A LOW-PHASE NOISE FREQUENCY MULTIPLIER CHAIN

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ABSTRACT

In this paper a low phase-noise frequency multiplier chain from 10MHz to 480MHz, built in two stages and intended for use as a base for the synthesis of frequencies upto the far infrared (FIR) region, is presented. Different methods of frequency multiplication were used and the associated circuit diagrams given. To investigate the performance of the chain, its spectral purity was measured. Measurements were made both in frequency and time domin and a set of graphs have been given. It has been verified that this chain is very stable and that it can be used for frequency synthesis upto 300GHz.

INTRODUCTION

The techniques of frequency multiplication are extensively used in many branches of Radio Engineering and applied physics [3]. In fact the synthesis of extremely accurate frequency upto the far infrared (FIR) region has put new vistas on the metrology of time and frequency. The measurements of time and frequency are basic operations which are influenced by the development of frequency synthesis techniques [10]. This development has impact not only on the measurement of time and frequency but also on different disciplines such as length standards and metrology, spectroscopy, cronomety, communication and relativistic experiments. The realization of microwave frequency standards (cesium, rubidium) and the subsequent development of new devices in the sub-millimeter (Mg beam, HCN laser) and the infrared region have imposed stringent specifications on frequency synthesis. Consequently, the driving crystal oscillators and the first multiplier stages must exhibt very reliable performance so as to be used in this field.

The realization of the desired accuracy and precision becomes easy and less expensive if there is careful optimization of the design regarding noise. The long term goal of frequency synthesis is to create a simple, reliable and less expensive model for synthesizing frequencies upto FIR and the visible radiation (VR) with good accuracy, stability and reproducibility as that of the driving source. In this way the accuracy and stability of excellent signal sources in the microwave region can be transferred into the FIR regions. Conversely, if in the future, it is discovered that most stable signal sources are in the FIR, then it will be simpler to transfer their stability and accuracy into other frequency spectrum region.

THEORETICAL DESCRIPTION OF FREQUENCY MULTIPLIERS

A. An Oscillator Model

For an ideal oscilator the output signal can be expressed by:

$$v(t) = V_o \sin(2\pi\nu_o t + \psi_o) \tag{1}$$

where V_o and ν_o are the amplitudes and the nominal frequency respectively. ψ_o is the initial phase angle. For such a signal the power spectral density is:

$$S_v(f) = P_o\,\delta(f-\nu_o) \tag{2}$$

where δ is Dirac's delta
P_o; is the emitted power

The total power is:

$$P_T = \int_0^\infty S_v(f) = \int_0^\infty P_0\,\delta(f-\nu_0)\,dt = P_0 \tag{3}$$

For real oscillators, pertur-bations introduced by noise has to be considered. In such a case:

$$v(t) = (V_0 + \varepsilon(t))\sin(2\pi\nu_0 + \phi(t)) \tag{4}$$

where ε and ϕ are the amplitude and phase fluctuations, respectively

The angular frequency is:

$$\nu(t) = \nu_0 + \frac{1}{2\pi}\left(\frac{d\phi}{dt}\right) \tag{5}$$

and $\frac{d\phi}{dt}$ if it exists, represents the angular frequency fluctuation.

Rewriting (5), $\nu(t) = \nu_0 + \Delta\nu(t)$ [5a]

where $$\Delta\nu(t) = \frac{1}{2\Pi}\frac{d\phi}{dt}$$ [5b]

is a random process characterizing the frequency noise. For high quality oscillators

$$/\Delta\nu(t)/ << \nu_0 \text{ and } \left|\frac{\epsilon(t)}{V_0}\right| << 1$$

An important parameter is the instantaneous frequency deviation $y(t)$

$$y(t) = \frac{\Delta\nu(t)}{\nu_0}$$

$y(t)$ remains unaltered in the process of frequency multiplication or division.

The amplitude noise, $\epsilon(t)$, is generally negligible with respect to the phase noise, $\phi(t)$, or the frequency noise $\frac{d\phi}{dt}$, so that equation (4) can be written as

$$v(t) = V_0 \sin(2\Pi\nu_0 t + \phi(t)) \tag{6}$$

The frequency stability of the signal in eqn. (6) is usually characterized by the spectral density of the phase flunctuation, $S_\phi(f)$, in the frequency domain and by the Allan variance, $\sigma_y^2(\tau)$ in the time domain.

Definition 1

The spectral density of the phase noise, $S_\phi(\nu_0, f)$ is defined as

$$S_\phi(\nu_0, f) = \int_{-\infty}^{\infty} R_\phi(\tau)\exp(-j2\Pi f\tau)d\tau$$

$$= 2\int_0^{\infty} R_\phi(\tau)\cos(2\Pi f\tau)d\tau \text{ (rad}^2\text{/hz)} \tag{7}$$

where $R_\phi(\tau) = < \phi(t), \phi(t+\tau) >$

$$= \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\phi(t)\phi(t+\tau)dt$$

Definition 2:

$$\text{Let } y_k = \frac{1}{\tau}\int_{t_k}^{t_k+\tau}(\theta)\,d\theta \tag{8}$$

It can be shown that:

$$\overline{y_k} = (\theta(t_k+\tau) - \phi(t_k))/2\Pi\nu_0\tau \tag{9}$$

where the numerator indicates the phase error accumulated over the interval t_k and $t_k + \tau$. A mesurement of duration τ provides one sample $\bar{y}_k$. Assuming $y(t)$ and hence $\bar{y}_k$ has a zero mean, the variance is equal to the mean square value of $\bar{y}_k$ (1):

$$\text{i.e.} \quad \sigma^2(\bar{y}_k) = \langle \bar{y}_k \rangle \tag{10}$$

In the literatures $\sigma^2(\bar{y}_k)$ is usually indicated by $I^2(\tau)$ and it is called the true variance.

The sample variance is defined in several ways; one of which is:

$$\sigma^2(N, T, \tau) = \frac{1}{N-1}\left\{\bar{y}_i - \frac{1}{N}\sum_{j=1}^{N}\bar{y}_j\right\}^2 \tag{11}$$

For white noise and for $T = \tau$, one has

$$\langle \sigma^2(N, T, \tau) = I^2(\tau) \tag{12}$$

with $N = 2$, $T = \tau$ we have the so called **Allan Variance,** defined as:

$$\langle\sigma_y^2(2,\tau,\tau)\rangle = \langle \sum_{i=1}^{2}\left\{\bar{y}_i - ½\sum_{j=1}^{2}\bar{y}_j\right\}^2 \rangle \tag{13}$$

B. Characterization of a frequency Multiplier Chain

B.1 The ideal frequency multiplier

Frequency multiplication is generally obtained by utilizing a non linear device which is capable of producing harmonics of the input signal. If such a device is ideal then the input/output characteristics is of the form

$$v_o = f(v_i) \tag{14}$$

For a sinusoidal input signal, $V_i = A_o \sin(2\Pi v_o t + \phi_o)$ the output is of the form (using power series):

$$v_0 = \sum_{n=0}^{\infty} a_n V_i^n(t) = \sum_{n=0}^{\infty} a_n A_o^n \sin^n(2\Pi \nu_o t + \phi_o) \tag{15}$$

The coefficients, a_n, depend on the non-linear device used and the frequency of the input signal. For a signal perturbated with noise:

$$V_i(t) = A_o \sin(2\Pi\nu_o t + \phi(t)) \tag{16}$$

The N^{th} harmonic can be written as:

$$V_n(t) = A'_o \sin[2\Pi N\nu_o t + N\phi(t)] \tag{17}$$

where A'_o depend on A_o and a_n

Thus the autocorrelation function of the phase noise of the N^{th} harmonic is

$$\begin{aligned} R_{n\phi(\tau)} &= \langle N\phi(t), N\phi(t+\tau)\rangle \\ &= N^2 R_\phi(\tau) \end{aligned} \tag{18}$$

and the spectral density of the phase noise is

$$S_\phi(N\nu_o, f) = N^2 S_\phi(\nu_o, f) \tag{19}$$

From equation [19] we observe that the spectral purity of a signal, even under the assumption of ideal devices, is being deteriorated by a factor of N^2, where N is the frequency multiplication factor. This assumption is valid as long as $N^2\phi^2 << 1\text{rad}^2$

B.2. The practical frequency multiplier

In the real case the frequency multipliers are affected by their own internal noise and like any other non-linear device, have the tendency of AM to PM conversion of the input signal.

The characterization of the noise of a frequency multiplier is normally conducted on the spectral density of the phase-noise of the multiplier chain referred to the input (2). Hence

$$\begin{aligned} S_\phi(\nu_o, f) = {} & S_\phi^{sig}(\nu_o, f)\, G^2(f) \\ & + S_\alpha^{sig}(\nu_o, f) K^2(f) + S_\phi^{ch}(\nu_o, f) \end{aligned} \tag{20}$$

where

ν_o	the input signal frequency
f	the spectral frequency referred to ν_o
$S_\phi^{sig}(\nu_o, f)$	Power spectral density of the phase of the input signal
$S_\alpha^{sig}(\nu_o, f)$	Power spectral density of the amplitude of the input signal
$G^2(f)$	transfer function of the chain at the Fourier frequency, f
$K^2(f)$	the AM to PM conversion of coefficient of the chain at the frequency f

$\bar{S}_{\phi}^{ch}(\nu_o, f)$ power spectral density of the phase-noise of the chain referred to the frequency ν_o.

The signal to noise ratio (S/N) deteriorates along the chain as the square of the frequency multipliction factor, n and for n sufficiently large, the phase-noise in the side bands reduces the carrier power further such that at a certain frequency, ν^{coll}, the S/N ratio collapses. The collapse frequency, ν^{coll} can be expressd as (5):

$$\nu^{coll} = \nu_o \left\{ \left(\frac{\pi\alpha}{\sin(\pi/\alpha)}\right) B S_{\phi}^{\omega}(\tau_o) \right\}^{-1/2} \quad (21)$$

where $S_{\phi}^{\omega}(\nu_0)$ Power spectral density of phase-noise in its "white" region B and α are respectively the upper cut off frequency of the system and its asymptotic slope.

At $\nu = \nu^{coll}$, the carrier power reduce to e^{-1} of the total power (the rest being transferred to the side bands of the noise). Eqn. (21) tells us that to obtain large ν^{coll}, it is necessary to have low $S_{\phi}^{\omega}(\nu)$ and/or a narrow noise band, 2B. To have a narrow noise band one has to use a quartz filter at some appropriate point in the multiplier chain. From theoretical results shown in (5):

$$\frac{S}{N_\phi} = \frac{1}{B_W} \frac{2}{n^{2} S_{\phi}^{\omega}(\nu_o)} \left(\frac{\phi}{1-e^{-\phi}}\right)^{\frac{\alpha}{\alpha^{-1}}} e^{-\phi} \quad (22)$$

(for $B_W << B$)

where:

$$\phi = \int_0^\infty S_\phi(\nu, f)df = \left(\frac{\nu}{\nu^{coll}}\right)^2$$

B_W = the bandwidth of the filter of the spectrum analyzer.

while the width of the phase noise pedestal is:

$$\Delta\nu_P = 2B\left(\frac{\phi}{1-e^{-\phi}}\right)^{\frac{-1}{\alpha^{-1}}} \quad (23)$$

The spectral purity is guaranteed at every point of the chain if and only if

$$\left(\frac{P}{KT_e B_W}\right) >> \frac{S}{N_\phi} \quad (24)$$

where P is the signal power at the desired frequency KT_eB_W= noise power in the circuit

T_e= effective noise temperture

It can also be shown that (5), the minimum power at the frequency ν must be:

$$P_{min} >> \frac{2KT_e}{n^2 S_{\phi}^{\omega}(\nu_o)}; \quad \nu < \nu^{coll} \quad (25)$$

THE FREQUENCY MULTIPLIER CHAIN FROM 10MHz to 480MHz

The frequency multiplier chain from 10MHz to 480MHz was built in two stages (Fig. 1). The first stage is from 10MHz to 60MHz and the second one is from 60MHz to 480MHz. The chain is driven with a 10MHz quartz oscillator. The collapse frequency of this oscillator is calculated (eqn 21) to be 6.7 THz. Before it can be used to drive the chain, the quartz oscillator has to be isolated and also amplified and therefore we need the isolation amplifier.

A. The 10MHz to 60MHz Multiplier Chain

The symmetrical transistor configuration was used to realize this chain. Two chains were built which will later be used to make phase-noise measurements by the method of comparison. The first one is in two stages (3x2) i.e. a tripler in the push-pull configuration followed by a doubler in the push-push configuration (Fig. 2). The second one is built in only one stage (1x6) using a push-push configuration (Fig. 3). The output is tuned on the 6th harmonic and then fed into a 60MHz quartz filter. The quartz filter is not only to filter the undesired harmonic but mainly to reduce the side band of the noise (it has α = 40dB/dec and 2B= 20KHz)

B. THE 60MHz to 480MHz Multiplier Chain

To realize this stage a step-recovery (SRD) circuit is used (Fig.4). The SRD has the advantage of generating strong harmonics and therefore has a high efficiency of multiplication in only one step. However, one has to keep in mind that the SRD must be used in the last and not in the first stage because it doesn't satisfy the necessary spectral purity required in the 1st stage.

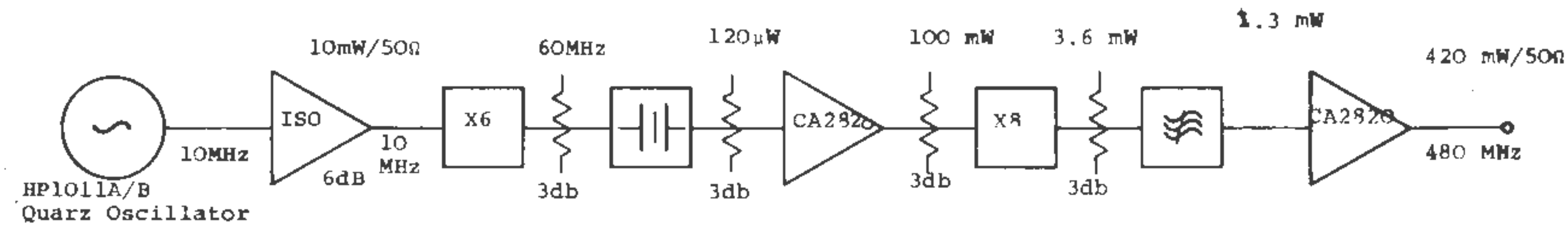


Fig.1: Block diagram of the frequency multiplier chain from 10MHz to 480MHz

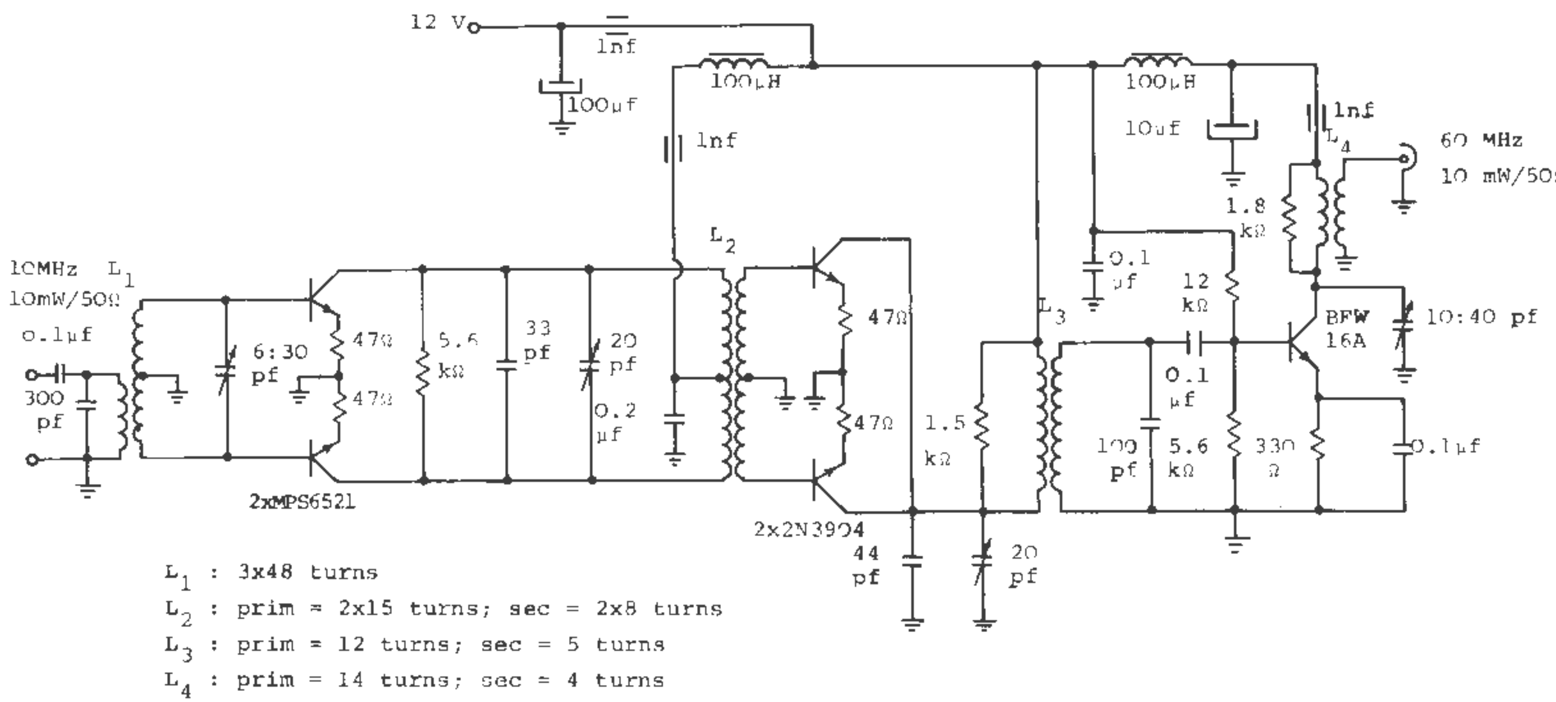


Fig. 2: The frequency multiplier chain from 10MHz to 60MHz in two steps.

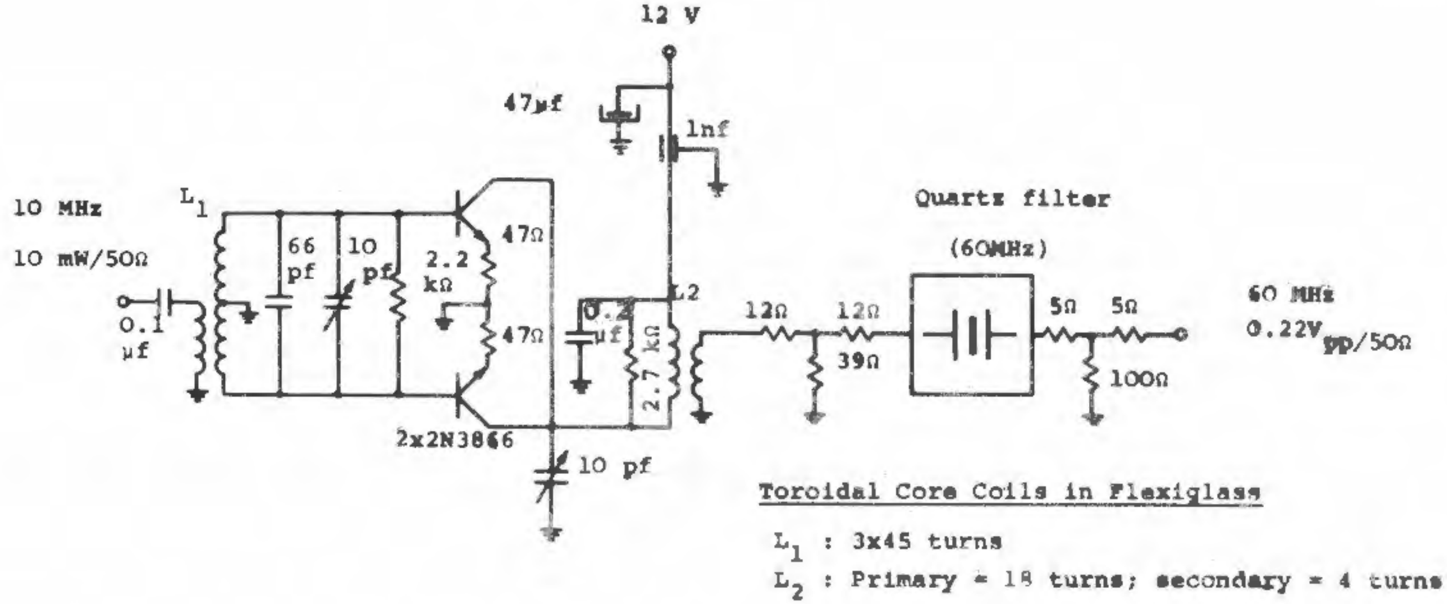


Fig.3: The frequency multiplier chain from 10MHz to 60MHz in one step.

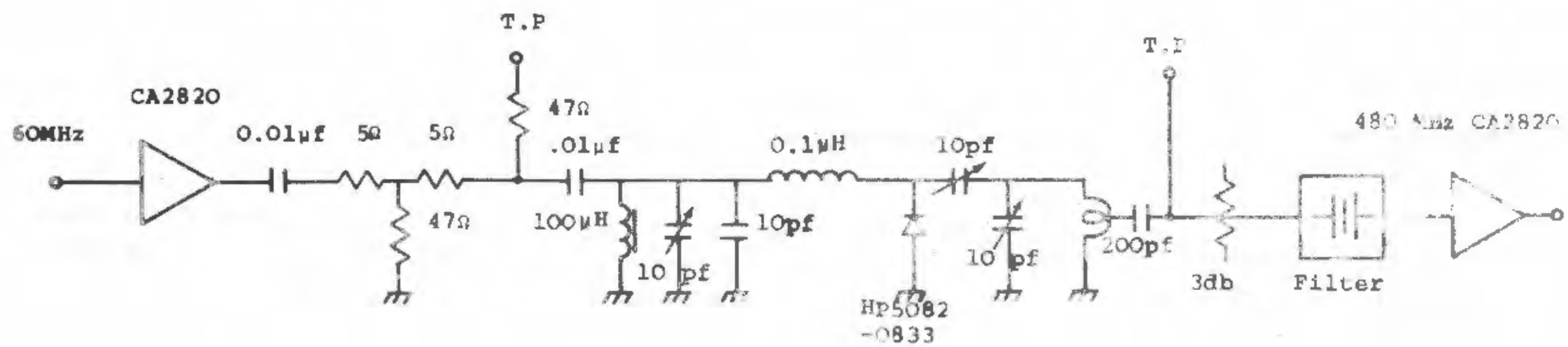


Fig. 4: The frequency multiplier chain from 60MHz to 480MHz

To eliminate spurious harmonics an interdigital filter tuned at 480MHz is used. This filter guarantees an output signal with spurious components not more thatn - 70 dBc inorder to avoid intermodulation in the following stage. Figures 5 (a) and (b) show respectively the 480 MHz signal before and after the interdigital filter.

THE PHASE NOISE MEASUREMENTS

A. Measurements in the Frequency Domain

What is being done here is to measure $S_\phi(\nu_O, f)$. The experimental set up is shown in figure 6 and the measurement is done on the two 60 MHz multiplier chains. The vector voltmenter measures the signal level at the inputs of the mixer (which has to be equal). is a phase displacing apparatus and it is varied until the two signals are in quadrature. The FFT 512/S is a low frequency spectrum analyzer and it measures the spectral density $\bar{S}_V$ (f) of the noise at various Fourier frequencies.

The sensitivity of the Measurement set up

By sensitivity of the measurement set up we mean the noise in the system

i.e. $$S_\phi(\nu_O, f) = \frac{1}{2K_D^2} S_V(f) \qquad (26)$$

where

$S_V(f)$ the noise spectrum at the output of the mixer (V^2_{rms}/Hz)

K_D the sensitivity of the mixer (V/rad)

This measurement is done in the absence of the two multiplier chains. The signal at the output of the isolation amplifier is fed into both inputs of the mixer.

On the screen of the spectrum analyzer we read the value of $S_1(f) = \gamma^2 S_V(f) G^2(f)$

where γ^2 is a calibration factor of the instrument and it depends on the type of noise (see table 1)

Table 1

	Type of noise	γ^2
f^0	white phase noise	1.6 (2dB)
f^{-1}	flicker phase noise	2 (3dB)
f^{-2}	white frequency noise	3.2 (5dB)
f^{-3}	flicker frequency noise	5 (7dB)

Expressing $S_\phi(\nu_O, f)$ in terms of S_V (f) we have

$$S_\phi(\nu_O, f) = \frac{1}{2} \frac{\bar{S}_V(f)}{K_D^2 \gamma^2 G^2(f)} \qquad (27)$$

where $S_V(f)$ is read on the spectrum analyzer

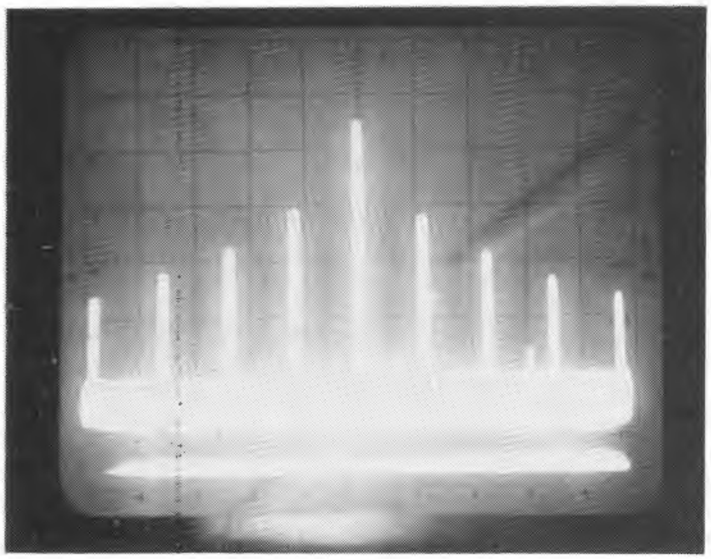
$$K_D = \frac{v_1 - v_2}{\psi_2 - \psi_2} = 0.2 \text{ volt/rad (measured)}$$

γ^2 = 2dB (from Table 1)

G(f) = 80dB (fixed)

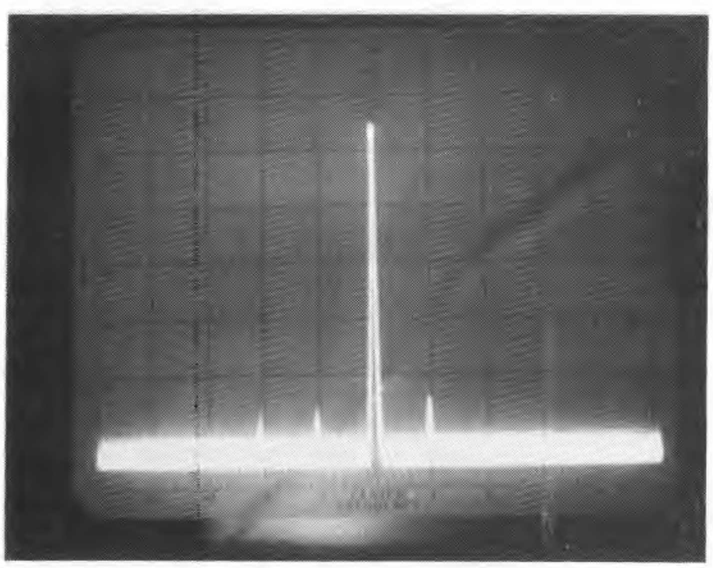
The value of $S_\phi(\nu_O, f)$ evaluated in this way for sufficient number of frequencies is plotted [Fig. 7 (a)].

The phase noise of the Isolation amplifier is also measured by applying the signal at the output of the two Isolation amplifiers into the two inputs of the mixer. The spectrum analyzer reads $\bar{S}_V$(f) and $S_\phi(\nu_O, f)$ is calculated as indicated in eqn. 27 and then plotted (Fig. 7 b). Finally the phase noise of the two multiplier chains at 60 MHz are measured and the result plotted [Fig. 7 (c)]. Since we are interested in the phase noise referred to the input, the measurement done at 60MHz has to be referred to the input (i.e. to 10MHz). Thus we have to subtract the contribution of the noise due to the multiplication factor N, which is 10 log N^2 = 10 log 36 = 15.6 dB. The result is plotted in Fig. 7. (d).



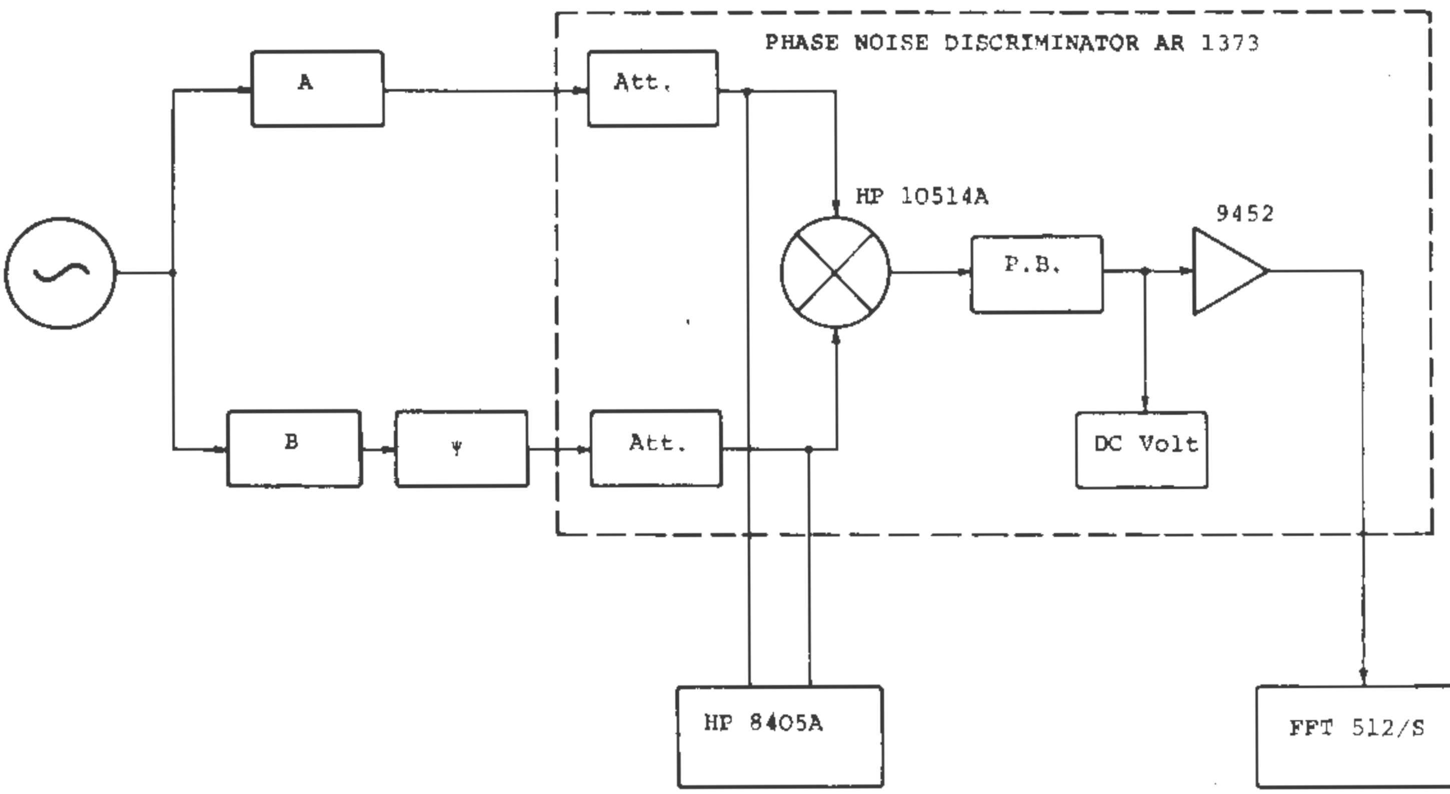
Vertical axis:10dB/div.
Horizontal axis:60MHz/div.
The 480MHz signal level:
3.6mW/50Ω

Fig.5(a): The 480MHz signal spectrum before the interdigital filter.



Vertical axis:10dB/div.
Horizontal axis:60MHz/div.
The 480MHz signal level:
1.3mW/50Ω

Fig.5(b): The 480MHz signal spectrum after the interdigital filter.


PHASE NOISE DISCRIMINATOR AR 1373
A
Att.
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DC Volt
HP 8405A
FFT 512/S

Fig. 6

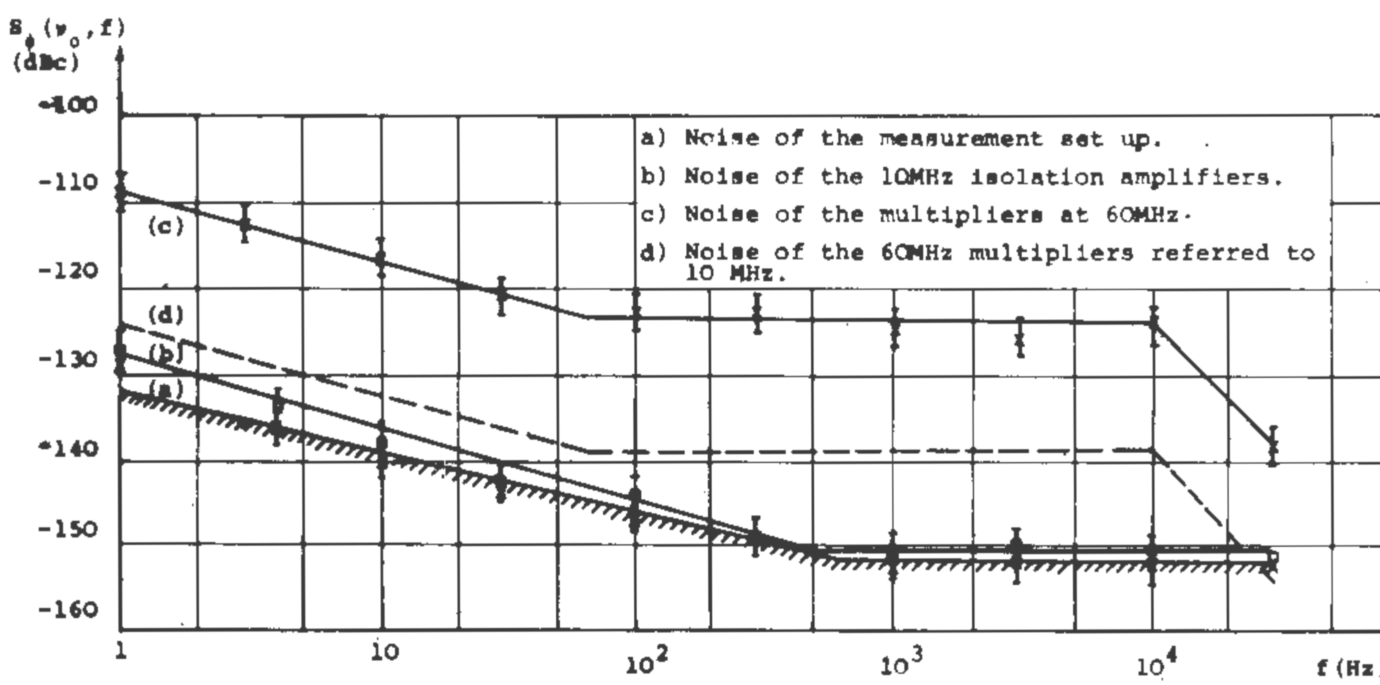
$S_\phi(\nu_0, f)$ (dBc)
-100
-110
-120
-130
-140
-150
-160
a) Noise of the measurement set up.
b) Noise of the 10MHz isolation amplifiers.
c) Noise of the multipliers at 60MHz.
d) Noise of the 60MHz multipliers referred to 10 MHz.
(c)
(d)
(b)
(a)
1
10
10^2
10^3
10^4
f (Hz)

Fig. 7

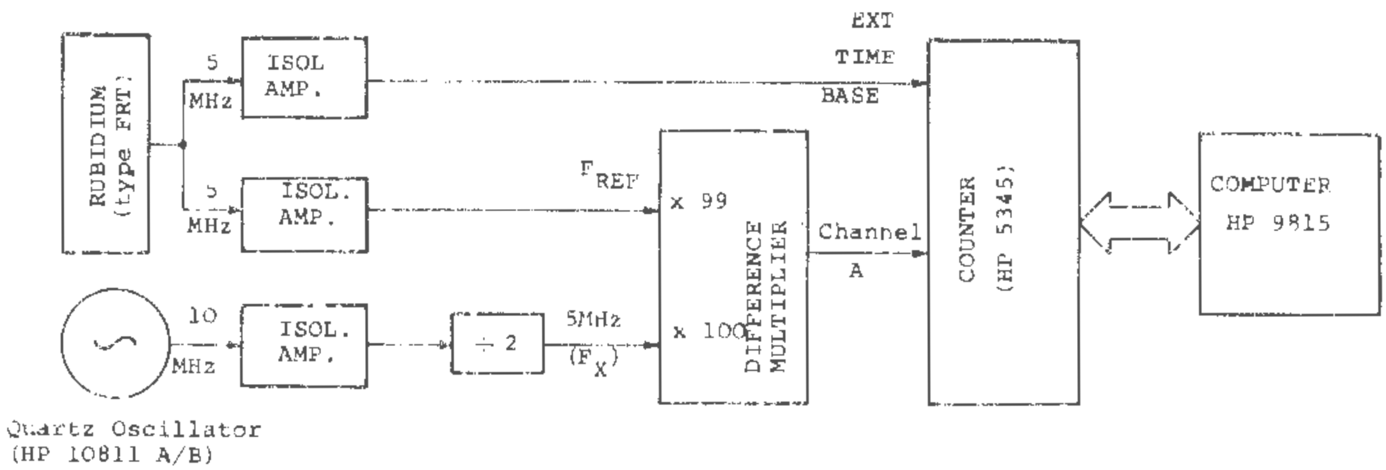

Fig.8: Block diagram for the measurement of the Allan variance of the Quartz Oscillator.

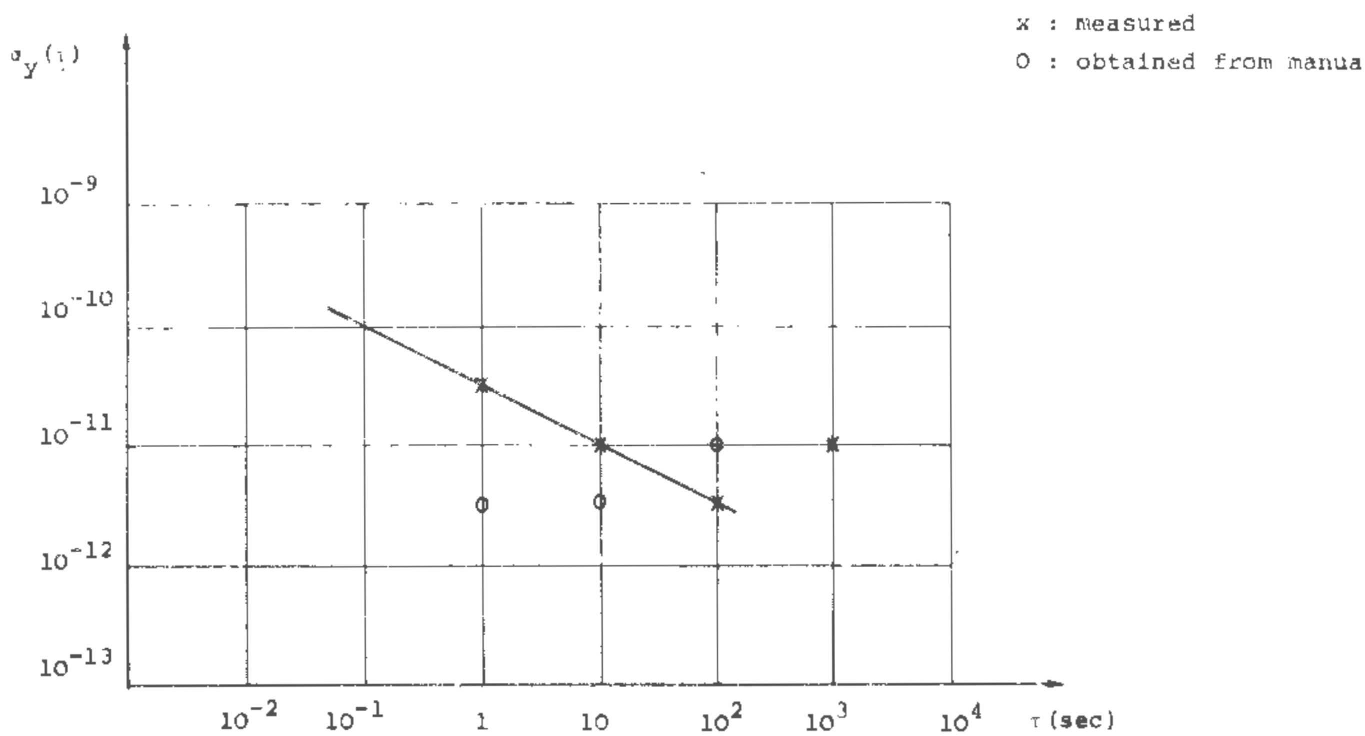

Fig.9: Allan Variance of the Quartz oscillator.

Table 2

(Gate Time)$^{(Sec)}$	1	10	100	1000
No. of samples (N)	100	10	10	10
σ_y (measured)	4.73×10^{-11}	1.33×10^{-11}	0.42×10^{-11}	1.33×10^{-11}
σ_y (from manual)	0.5×10^{-11}	0.5×10^{-11}	1×10^{-11}	

B. Measurement in the Time Domain

In the frequency domain measurement, the accuracy of the spectrum analyzer in the low Fourier frequency (very near the carrier) is not reliable. For this reason time domain measurement is necessay. The stability measurement in the time domain essentially boils down to measuring the Allan Variance, $\sigma_y(\tau)$.

Since the flicker noise of the frequency multiplier is $\alpha(1/f)$ while that of the quartz oscillator is $\alpha\frac{1}{3}$, it means that near the carier frequency, the dominant noise is that of the quartz. Consequently, in the time domain it is sufficient to measure the Allan Variance of the quartz oscillator. The measurement set up is given in Fig. 8.

The measurement of the Allan Variance is done with the help of a computer (HP9815) using a program intended for this purpose. The data required by the computer are; the nominal frequency (5MHz), the order of multipliction of the "difference" multiplier (M = 100) and the number of samples (N) desired for each gate time (τ). By taking the average for a certain number of samples, the following results are obtained (Table 2) which are then plotted (Fig. 9)

CONCLUSION

In the circuits built transistors with high gain and low noise are used. Moreover, the currents in the multipliers are made as low as possible to reduce noise. Inspite of these precautions, the phase noise of the multipliers at 60MHz has been 11dB (in the white noise region) above the sensitivity of the measurement set up utilized. Part of this noise is due to the power amplifiers used externally to raise the power level inorder to drive the mixer.

It has been verified that this chain is very stable and can be used for frequency synthesis upto 300GHz. To go higher in frequency it is necessary to reduce further the noise in the first stage. One way of doing it may be to realize the 1st stage with a tripler using a Schottky diode followed by a normal doubler.

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