THE USE OF SQUARE CROSS SECTIONS OF UNIT-LENGTH SIDE FOR THE ANALYSIS OF RECTANGULAR SOLID AND HOLLOW SECTIONS UNDER BIAXIALLY ECCENTRIC LOADS

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ABSTRACT

This paper deals with the analytical proof of the equivalence between the relative design axial load and biaxial bending resistance of a solid and hollow rectangular section, and the design axial load and biaxial bending resistance of the associated square solid and hollow cross sections of unit-length side. The results of the proof showed that the equivalence exists only if the design compressive strength of the concrete in the square cross section of unit-length side is equal to one for solid sections and $1/\alpha$ for hollow rectangular sections, where α is defined as the ratio of the solid to the gross cross sectional area. The new sets of compressive strengths and transformations of rectangular sections into square cross section of unit-length side have been shown to be matched with consistent transformations of the design yield strength of reinforcing bars, locations of concrete fibre and rebar, and area of reinforcement.

Keywords*: Biaxial eccentric load, Hollow section, Solid sections, Square cross section.*

INTRODUCTION

The results of cross section analysis show that the relative design axial load and biaxial bending resistance of a solid rectangular reinforced concrete section is identical to the design axial load and biaxial bending resistance of an associated square cross section of unit-length side, provided that, (1) the design compressive strength of the concrete in the square cross section of unit-length side is equal to one, (2) the resulting rebar location and area of reinforcement is in conformity with the transformation of the rectangular cross-section into the square cross section of unit-length side, and (3) the resulting design yield strength of the reinforcement is in conformity with the transformation of the design compressive strength of concrete.

Moreover the total amount of reinforcement in the transformed section can be replaced by the mechanical reinforcement ratio, ω, if the design yield strength of the reinforcement is taken to be equal to one.

Keeping the transformation requirements in respect of rebar locations, concrete fibre locations, and amount of reinforcement the same as in the solid cross section, it can be shown that the relative values of the design axial load and biaxial bending resistance of a rectangular hollow section with solidity ratio α is identical to the design axial load and biaxial bending resistance of the corresponding square hollow section of unit-length side, provided that the design compressive strength of concrete used in the analysis of the latter is equal to $1/\alpha$.

Comparisons of cross section analysis results have shown the equivalence between the relative design axial load and biaxial bending resistance of a rectangular reinforced concrete section and the design axial load and biaxial bending resistance of an associated square cross section of unit-length side. However, analytical proof for its justification is hardly available in the literature. More recently, the square cross section of unit-length side is used to calculate interaction diagrams for load eccentricities along axes parallel to the axes of symmetry and to a diagonal of a solid rectangular cross section for the derivation of approximate analytical expressions of the moment contours based on the ACI Code Specification [1] [2]. Analytical proof of the equivalence between the dimensionless expressions for the solid rectangular section with four corner reinforcement and the ultimate bending moments and axial force of an equivalent square cross section of unit length-side is also provided in Ref [1].

RESEARCH SIGNIFICANCE

The purpose of this paper is to present a new approach for the analytical proof of the equivalence between the relative design axial load and biaxial bending resistance of:

i. Solid rectangular section with arbitrary reinforcement arrangement and the design axial load and biaxial bending resistance of the associated square cross section of unit length side.

ii. Hollow rectangular section with arbitrary reinforcement arrangement and the design axial load and biaxial bending resistance of the associated square hollow cross section of unit length side.

iii. Same as in (i) and (ii), but using net cross section for high strength concrete as recommended in modern building codes [3] [4].

EQUIVALENCE BETWEEN THE RELATIVE DESIGN AXIAL LOAD AND BIAXIAL BENDING RESISTANCES OF A RECTANGULAR SECTION AND THE ASSOCIATED SQUARE CROSS SECTION OF UNIT-LENGTH SIDE

Biaxial interaction diagrams for solid rectangular cross section made of reinforced concrete are presented in non-dimensional form as:

$$\nu_{Rd} = \frac{N_{Rd}}{f_{cd} \cdot b \cdot h} \tag{1}$$

$$\mu_{Rd\,y} = \frac{M_{Rd\,y}}{f_{cd} \cdot b \cdot h^2} \tag{2}$$

$$\mu_{Rd\,z} = \frac{M_{Rd\,z}}{f_{cd} \cdot h \cdot b^2} \tag{3}$$

where,

ν_{Rd}, $\mu_{Rd\,y}$, $\mu_{Rd\,z}$	are the relative values of the design axial load and biaxial bending resistance of the rectangular cross section.
N_{Rd}, $M_{Rd\,y}$, and $M_{Rd\,z}$	are the design axial load and biaxial bending resistance of he rectangular cross section.
f_{cd}	is the design compressive strength of concrete.
b, and h	are the side lengths of the rectangular cross-section

It can be seen from Eq. (1) to (3) that the design axial load and biaxial bending resistance of a different cross section whose side lengths and design compressive strength of concrete satisfy Eq. (4) to (6) would be identical to the required relative values of the design axial load and biaxial bending resistance of the original rectangular section.

$$f_{cd} \cdot b \cdot h = 1 \tag{4}$$

$$f_{cd} \cdot b \cdot h^2 = 1 \tag{5}$$

$$f_{cd} \cdot h \cdot b^2 = 1 \tag{6}$$

Equation (4) to (6) represents a system of independent simultaneous equations in b, h, and f_{cd}. The solutions are:

$$b = 1,\ h = 1, \text{ and } f_{cd} = 1. \tag{7}$$

Thus given a rectangular reinforced concrete section with side lengths (b/h) and design compressive strength f_{cd}, there is an alternative way of determining its relative values of the design axial load and biaxial bending resistance. It is achieved through the transformation of the side lengths of the original cross section and the design compressive strength of concrete into unity. The design values of the axial load and biaxial bending resistance of the transformed section will then give the relative design axial load and biaxial bending resistance ν_{Rd}, $\mu_{Rd\,y}$, and $\mu_{Rd\,z}$ of the original cross section directly. It is to be recognized that in the process, locations of concrete fibre and rebar, area of reinforcement, and the design yield strength of reinforcing bars undergo consistent transformations as will be described in more detail in the following sections.

In summary all rectangular sections with arbitrary side lengths b and h possess the same relative design axial load and biaxial bending resistances as that of the associated square cross section of unit-length side, provided that,

i. The design compressive strength of the concrete f_{cd} in the square cross section of unit-length side is equal to 1.

ii. The resulting rebar location and area of reinforcement in the square cross section of unit-length side is in conformity with the coordinate and area transformation of the rectangular cross-section.

iii. The resulting design yield strength of the reinforcement in the square cross section of unit-length side is in conformity with the transformation of the design compressive strength of concrete.

COORDINATE TRANSFORMATION MATRIX

Rectangular Solid Cross-Sections

The transformation of a rectangular cross-section with side lengths b and h into a square cross section of unit-length side can be expressed in matrix form as:

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} b \\ h \end{Bmatrix} = \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \quad (8)$$

Thus given a rectangular reinforced concrete section (Fig. 1(a)), the transformation matrix in Eq. (8), will be used to determine the coordinates of any desired point in the associated square cross section of unit-length side (Fig. 1(b)). As an example, the coordinate of the corner concrete fibre and rebar locations in the positive quadrant, are determined in Eqs. (9) and (10) respectively.

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} b/2 \\ h/2 \end{Bmatrix} = \begin{Bmatrix} 0.5 \\ 0.5 \end{Bmatrix} \quad (9)$$

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} (b/2) - b' \\ (h/2) - h' \end{Bmatrix} = \begin{Bmatrix} 0.5 - (b'/b) \\ 0.5 - (h'/h) \end{Bmatrix} \quad (10)$$

Therefore equivalent square cross sections of unit-length side can be made to represent all rectangular sections regardless of their aspect ratios, provided that they satisfy the conditions of equal dimensionless cover in the *y*- and *z*- directions. This property is exploited in the preparation of biaxial interaction diagrams, although it is not in accordance with the usual practice where equal absolute cover is provided in both directions. The following example shows the differences observed in a square cross section of unit-length side when equal absolute cover is provided in the actual rectangular cross section instead of equal dimensionless cover.

Assuming that the cover in the rectangular section (Fig. 1 (a)) is equal in both directions with $b' = h' = c'$, the rebar location in the positive quadrant in the square cross section of unit-length side is determined by the transformation given in Eq. (11).

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} (b/2) - c' \\ (h/2) - c' \end{Bmatrix} = \begin{Bmatrix} 0.5 - (c'/b) \\ 0.5 - (c'/h) \end{Bmatrix} \quad (11)$$

Assuming further that $c' = 0.2 \cdot b$ and that the aspect ratio b/h = 0.5, the coordinate of the location of the reinforcement in the transformed section is:

$$\begin{Bmatrix} 0.5 - (c'/b) \\ 0.5 - (c'/h) \end{Bmatrix} = \begin{Bmatrix} 0.3 \\ 0.4 \end{Bmatrix} \quad (12)$$

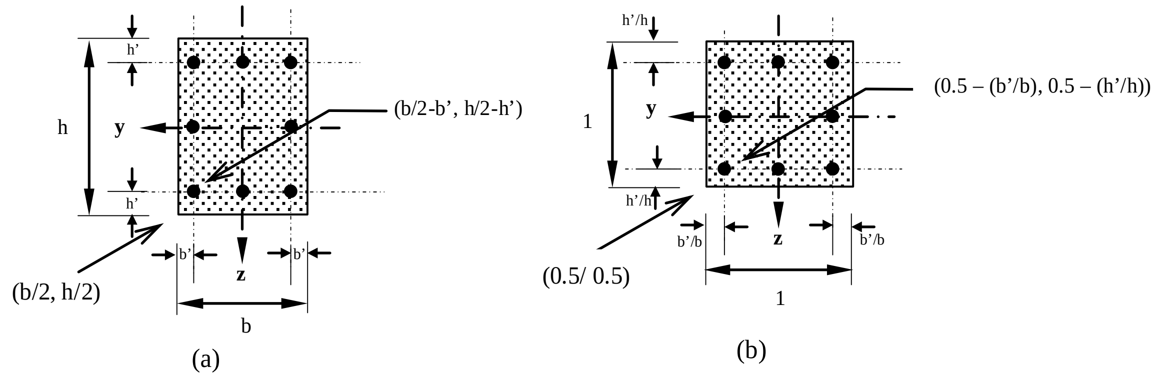

Figure 1(a) Rectangular solid section; and (b) Square cross section of unit-length side

For a given size of concrete cover that can be expressed as a fraction of the length of one side of a rectangular cross section, it can be shown that the coordinates of the location of the reinforcement in the transformed section is a function of the aspect ratio of the original rectangular section. On the other hand it can be observed from Eq. (10) that, the rebar location can be made independent of the aspect ratio by choosing equal dimensionless covers, $b'/b = h'/h$, in both directions.

The result of the transformation shows that, the rebar are placed further apart in the direction of the larger dimension, i.e. *h*, as opposed to the symmetrical location of the reinforcement with equal y- and z- coordinates for the case of equal dimensionless cover.

Rectangular Hollow Cross Sections

Figure 2(a) and (b) show the actual rectangular hollow section with uniformly distributed reinforcement along the edges and the associated square hollow section of unit-length side respectively. Compared to the solid rectangular section, the hollow rectangular section has one additional parameter, which is the ratio of the wall thickness to the corresponding edge length.

$$f_{cd} \cdot \alpha \cdot b \cdot h = 1 \tag{16}$$

$$f_{cd} \cdot \alpha \cdot b \cdot h^2 = 1 \tag{17}$$

$$f_{cd} \cdot \alpha \cdot h \cdot b^2 = 1 \tag{18}$$

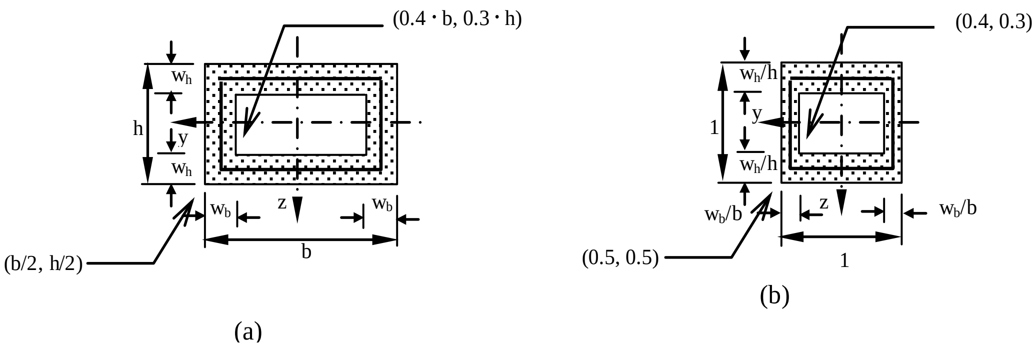

Figure 2(a) Rectangular hollow section; and (b) Square cross section of unit-length side

Biaxial interaction diagrams for hollow rectangular cross section made of reinforced concrete are presented in non-dimensional form as:

$$\nu_{Rd} = \frac{N_{Rd}}{f_{cd} \cdot \alpha \cdot b \cdot h} \tag{13}$$

$$\mu_{Rd\,y} = \frac{M_{Rd\,y}}{f_{cd} \cdot \alpha \cdot b \cdot h^2} \tag{14}$$

$$\mu_{Rd\,z} = \frac{M_{Rd\,z}}{f_{cd} \cdot \alpha \cdot h \cdot b^2} \tag{15}$$

where,

α is the fraction of the solid part of the cross section which will be referred to as "solidity ratio" in short and the definitions of other variables are as in Eq. (1) to (3).

It can be seen from Eq. (13) to (15) that the design axial load and biaxial bending resistance of an associated rectangular hollow cross section whose side lengths and design compressive strength of concrete f_{cd} satisfy Eq. (16) to (18) would be identical to the required relative values of the design axial load and biaxial bending resistance ν_{Rd}, $\mu_{Rd\,y}$, and $\mu_{Rd\,z}$, of the original rectangular hollow section.

Equation (16) to (18) represents a system of independent simultaneous equations in b, h, and f_{cd}. The solutions are:

$$b = 1,\ h = 1, \text{ and } f_{cd} = 1/\alpha \tag{19}$$

Thus the relative values of the design axial load and biaxial bending resistance of a rectangular hollow section with design compressive strength f_{cd} can be determined by using a transformed square hollow section of unit length side with the corresponding design compressive strength of concrete equal to $1/\alpha$. Moreover the solutions indicate that the coordinate transformation matrix for rectangular hollow sections is the same as that for rectangular solid cross section described in section 3.1. Therefore for the example hollow cross-section shown in Fig. 2(a) with $b/h = 2.0$, $w_b = w_h = 0.2 \cdot h$, the outer and inner corner points of the transformed concrete section in the positive quadrant have the coordinates (0.5/0.5) and (0.4/0.3) respectively as determined by the transformations shown in Eqs. (20) and (21).

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} b/2 \\ h/2 \end{Bmatrix} = \begin{Bmatrix} 0.5 \\ 0.5 \end{Bmatrix} \tag{20}$$

$$\begin{bmatrix} 1/b & 0 \\ 0 & 1/h \end{bmatrix} \begin{Bmatrix} 0.4 \cdot b \\ 0.3 \cdot h \end{Bmatrix} = \begin{Bmatrix} 0.4 \\ 0.3 \end{Bmatrix} \tag{21}$$

The locations of individual rebar in the square hollow section of unit-length side are also determined using the same transformation matrix.

AREA AND STEEL STRENGTH TRANSFORMATION

Rectangular solid sections

The transformation of the rectangular cross section into the equivalent square cross section of unit-length side and the transformation of the design compressive strength of concrete f_{cd} into unity have consequences on the magnitudes of cross sectional areas and material strengths that will be used in the transformed section.

According to the solutions in Eq. (7), the gross cross sectional area $b \cdot h$ of a solid reinforced concrete section is transformed into the unit area, $1 \cdot 1 = 1$, while the design compressive strength of concrete f_{cd} is transformed into unity. Therefore the transformation coefficients for areas and design strengths of materials are $1/(b \cdot h)$ and $1/f_{cd}$ respectively.

Thus using the superscript "*u*" to designate the square cross section of unit-length side, the transformed area of reinforcement A_s^u, and design yield strength of reinforcement f_{yd}^u are:

$$A_s^u = \frac{A_s}{b \cdot h} \tag{22}$$

$$f_{yd}^u = \frac{f_{yd}}{f_{cd}} \tag{23}$$

Further the geometric reinforcement ratio in the transformed section is:

$$\rho^u = \frac{A_s^u}{b^u \cdot h^u} \tag{24}$$

Substituting $b^u = h^u = 1$ and the expression for A_s^u from Eq. (22)

$$\rho^u = \frac{A_s}{b \cdot h} = \rho \tag{25}$$

Equation (25) indicates that the geometric reinforcement ratio is invariant under the transformation.

Similarly the mechanical reinforcement ratio in the transformed section is:

$$\omega^u = \rho^u \cdot \frac{f_{yd}^u}{f_{cd}^u} \tag{26}$$

Substituting $f_{cd}^u = 1$

$$\omega^u = \rho^u \cdot f_{yd}^u \tag{27}$$

Substituting further for ρ^u and f_{yd}^u from Eqs. (25) and (23)

$$\omega^u = \rho \cdot \frac{f_{yd}}{f_{cd}} = \omega \tag{28}$$

Equation (28) indicates that ω is also invariant under the transformation.

Finally from Eqs. (24), (25), and (28):

$$A_s^u = \omega \cdot \frac{f_{cd}}{f_{yd}} \tag{29}$$

Equation (29) gives the transformed area of steel in the square cross section of unit-length side in terms of the mechanical reinforcement ratio ω, the design compressive strength of the concrete, and yield strength of the reinforcement in the original cross section. This same amount of concrete area is to be deducted if the analysis would be based on net cross section. Usually analysis is based on gross cross sections as the use of net cross sections does not affect the result significantly. The effect of the displaced amount of concrete on the cross section capacity may however be significant if high strength of concrete is used requiring analysis on the basis of net cross section for high strength concrete [3] [4] .

The transformed area of reinforcement A_s^u can also be expressed in terms of the transformed design yield strength of reinforcement f_{yd}^u as:

$$A_s^u = \frac{\omega}{f_{yd}^u} \tag{30}$$

Additional analytical advantage can be gained by setting $f_{yd}^u = 1$, because it allows the direct substitution of the reinforcement data by the mechanical reinforcement ratio *ω*. It is to be noted that this is not a consequence of the transformations discussed so far. It is rather an isolated action that allows the substitution of the amount of reinforcement A_s^u in the square cross section of unit-length side by *ω*, provided that f_{yd}^u =1. The direct use of ω as reinforcement data can be used advantageously in the calculation of biaxial interaction diagrams where it can be systematically varied to cover the practical range of the mechanical reinforcement ratio.

Rectangular Hollow Sections

The gross concrete area plus the hollow part of the cross section constitute the total area equal to $b \cdot h$. According to the solutions in Eq. (19), this area is transformed into the unit area, $1 \cdot 1 = 1$, while the design compressive strength of concrete f_{cd} is transformed into $1/\alpha$. Therefore the transformation coefficients for areas and design strengths of materials are $1/(b \cdot h)$ and $1/(\alpha \cdot f_{cd})$ respectively.

Using the superscript "*u*" to designate the square cross section of unit-length side, the transformed area of reinforcement A_s^u and design yield strength of reinforcement f_{yd}^u are:

$$A_s^u = \frac{A_s}{b \cdot h} \tag{31}$$

$$f_{yd}^u = \frac{f_{yd}}{\alpha \cdot f_{cd}} \tag{32}$$

Similarly the transformed area of concrete A_c^u is:

$$A_c^u = \frac{A_c}{b \cdot h} = \frac{\alpha \cdot b \cdot h}{b \cdot h} = \alpha \tag{33}$$

Since Eq. (33) can be rewritten as $A_c^u = \alpha \cdot 1 \cdot 1$, it can be concluded that the "solidity" ratio does not change under the transformation.

The geometric reinforcement ratio in the transformed section is:

$$\rho^u = \frac{A_s^u}{\alpha \cdot b^u \cdot h^u} \tag{34}$$

Substituting $b^u = h^u = 1$ and the expression for A_s^u from Eq. (31)

$$\rho^u = \frac{A_s}{\alpha \cdot b \cdot h} = \rho \tag{35}$$

Equation (35) indicates that the geometric reinforcement ratio is invariant under the transformation.

Similarly the mechanical reinforcement ratio in the transformed section is:

$$\omega^u = \rho^u \cdot \frac{f_{yd}^u}{f_{cd}^u} \tag{36}$$

Substituting $f_{cd}^u = 1/\alpha$

$$\omega^u = \alpha \cdot \rho^u \cdot f_{yd}^u \tag{37}$$

Substituting further for ρ^u and f_{yd}^u from Eqs. (35) and (32)

$$\omega^u = \rho \cdot \frac{f_{yd}}{f_{cd}} = \omega \tag{38}$$

Equation (38) indicates that ω is also invariant under the transformation.

Finally from Eqs. (34), (35), and (38):

$$A_s^u = \alpha \cdot \omega \cdot \frac{f_{cd}}{f_{yd}} \tag{39}$$

Equation (39) gives the transformed area of steel in the square hollow section of unit-length side in terms of the solidity ratio, mechanical reinforcement ratio ω, design compressive strength of the concrete, and yield strength of the reinforcement in the original rectangular hollow section. This same amount of concrete area is to be deducted if the analysis would be based on net cross section.

The transformed area of reinforcement A_s^u can also be expressed in terms of the transformed design yield strength of reinforcement f_{yd}^u as:

$$A_s^u = \frac{\omega}{f_{yd}^u} \tag{40}$$

Thus the amount of reinforcement in the hollow square cross section of unit length side can be replaced by ω by setting $f_{yd}^u = 1$.

CONCLUSIONS

The following conclusions can be drawn from this study:

1. The relative values of the design axial load and biaxial bending resistance of a rectangular reinforced concrete section is identical to the design axial load and biaxial bending resistance of an associated square cross section of unit-length side, provided that the design

compressive strength of concrete f_{cd}^{u} that is used in the analysis of the square cross section of unit-length side is equal to one and its rebar locations are in conformity with the transformation of the side lengths of the actual rectangular cross section.

2. Design axial load and biaxial bending resistance are calculated on the basis of net cross section for high strength concrete by deducting the amount of the transformed area of steel $A_s^u = \omega \cdot \frac{f_{cd}}{f_{yd}}$ from the square cross section of unit-length side.

3. The relative design axial load and biaxial bending resistance of a rectangular hollow reinforced concrete section with "solidity ratio" α is identical to the design axial load and biaxial bending resistance of an associated square hollow section of unit-length side, provided that the design compressive strength of concrete, f_{cd}^{u} that is used in the analysis of the square cross section of unit-length side is equal to $1/\alpha$, and its rebar locations are in conformity with the transformation of the side lengths of the actual rectangular hollow section.

4. Design axial load and biaxial bending resistance are calculated on the basis of net cross section for high strength concrete by deducting the amount of the transformed area of steel $A_s^u = \alpha \cdot \omega \cdot \frac{f_{cd}}{f_{yd}}$ from the square hollow section of unit-length side.

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Appendix-Verification Calculations of Axial Load and Biaxial Bending Resistance of Rectangular and Equivalent Square Cross Section of Unit Length-side

Table 1: Input Data for the Calculation of a Typical Interaction Curve-Normal Strength Concrete

Square cross section of unit length-side – $C$12/15 up to $C$50/60; dimensionless cover = 0.1; ν_{Ed} = -0.8, and ω = 0.5 (4- corner reinforcement arrangement based on gross cross section)

	2					
f_{cd}	1.	-2.000	-3.500	.0000	.0000	2.000
f_{yd}	1.	-2.174	25.000	.0000	2.174	1.000
	2 3 1					
	1 1 5					
	.5000	.5000	.0000			
	1 2 5					
	.4000	.4000	.125	**ω/4**		
	3					
	-.8	1.0	1.0	0 0		
	0					

Table 2: Analysis Result of the Square Cross Section of Unit-length Side Used as Coordinates for the Example Interaction Curve and Design Normal Load and Moment Resistances of the Actual Cross Section [5]

ν	**μ_{Edy}**	**μ_{Edz}**	**N**	**M_{Edy}**	**M_{Edz}**	**R_u/R**
-0.8	0.2409	0	-2901.33	698.9312	0	1
-0.8	0.2394	0.0084	-2901.33	694.5792	12.1856	1
-0.8	0.2377	0.0166	-2901.33	689.6469	24.08107	1
-0.8	0.2357	0.0248	-2901.33	683.8443	35.97653	1
-0.8	0.2335	0.0328	-2901.33	677.4613	47.58187	1
-0.8	0.231	0.0407	-2901.33	670.208	59.04213	1
-0.8	0.2264	0.0481	-2901.33	656.8619	69.77707	1
-0.8	0.2213	0.0552	-2901.33	642.0651	80.0768	1
-0.8	0.2163	0.062	-2901.33	627.5584	89.94133	1
-0.8	0.2114	0.0687	-2901.33	613.3419	99.6608	1
-0.8	0.2065	0.0752	-2901.33	599.1253	109.0901	1
-0.8	0.2017	0.0815	-2901.33	585.1989	118.2293	1
-0.8	0.1968	0.0876	-2901.33	570.9824	127.0784	1
-0.8	0.192	0.0936	-2901.33	557.056	135.7824	1
-0.8	0.1871	0.0995	-2901.33	542.8395	144.3413	1
-0.8	0.1823	0.1052	-2901.33	528.9131	152.6101	1
-0.8	0.1774	0.1109	-2901.33	514.6965	160.8789	1
-0.8	0.1726	0.1164	-2901.33	500.7701	168.8576	1
-0.8	0.1677	0.1218	-2901.33	486.5536	176.6912	1
-0.8	0.1628	0.1272	-2901.33	472.3371	184.5248	1
-0.8	0.1578	0.1324	-2901.33	457.8304	192.0683	1
-0.8	0.1528	0.1376	-2901.33	443.3237	199.6117	1
-0.8	0.1478	0.1427	-2901.33	428.8171	207.0101	1
-0.8	**0.1428**	**0.1478**	**-2901.33**	**414.3104**	**214.4085**	**1**
-0.8	0.1376	0.1528	-2901.33	399.2235	221.6619	1
-0.8	0.1324	0.1578	-2901.33	384.1365	228.9152	1
-0.8	0.1272	0.1628	-2901.33	369.0496	236.1685	1
-0.8	0.1218	0.1677	-2901.33	353.3824	243.2768	1
-0.8	0.1164	0.1726	-2901.33	337.7152	250.3851	1

-0.8	0.1109	0.1774	-2901.33	321.7579	257.3483	1
-0.8	0.1052	0.1823	-2901.33	305.2203	264.4565	1
-0.8	0.0995	0.1871	-2901.33	288.6827	271.4197	1
-0.8	0.0936	0.192	-2901.33	271.5648	278.528	1
-0.8	0.0876	0.1968	-2901.33	254.1568	285.4912	1
-0.8	0.0815	0.2017	-2901.33	236.4587	292.5995	1
-0.8	0.0752	0.2065	-2901.33	218.1803	299.5627	1
-0.8	0.0687	0.2114	-2901.33	199.3216	306.6709	1
-0.8	0.062	0.2163	-2901.33	179.8827	313.7792	1
-0.8	0.0552	0.2213	-2901.33	160.1536	321.0325	1
-0.8	0.0481	0.2264	-2901.33	139.5541	328.4309	1
-0.8	0.0407	0.231	-2901.33	118.0843	335.104	1
-0.8	0.0328	0.2335	-2901.33	95.16373	338.7307	1
-0.8	0.0248	0.2357	-2901.33	71.95307	341.9221	1
-0.8	0.0166	0.2377	-2901.33	48.16213	344.8235	1
-0.8	0.0084	0.2394	-2901.33	24.3712	347.2896	1
-0.8	0	0.2409	-2901.33	0	349.4656	1

Table 3: Example Input Data for Verification Calculation of the Actual Cross Section

Actual cross section – *C*20/25 - based on gross cross section (*b*/ *h* = 0.4m/ 0.8m)
2
f_{cd} 11333.3 -2.000 -3.500 .0000 .0000 2.000
f_{yd} 434782.6 -2.174 25.000 .0000 2.174 1.000
2 3 1
1 1 5
.2000 .4000 .0000
1 2 5
.1600 .3200 0.0010426667 **$A_s/4$**
3
-2901.33 414.3104 214.4085 0 0
0

Table 4: Input Data for the Calculation of a typical Interaction Curve-High Strength Concrete

Square cross section of unit length-side – *C*100/115; dimensionless cover = 0.1; ν_{Ed} = -0.8, and ω = 0.5 (4-corner reinforcement arrangement based on net cross section)
2
f_{cd} 1. -2.200 -2.200 .0000 .0000 1.550
f_{yd} 1. -2.174 25.000 .0000 2.174 1.000
3 3 1
1 1 5
.5000 .5000 .0000
4 1 1
.4000 .4000 -.004692
-.4000 .4000 -.004692
-.4000 -.4000 -.004692
.4000 -.4000 -.004692
1 2 5
.4000 .4000 .125 **ω/4**
3
-.8 1.0 1.0 0 0
0

Table 5: Analysis Result of the Square Cross Section of Unit-length Side Used as Coordinates for the Example Interaction Curve and Design Normal Load and Moment Resistances of the Actual Cross Section [5]

ν	**μ_{Edy}**	**μ_{Edz}**	**N**	**M_{Edy}**	**M_{Edz}**	**Ru/R**
-0.8	0.1635	0	-13056	2134.656	0	1
-0.8	0.1589	0.0055	-13056	2074.598	35.904	1
-0.8	0.1545	0.0108	-13056	2017.152	70.5024	1
-0.8	0.1502	0.0158	-13056	1961.011	103.1424	1
-0.8	0.1461	0.0205	-13056	1907.482	133.824	1
-0.8	0.1422	0.0251	-13056	1856.563	163.8528	1
-0.8	0.1383	0.0294	-13056	1805.645	191.9232	1
-0.8	0.1346	0.0336	-13056	1757.338	219.3408	1
-0.8	0.131	0.0376	-13056	1710.336	245.4528	1
-0.8	0.1275	0.0414	-13056	1664.64	270.2592	1
-0.8	0.124	0.0451	-13056	1618.944	294.4128	1
-0.8	0.1207	0.0487	-13056	1575.859	317.9136	1
-0.8	0.1174	0.0523	-13056	1532.774	341.4144	1
-0.8	0.1142	0.0557	-13056	1490.995	363.6096	1
-0.8	0.111	0.059	-13056	1449.216	385.152	1
-0.8	0.1079	0.0623	-13056	1408.742	406.6944	1
-0.8	0.1048	0.0655	-13056	1368.269	427.584	1
-0.8	0.1018	0.0686	-13056	1329.101	447.8208	1
-0.8	0.0988	0.0717	-13056	1289.933	468.0576	1
-0.8	0.0958	0.0748	-13056	1250.765	488.2944	1
-0.8	0.0928	0.0779	-13056	1211.597	508.5312	1
-0.8	0.0898	0.0809	-13056	1172.429	528.1152	1
-0.8	0.0868	0.0839	-13056	1133.261	547.6992	1
-0.8	0.0839	0.0868	-13056	1095.398	566.6304	1
-0.8	0.0809	0.0898	-13056	1056.23	586.2144	1
-0.8	0.0779	0.0928	-13056	1017.062	605.7984	1
-0.8	0.0748	0.0958	-13056	976.5888	625.3824	1
-0.8	**0.0717**	**0.0988**	**-13056**	**936.1152**	**644.9664**	**1**
-0.8	0.0686	0.1018	-13056	895.6416	664.5504	1
-0.8	0.0655	0.1048	-13056	855.168	684.1344	1
-0.8	0.0623	0.1079	-13056	813.3888	704.3712	1
-0.8	0.059	0.111	-13056	770.304	724.608	1
-0.8	0.0557	0.1142	-13056	727.2192	745.4976	1
-0.8	0.0523	0.1174	-13056	682.8288	766.3872	1
-0.8	0.0487	0.1207	-13056	635.8272	787.9296	1
-0.8	0.0451	0.124	-13056	588.8256	809.472	1
-0.8	0.0414	0.1275	-13056	540.5184	832.32	1
-0.8	0.0376	0.131	-13056	490.9056	855.168	1
-0.8	0.0336	0.1346	-13056	438.6816	878.6688	1
-0.8	0.0294	0.1383	-13056	383.8464	902.8224	1
-0.8	0.0251	0.1422	-13056	327.7056	928.2816	1
-0.8	0.0205	0.1461	-13056	267.648	953.7408	1
-0.8	0.0158	0.1502	-13056	206.2848	980.5056	1
-0.8	0.0108	0.1545	-13056	141.0048	1008.576	1
-0.8	0.0055	0.1589	-13056	71.808	1037.299	1
-0.8	0	0.1635	-13056	0	1067.328	1

Table 6: Example Input Data for Verification Calculation of the Actual Cross Section-High Strength Concrete

Actual cross section - *C*100/115 – (based on net cross section (*b*/ *h* = 0.4m/ 0.8m)
2
f_{cd} 51000 -2.200 -2.200 .0000 .0000 1.550
f_{yd} 434782.61 -2.174 25.000 .0000 2.174 1.000
3 3 1
1 1 5
.2000 .4000 .0000
4 1 1
.1600 .3200 -.004692
-.1600 .3200 -.004692
-.1600 -.3200 -.004692
.1600 -.3200 -.004692
1 2 5
.1600 .3200 .004692 **$A_s/4$**
3
-13056. 936.1152 644.9664 0 0
0

Table 7: Input Data for the Calculation of a typical Interaction Curve for Rectangular Hollow Section (b/h = 2.0; $w_h/h = w_b/h$ = 0.2; $A_c = \alpha \cdot b \cdot h$; α = 0.52) - High strength concrete

Square hollow section of unit-length side; *C*100/115; v_{Ed} = -0.8 and ω = 2.0 - based on net cross section

```
2
fcd  1.923076923 -2.20    -2.200    .0000    .0000    1.550
fyd  1.          -2.174   25.000    .0000    2.174    1.000
4 3 1
4 1 1
-.5000    .5000    .0000
-.5000    -.5000   .0000
.5000     -.5000   .0000
.5000     .5000    .0000
4 1 1
.4000     .3000    .0000
.4000     -.3000   .0000
-.4000    -.3000   .0000
-.4000    .3000    .0000
36 1 1
.4500     .4000    -.0033886666565006667          α · (ω/36) · fcd/ fyd
.3750     .4000    -.0033886666565006667
.3000     .4000    -.0033886666565006667
.2250     .4000    -.0033886666565006667
.1500     .4000    -.0033886666565006667
.0750     .4000    -.0033886666565006667
.0000     .4000    -.0033886666565006667
.4500     .2666666667 -.0033886666565006667
.4500     .1333333333 -.0033886666565006667
.4500     .0000    -.0033886666565006667
-.4500    .4000    -.0033886666565006667
-.3750    .4000    -.0033886666565006667
-.3000    .4000    -.0033886666565006667
-.2250    .4000    -.0033886666565006667
-.1500    .4000    -.0033886666565006667
-.0750    .4000    -.0033886666565006667
-.4500    .2666666667 -.0033886666565006667
-.4500    .1333333333 -.0033886666565006667
-.4500    .0000    -.0033886666565006667
-.4500    -.4000   -.0033886666565006667
-.3750    -.4000   -.0033886666565006667
-.3000    -.4000   -.0033886666565006667
-.2250    -.4000   -.0033886666565006667
-.1500    -.4000   -.0033886666565006667
-.0750    -.4000   -.0033886666565006667
.0000     -.4000   -.0033886666565006667
-.4500    -.2666666667 -.0033886666565006667
-.4500    -.1333333333 -.0033886666565006667
.4500     -.4000   -.0033886666565006667
.3750     -.4000   -.0033886666565006667
.3000     -.4000   -.0033886666565006667
.2250     -.4000   -.0033886666565006667
.1500     -.4000   -.0033886666565006667
.0750     -.4000   -.0033886666565006667
.4500     -.2666666667 -.0033886666565006667
.4500     -.1333333333 -.0033886666565006667
10 2 5
.4500     .4000    .0555555555555555          ω / 36
.3750     .4000    .0555555555555555
.3000     .4000    .0555555555555555
.2250     .4000    .0555555555555555
.1500     .4000    .0555555555555555
.0750     .4000    .0555555555555555
.0000     .4000    .0555555555555555
.4500     .2666666667 .0555555555555555
.4500     .1333333333 .0555555555555555
.4500     .0000    .0555555555555555
3
-.8       .185     .2961538     0  0
0
```

Table 8: Analysis Result of a Square Hollow Cross Section of Unit-length Side Used as Coordinates for the Example Interaction Curve and Design Normal Load and Moment Resistances of the Actual Cross Section [5].

ν_{Ed}	μ_{Edy}	μ_{Edz}	N	M_{Edy}	M_{Edz}	R_u/R
-0,8	0,4938461	0,01730769	-42432	26193,5971	1835,99976	1
-0,8	0,4763462	0,03326923	-42432	25265,4024	3529,19992	1
-0,8	0,46	0,04826923	-42432	24398,4	5120,39992	1
-0,8	0,4446154	0,0625	-42432	23582,4008	6630	1
-0,8	0,43	0,07576923	-42432	22807,2	8037,59992	1
-0,8	0,4161538	0,08846154	-42432	22072,7976	9384,00016	1
-0,8	0,4028846	0,1005769	-42432	21368,9992	10669,1976	1
-0,8	0,3903846	0,1119231	-42432	20705,9992	11872,8024	1
-0,8	0,3784615	0,1230769	-42432	20073,598	13055,9976	1
-0,8	0,3669231	0,1336538	-42432	19461,6012	14177,9951	1
-0,8	**0,3559616**	**0,1438462**	**-42432**	**18880,2033**	**15259,2049**	**1**
-0,8	0,3451923	0,1536538	-42432	18308,9996	16299,5951	1
-0,8	0,3348077	0,1632692	-42432	17758,2004	17319,5967	1
-0,8	0,3246154	0,1726923	-42432	17217,6008	18319,1992	1
-0,8	0,3148077	0,1817308	-42432	16697,4004	19278,0033	1
-0,8	0,305	0,1905769	-42432	16177,2	20216,3976	1
-0,8	0,2953846	0,1992308	-42432	15667,1992	21134,4033	1
-0,8	0,2859615	0,2078846	-42432	15167,398	22052,3984	1
-0,8	0,2767308	0,2161538	-42432	14677,8016	22929,5951	1
-0,8	0,2675	0,2244231	-42432	14188,2	23806,8024	1
-0,8	0,2582692	0,2325	-42432	13698,5984	24663,6	1
-0,8	0,2492308	0,2405769	-42432	13219,2016	25520,3976	1
-0,8	0,2401923	0,2486538	-42432	12739,7996	26377,1951	1
-0,8	0,2309615	0,2565385	-42432	12250,198	27213,6041	1
-0,8	0,2219231	0,2644231	-42432	11770,8012	28050,0024	1
-0,8	0,2126923	0,2723077	-42432	11281,1996	28886,4008	1
-0,8	0,2036538	0,2801923	-42432	10801,7976	29722,7992	1
-0,8	0,1944231	0,2880769	-42432	10312,2012	30559,1976	1
-0,8	0,185	0,2961538	-42432	9812,4	31415,9951	1
-0,8	0,1755769	0,3040385	-42432	9312,59878	32252,4041	1
-0,8	0,1659615	0,3121154	-42432	8802,59796	33109,2016	1
-0,8	0,1561539	0,3203846	-42432	8282,40286	33986,3984	1
-0,8	0,1463462	0,3286538	-42432	7762,20245	34863,5951	1
-0,8	0,1361538	0,3371154	-42432	7221,59755	35761,2016	1
-0,8	0,1257692	0,3457692	-42432	6670,79837	36679,1967	1
-0,8	0,1151923	0,3546154	-42432	6109,79959	37617,6016	1
-0,8	0,1042308	0,3636538	-42432	5528,40163	38576,3951	1
-0,8	0,09307692	0,3730769	-42432	4936,79984	39575,9976	1
-0,8	0,08134615	0,3826923	-42432	4314,5998	40595,9992	1
-0,8	0,06923077	0,3928846	-42432	3672,00004	41677,1984	1
-0,8	0,05673077	0,4032692	-42432	3009,00004	42778,7967	1
-0,8	0,04346154	0,4140384	-42432	2305,20008	43921,1935	1
-0,8	0,02980769	0,4253846	-42432	1580,99988	45124,7984	1
-0,8	0,01519231	0,4373077	-42432	805,800122	46389,6008	1
-0,8	0	0,45	-42432	0	47736	1

Table 9: Example Input Data Verification of the Actual Cross Section

Actual Cross Section; b = 2.0 m; h = 1.0 m; α = 0.52; w_b/h=w_h/h=0.20 $C$100/115 (based on net cross section)

2
f_{cd} 51000 -2.200 -2.200 .0000 .0000 1.550
f_{yd} 434782.61 -2.174 25.000 .0000 2.174 1.000
4 3 1
4 1 1
-1.000 .5000 .0000
-1.000 -.5000 .0000
1.000 -.5000 .0000
1.000 .5000 .0000
4 1 1
.8000 .3000 .0000
.8000 -.3000 .0000
-.8000 -.3000 .0000
-.8000 .3000 .0000
36 1 1
.9000 .4000 -.006777333333333 A_s / 36
.7500 .4000 -.006777333333333
.6000 .4000 -.006777333333333
.4500 .4000 -.006777333333333
.3000 .4000 -.006777333333333
.1500 .4000 -.006777333333333
.0000 .4000 -.006777333333333
.9000 .2666666667 -.006777333333333
.9000 .1333333333 -.006777333333333
.9000 .0000 -.006777333333333
-.9000 .4000 -.006777333333333
-.7500 .4000 -.006777333333333
-.6000 .4000 -.006777333333333
-.4500 .4000 -.006777333333333
-.3000 .4000 -.006777333333333
-.1500 .4000 -.006777333333333
-.9000 .2666666667 -.006777333333333
-.9000 .1333333333 -.006777333333333
-.9000 .0000 -.006777333333333
-.9000 -.4000 -.006777333333333
-.7500 -.4000 -.006777333333333
-.6000 -.4000 -.006777333333333
-.4500 -.4000 -.006777333333333
-.3000 -.4000 -.006777333333333
-.1500 -.4000 -.006777333333333
.0000 -.4000 -.006777333333333
-.9000 -.2666666667 -.006777333333333
-.9000 -.1333333333 -.006777333333333
.9000 -.4000 -.006777333333333
.7500 -.4000 -.006777333333333
.6000 -.4000 -.006777333333333
.4500 -.4000 -.006777333333333
.3000 -.4000 -.006777333333333
.1500 -.4000 -.006777333333333
.9000 -.2666666667 -.006777333333333
.9000 -.1333333333 -.006777333333333
10 2 5
.9000 .4000 .006777333333333 A_s / 36
.7500 .4000 .006777333333333
.6000 .4000 .006777333333333
.4500 .4000 .006777333333333
.3000 .4000 .006777333333333
.1500 .4000 .006777333333333
.0000 .4000 .006777333333333
.9000 .2666666667 .006777333333333
.9000 .1333333333 .006777333333333
.9000 .0000 .006777333333333
3
-42432 18880.2033 15259.2049 0 0
0