm of $\overline{\mathrm{C}}^n$ tends to vanish as n increases; that is,

$$\lim_{n\to\infty} \left\| \overline{\mathrm{C}}^n \right\| = 0 \qquad (9)$$

The quality of approximations by the binomial series models is sensitive to changes in the flexibility matrix. This reanalysis model exhibit poor performance if large design modifications are introduced. In view of this, any improvement on these models has its root in manipulating the flexibility matrix and, hence, the scaling operation is performed on the initial flexibility matrix.

Improved Binomial Series Approximation Model

In order to improve the response-prediction quality of the binomial series model of Eq. (8), a linear scaling of the initial flexibility matrix is suggested as follows:

$$\mathbf{F}_{0,s} = \mathrm{f}\,\mathbf{F}_0 \qquad (10)$$

in which $\mathbf{F}_{0,s}$ is the scaled flexibility matrix and f is a positive scaling factor for the initial flexibility matrix $\mathbf{F}_0$. This scaling operation in general need not produce a scaled flexibility matrix that corresponds to any specific design. For a general change in design, the matrix $\mathbf{F}$ at the new design coordinate can then be expressed as:

$$\mathrm{F} = \mathrm{F}_0 + \Delta\overline{\mathrm{F}} = \mathrm{f}\,\mathrm{F}_0 + \Delta\mathrm{F}_f$$

The scalar multiplier f is chosen such that the quality of response approximations by the binomial series model of Eq. (8) is improved.

To improve the quality of approximations by the binomial series model of Eq. (8), the proposed scaling procedure is employed. The compatibility equation, Eq. (2), as a function of the scaled flexibility matrix can be posed as:

$$\left(f\,\mathbf{F}_0 + \Delta\mathbf{F}_f \right)\mathbf{N} = \mathbf{D}_0 + \Delta\overline{\mathbf{D}} \qquad (12)$$

Substituting for $\Delta\mathbf{F}_f$ from Eq. (11) into Eq. (12) and employing the notations used along with Eq. (5) with some rearrangement gives:

$$\left(\mathbf{I} + \frac{1-f}{f}\mathbf{I} + \frac{1}{f}\overline{\mathbf{C}} \right) f\,\mathbf{N} = \overline{\mathbf{N}} \qquad (13)$$

Define, for brevity,

$$\mathbf{C}_f = \frac{1-f}{f}\mathbf{I} + \frac{1}{f}\overline{\mathbf{C}} \qquad (14)$$

Making use of this last expression of Eq. (14) in Eq. (13), and expanding $\left(\mathbf{I}+\mathbf{C}_f\right)^{-1}$ under certain conditions to be presented subsequently, the following improved binomial series approximation model for the redundant forces **N** is obtained:

$$\mathbf{N} = \frac{1}{f}\left(\mathbf{I} - \mathbf{C}_f + \mathbf{C}_f^2 - \mathbf{C}_f^3 + \ldots \right)\overline{\mathbf{N}} \qquad (15)$$

It is now required to determine an effective and efficient scaling factor f that will improve the convergence properties of Eq. (15) over those of Eq. (8).

A simple criterion based on norm-minimization of the matrix $\overline{\mathbf{C}}$ is now proposed to find an effective and efficient scaling factor f. If the binomial series expansion of Eq. (15) is valid, the resulting approximation is greatly influenced by the first few terms. If the effect of these dominant terms of the series is minimized, that of the rest of the terms in the series vanishes progressively. The criteria to be proposed take this fact into consideration and norm-minimization is carried out on the most influential term of the series of Eq. (15) which is the second term of the series.

The criterion to be proposed for the selection of an effective scaling factor f is that of minimizing the Euclidean norm of the matrix $\mathbf{C}_f$, that is,

$$\left\| \mathbf{C}_f \right\| = \left(\sum\sum \mathbf{C}_{f,ij}^2 \right)^{1/2} \rightarrow \min \qquad (16)$$

Substituting for $\mathbf{C}_f$ from Eq. (14) into Eq. (16), one obtains:

$$\left[\sum\sum \left(\frac{1-f}{f}\delta_{ij} + \frac{1}{f}\overline{C}_{ij} \right)^2 \right]^{1/2} \rightarrow \min \qquad (17)$$

where δ_{ij} is the Kronecker's delta.

Differentiating Eq. (17) with respect to f, noting that $f \neq 0$ and assuming $\sum\sum\left(\frac{1-f}{f}\delta_{ij} + \frac{1}{f}\overline{C}_{ij} \right)^2 \neq 0$, finally setting the resulting expression to zero:

$$-\sum\sum\left[(1-f)\delta_{ij} + \overline{C}_{ij} \right]\left(\delta_{ij} - \overline{C}_{ij} \right) = 0 \qquad (18)$$

Solving for f from Eq. (18),

$$f = \frac{\sum\sum\left(\delta_{ij} + \overline{C}_{ij} \right)^2}{\sum\sum \delta_{ij}\left(\delta_{ij} + \overline{C}_{ij} \right)} \qquad (19a)$$

or

$$f = \frac{\sum\sum\left(\delta_{ij} + \overline{C}_{ij} \right)^2}{\sum\sum\left(1 + \overline{C}_{ii} \right)} \qquad (19b)$$

The criterion of Eqs. (19) provides a model with large quality zone from which the most efficient scaling factor can be extracted. Determination of the redundant forces follows by employing Eq. (15).

As it will be demonstrated subsequently by numerical examples, the proposed reanalysis model almost always improves convergence properties, including numerical stability, and usually only the first few terms of the series are sufficient to obtain high quality approximations of the forces. In most cases, even for relatively large changes in the design coordinate, the first two terms of the model often provide high level of accuracy so that:

$$\mathbf{N} \approx \frac{1}{f}\left(\mathbf{I} - \mathbf{C}_f \right)\overline{\mathbf{N}} \qquad (20)$$

The determination of the scaling factors f as a means of improving convergence of the matrix $\overline{\mathbf{C}}$ has been based on the Euclidean norm of the latter. The choice of the Euclidean norm for this purpose is based on the ease and efficiency of its determination. Euclidean norms constitute upper bounds to the spectral radius [8]; that is:

$$\rho\left(\mathbf{C}_f\right) = \max_{\mathbf{X}\neq 0} \frac{\left\| \mathbf{C}_f\,\mathbf{X} \right\|}{\left\| \mathbf{X} \right\|} \leq \left\| \mathbf{C}_f\,\mathbf{X} \right\|_F \qquad (21)$$

where $\|C_f X\|_F$ is the Euclidean norm of C_f and the proposed norm-minimization approach minimizes this upper bound. It will be shown that the scaling multiplier f selected by the criterion of Eqs. (19) and under a variety of changes in design coordinates almost invariably improves convergence properties and is capable of a converting non-convergent matrix $\bar{C}$ to a corresponding convergent C_f.

NUMERICAL EXAMPLES

Example No. 1: The potential capabilities of the proposed reanalysis model is tested on flexural system using the rigid plane frame structure with six degrees of static indeterminacy, Fig. 1.1. The structural layout, designation of redundant forces and the set of external actions are given in Tables 1.1, 1.2, and 1.3 respectively. Test data for all the three case are given in Table 1.4.

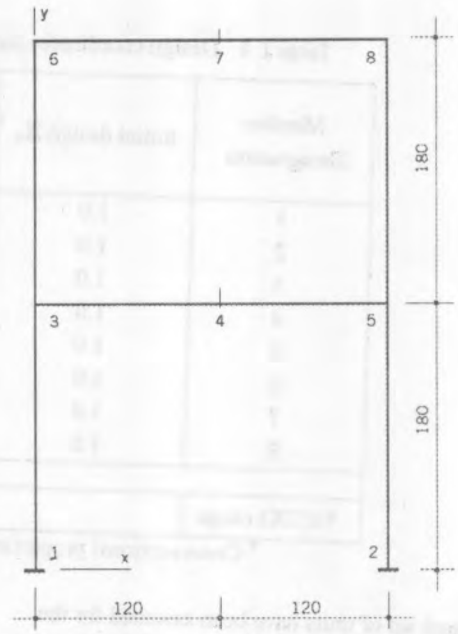


Fig. 1.1 Eight-member test model.

Table 1.1: Member designation and orientation.

Member	Orientation
1	1-3
2	3-6
3	6-7
4	7-8
5	3-4
6	4-5
7	2-5
8	5-8

Table 1.2: Redundant designation.

Designation	Moment
1	M_{13}
2	M_{25}
3	M_{45} (M_{43})
4	M_{36}
5	M_{58}
6	M_{78} (M_{76})

Table 1.3: Load components

Node	Loading components		Member	Loading components	
	x	y		x	y
3	45.0	0.0	3	0.0	-6.0
6	45.0	0.0	4	0.0	-6.0
			5	0.0	-6.0
			6	0.0	-6.0

Table 1.4: Design coordinates for initial and modified designs[†].

Member Designation	Initial design $\mathbf{X}_0$	Design coordinates for reanalysis	
		Case 1	
		X	% (□**X**)
1	1.0	0.51013	-48.99
2	1.0	0.14084	-85.92
3	1.0	0.12523	-87.48
4	1.0	0.12523	-87.48
5	1.0	0.40605	-59.40
6	1.0	0.40605	-59.40
7	1.0	0.14048	-85.95
8	1.0	0.51053	-48.95
%(□**X**) range			-87.48 ~ -48.98

[†] Cross sectional properties **X** represent moment of inertia.

Consistent set of units have been assumed for the numerical example.

Example No. 2: The twenty-five-bar tower with seven degrees of static indeterminacy, Fig. 2.1, is presented as a structural problem upon which the potential capabilities of the proposed reanalysis model is tested.

The proposed reanalysis model is tested on two sets of new design coordinates that vary in magnitude and in the nature of changes of the design coordinates. These include a design that changes about 9000% from the base analysis point with perturbation of ±1000% of the initial design coordinate (Case 1) and another design in which different degree of random changes have been introduced (Case 2). The structural layout designation of redundant forces are given in Tables 1 and 2, respectively. Test data for all the three case are given in Table 3.

Consistent set of units have been assumed for the numerical example.

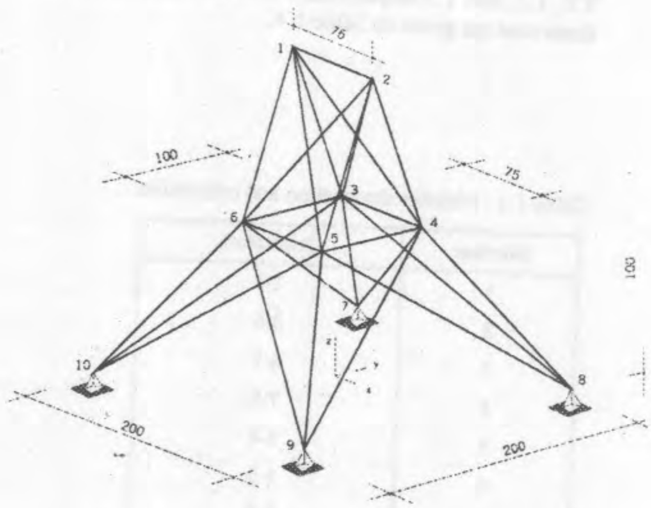


Figure 2.1 Twenty-five-bar transmission tower test model.

Table 2.1: Member designation and orientation.

Member	Orientation	Member	Orientation	Member	Orientation	Member	Orientation	Member	Orientation
1	1-2	6	2-4	11	4-5	16	4-9	21	6-9
2	1-4	7	2-5	12	3-4	17	5-8	22	6-10
3	2-3	8	1-3	13	5-6	18	4-7	23	3-7
4	1-5	9	1-6	14	3-10	19	3-8	24	4-8
5	2-6	10	3-6	15	6-7	20	5-10	25	5-9

Table 2.2: Redundant force designation.

Designation	Force	Designation	Member
1	F_1	5	F_{23}
2	F_{12}	6	F_{24}
3	F_{13}	7	F_{25}
-4	F_{22}		
4	F_{22}		

Table 2.3: Nodal load components.

Node	Loading components		
	x	y	z
1	100.0	1000.0	-500.0
2	0.0	1000.0	-500.0
3	50.0	0.0	0.0
4	50.0	0.0	0.0

Table 2.4: Design coordinates for initial and modified designs†.

Member Designation	Initial design $\mathbf{X}_0$	Design coordinates for reanalysis			
		Case 1		Case 2	
		X	% (Δ**X**)	**X**	% (ΔX)
1	10.0	101.0	910.0	3.0	-70.0
2	10.0	105.0	950.0	16.0	60.0
3	10.0	110.0	1000.0	4.0	-60.0
4	10.0	92.0	820.0	22.0	120.0
5	10.0	93.0	830.0	20.0	100.0
6	10.0	90.0	800.0	16.0	60.0
7	10.0	109.0	990.0	3.0	-70.0
8	10.0	98.0	880.0	19.0	90.0
9	10.0	105.0	950.0	9.0	-10.0
10	10.0	106.0	960.0	18.0	80.0
11	10.0	98.0	880.0	20.0	100.0
12	10.0	96.0	860.0	16.0	60.0
13	10.0	105.0	950.0	12.0	20.0
14	10.0	107.0	970.0	4.0	-60.0
15	10.0	92.0	820.0	6.0	-40.0
16	10.0	91.0	810.0	2.0	-80.0
17	10.0	104.0	940.0	15.0	50.0
18	10.0	98.0	880.0	12.0	20.0
19	10.0	98.0	880.0	17.0	70.0
20	10.0	106.0	960.0	23.0	130.0
21	10.0	103.0	930.0	8.0	-20.0
22	10.0	97.0	870.0	15.0	50.0
23	10.0	102.0	920.0	5.0	-50.0
24	10.0	94.0	840.0	20.0	100.0
25	10.0	95.0	850.0	4.0	-60.0
%(□X) range			800.0 ~ 1000.0		-80.0 ~ +130.0

† Cross sectional properties **X** represent area.

TEST RESULTS

The responses of the structure to the set of external actions have been determined using both the series of Eq. (8) and its improved version given in Eq. (15). In both cases, tabulated values are those obtained using only the first few terms of the series as noted in the corresponding tables. Exact analyses have also been carried out using STAAD.Pro 2004 [8] for the purpose of comparison.

Test results for the eight-member plane frame of Example No. 1 are given in Table 3.

Convergence of the reanalysis models is given in Fig. 3. Since the model given by Eq. (8) is not a converging one, only the convergence history of the improved reanalysis model given by Eq. (15) is given.

Similarly, test results for both case of the twenty-five bar tower of Example No. 2 are given in Tables 4 while convergence history of of the reanalysis models is given in Figs. 4. Table 4a provides results for test Case No. 1. while Table 4b is for test Case No. 2. In, the same manner, Table 4a provides convergence history of both models given by Eq. (8) and Eq. (15) for test Case No. 1. On the other hand, since the model given by Eq. (8) is not a converging for test Case No. 2, only the convergence history of the improved reanalysis model given by Eq. (15) is shown in Fig. 4b.

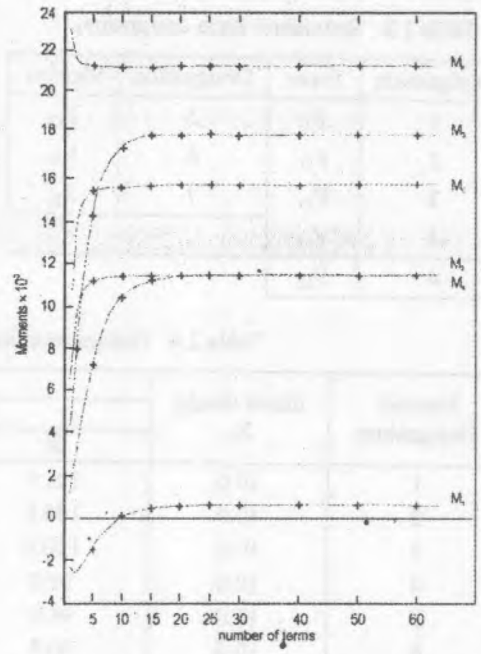


Figure 3 Convergence history - Eight-member test model

Table 3: Reanalysis results for plane frame of Example No. 1.

Redundant M	Initial analysis M_0	Reanalysis results				
		Model of Eq. 8		Model of Eq. 15 †		Exact M (STAAD.Pro 2004)
		M	% deviation.	M	% deviation	
1	5028.02	Does not converge.		515.692	-0.1	516.14
2	5112.63			11314.740	-0.0	11315.32
3	175.87			17928.886	-0.0	17928.93
4	1386.08			11306.853	0.0	11306.86
5	1732.19			15578.711	-0.0	15578.72
6	211.10			21201.237	0.0	21201.24

† Twenty-term approximation.

Table 4a: Reanalysis results for twenty-five bar tower of Example No. 2, Case 1.

Redundant N	Initial analysis N_0	Reanalysis results for Case 1				
		Model of Eq. 8‡		Model of Eq. 15‡		Exact N (STAAD.Pro 2004)
		N	% deviation	N	% deviation	
1	72.06	32.575	-74.3	126.652	0.0	126.61
2	166.11	69.471	-64.0	193.203	0.0	193.22
3	-135.11	-91.588	37.3	-146.005	0.0	-146.02
4	998.44	462.976	-54.1	1008.460	0.0	1008.45
5	-1239.86	-585.681	53.6	-1261.874	0.0	-1261.86
6	-1402.21	-664.416	51.9	-1382.017	0.0	-1382.00
7	880.97	389.694	-38.4	830.766	0.0	830.75

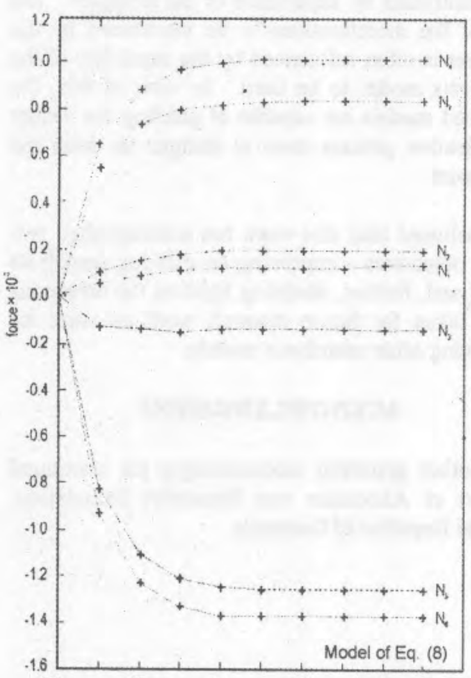
‡ Six-term approximation.

Table 4b: Reanalysis results for twenty-five bar tower of Example No. 2, Case 2.

Redundant N	Initial analysis N_0	Reanalysis results for Case 2				
		Model of Eq. 8		Model of Eq. 15[‡]		Exact N (STAAD.Pro 2004)
			% deviation	N	% deviation	
1	72.06	Does not converge.		293.431	0.0	290.40
2	166.11			382.606	0.0	317.84
3	-135.11			-415.902	0.0	-474.37
4	998.44			907.869	0.0	828.68
5	-1239.86			-894.021	0.0	-892.53
6	-1402.21			-1522.901	0.0	-1660.16
7	880.97			600.743	0.0	619 82

[‡] Six-term approximation.

Convergence of the reanalysis models is given in Figs. 4. Both model of Eq. (8) and Eq. (15) are convergent for test Case No. 1. However, the model given by Eq. (8) is not a converging for Case 2; accordingly, only the convergence history of the improved reanalysis model given by Eq. (15) is given for this latter case. In order to give more insight into the convergence properties of the models, several terms of the series have been included in the convergence history.



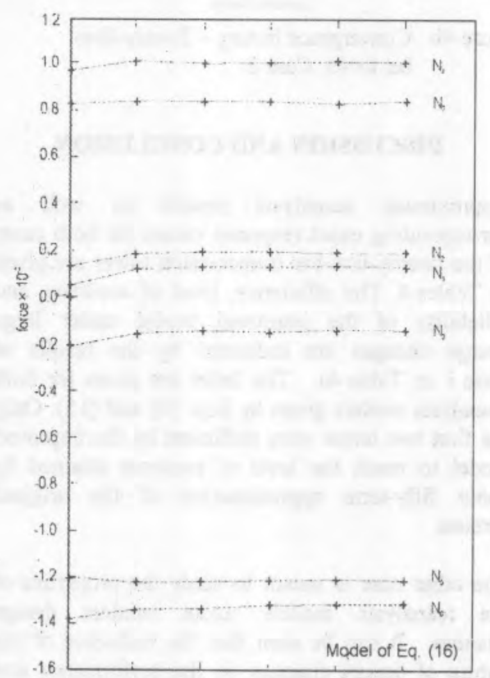


Figure 4a Convergence history - Twenty-five-bar tower, Case 1.

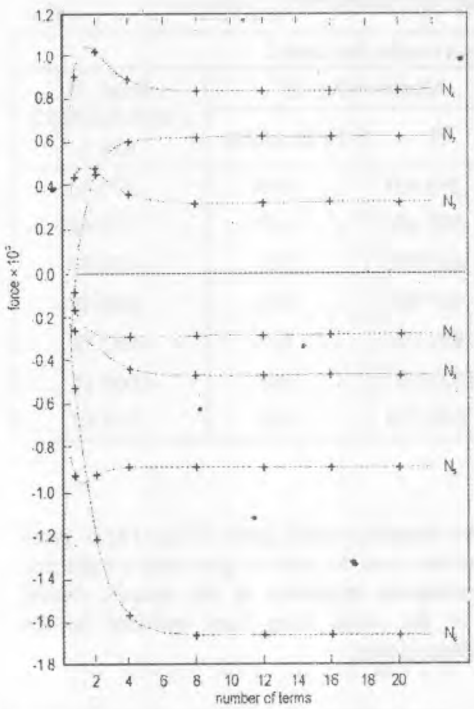


Figure 4b Convergence history – Twenty-five-bar tower, Case 2.

DISCUSSION AND CONCLUSION

Approximate reanalysis results as well as corresponding exact response values for both cases of the twenty-five-bar transmission tower are given in Tables 4. The efficiency, level of accuracy, and reliability of the improved model under large design changes are indicated by the results of Case 1 in Table 4a. The latter are given for both reanalysis models given by Eqs. (8) and (15). Only the first two terms were sufficient by the improved model to reach the level of accuracy attained by about fifty-term approximation of the original version.

The other case is meant to study the properties of the reanalysis models under random design changes. It can be seen that the influence of the nature of design changes on the performance and prediction quality of reanalysis models is no less than that of the magnitude of the changes. A few more terms of the improved model produce exact results.

In general, the improved model exhibits highly improved quality in response approximation when large changes are introduced. The test problems have indicated that the reanalysis model of Eq. (15) is always exceedingly better than its predecessor, Eq. (8). High quality approximations have been obtained by using only the first few terms of the infinite series.

In very few cases, these models have exhibited slow rate of convergence; however, once convergence is guaranteed, further efficiency can be introduced by employing dynamic acceleration techniques whereby the asymptotic limit of the response quantities **N** can be predicted based on a few terms of the series . This can be carried out either for individual N_i by Aitken's [9] acceleration or by introducing a common acceleration parameter for all response quantities **N** by the modified Aitken's method [10].

The implication of these high-quality reanalysis models is off-hand clear. The size of changes in design coordinates in any design modification is in part controlled by experience of the designer. The size of the modifications to be introduced by the designer is often influenced by the capability of the reanalysis model to be used. In view of this, the proposed models are capable of guiding the design modification process even at designs far from the base point.

It is believed that this work has accomplished two major objectives – improving an existing reanalysis model and, further, shedding light on the directions to be taken for future research work on ways for improving other reanalysis models.

ACKNOWLEDGMENT

The author gratefully acknowledges the continued support of Alexander von Humboldt Foundation, Federal Republic of Germany.

ANNEX

Review of the Generalized Force Method of Structural Analysis

In the finite element force method of structural analysis, a structure is discretized into a number of finite elements connected at nodal points and nodal forces are taken as the unknowns. This method of analysis [5] is based on an overall enforcement of equilibrium conditions in the structural system and a subsequent satisfaction of compatibility.

The equations of equilibrium relate externally applied loads **P** to internal reaction forces **T**, and for the entire structure these equations can be assembled to a form:

$$\mathbf{E}\mathbf{T}=\mathbf{P} \qquad (A.1)$$

where **E** the equilibrium matrix.

The equilibrium matrix **E** is a function of structural geometry and it is independent of the elements' sizes. In statically indeterminate structures, the number of unknown internal forces **T** is larger than the number of available equilibrium equations. Consequently, the set of equations Eq (A.1) is not sufficient for an explicit determination of **T**.

The deficiency in solving for the internal force distribution is supplied for by a second set of equations from consideration of displacement compatibility. The compatibility equation system is:

$$\mathbf{F}\,\mathbf{N}=\mathbf{D} \qquad (A.2)$$

where **F** is the flexibility matrix, **N** is the vector of redundant forces, and **D** is the displacement compatibility vector corresponding to the redundant forces.

In a statically indeterminate structure, the internal force system **T** can be expressed as a sum of two component distributions: the first due to externally applied loads on the base structure, and the second due to the redundancies assumed to be acting along fictitious cuts. This can be expressed as:

$$\mathbf{T}=\mathbf{T}_p\,\mathbf{P}+\mathbf{T}_n\,\mathbf{P} \qquad (A.3)$$

in which $\mathbf{T}_p$

represents the internal force distribution due to unit external forces on the base structure, and $\mathbf{T}_n$ is the internal force distribution due to unit redundant forces applied across fictitious cuts.

The internal force distribution matrices $\mathbf{T}_p$, $\mathbf{T}_n$ independently satisfy equilibrium; however, they individually violate compatibility. The matrix $\mathbf{T}_n$ can be interpreted as self-equilibrating forces system whose magnitudes **N** need to be determined.

Both $\mathbf{T}_p$ and $\mathbf{T}_n$ show specific patterns. The former contains a generally non-zero submatrix $\mathbf{T}_p$ associated with the determinate base structure and a zero submatrix with as many rows as there are indeterminacies. This can be expressed as:

$$\mathbf{T}_p=\left[\frac{\mathbf{T}_p'}{0}\right] \qquad (A.4)$$

On the other hand, the $\mathbf{T}_n$ matrix incorporates an identity matrix corresponding to the ' cut ' redundant members, besides a generally nonzero submatrix $\mathbf{T}_n$ related to the base structure. This can be expressed as:

$$\mathbf{T}_n=\left[\frac{\mathbf{T}_n}{\mathbf{I}}\right] \qquad (A.5)$$

The internal force distribution of Eq. (A.3) in a redundant structural system can then be expressed in terms of Eqs (A.4) and (A.5) as:

$$\mathbf{T}=\left[\begin{array}{cc}\mathbf{T}_P & \mathbf{T}_n\\ \hline \mathbf{0} & \mathbf{I}\end{array}\right]\left[\frac{\mathbf{P}}{\mathbf{N}}\right] \qquad (A.6)$$

The set of system equilibrium equations (A.1) with a modification that the internal redundants are now being considered as externally applied loads attains the form:

$$\left[\begin{array}{cc}\mathbf{T}_P'^{-1} & \mathbf{T}_n'^{-1}\mathbf{T}_n'\\ \hline \mathbf{0} & \mathbf{I}\end{array}\right]\mathbf{T}=\left[\frac{\mathbf{P}}{\mathbf{N}}\right] \qquad (A.7)$$

with the partitioning exactly as in Eq. (A.6).

Satisfaction of compatibility conditions demands that the magnitude of the redundant forces **N** be selected to ensure zero relative displacements at the fictitious cuts. The compatibility condition of zero relative displacements would be:

$$\mathbf{T}_n^t\,\mathbf{f}\,\mathbf{T}=\mathbf{0} \qquad (A.8)$$

where **f** is diagonal matrix of element flexibilities.

This is in fact equivalent to Eq (A.2). Substituting for **T** from Eq. (A.3) into Eq. (A.8), the latter becomes:

$$\mathbf{T}_n^t\,\mathbf{f}\,\left(\mathbf{T}_p\,\mathbf{P}+\mathbf{T}_n\,\mathbf{N}\right)=\mathbf{0} \qquad \text{(A.9)}$$

Comparing this latest expression with Eq (A.2), it can be seen that:

$$\mathbf{F}=\mathbf{T}_n^t\,\mathbf{f}\,\mathbf{T}_n \qquad \text{(A.10a)}$$

and
$$\mathbf{D}=-\mathbf{T}_n^t\,\mathbf{f}\,\mathbf{T}_p\,\mathbf{P} \qquad \text{(A.10b)}$$

Solving for the redundant unknowns in Eq (A.9):

$$\mathbf{N}=-\left(\mathbf{T}_n^t\,\mathbf{f}\,\mathbf{T}_n\right)^{-1}\left(\mathbf{T}_n^t\,\mathbf{f}\,\mathbf{T}_p\mathbf{P}\right) \qquad \text{(A.11)}$$

Equation (A.11) is the common expression for computing the redundant forces.

It is noteworthy in passing that the choice of a determinate base structure is a crucial point, especially when manual selection is made. Care should be exercised as inappropriate choices may create singularities and large numerical errors in solving the equations. Several proposals to this effect are now available for automatic selection of redundancies such as elimination or matrix decomposition methods [11, 12], or methods based upon solving a sub-problem by mathematical programming [13]. Once the redundant forces are determined, any other element force of interest can be evaluated easily by the equilibrium equations.

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