TOTAL THRUST ON EARTH-RETAINING STRUCTURES DUE TO SURCHARGES

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ABSTRACT

In the presence of various surcharges, it is often necessary to use a rapid and reliable method to estimate the magnitude of the resultant lateral pressure i.e. the total thrust and its point of application on an earth-retaining structure due to surcharges.

The tests performed by Spangler (1936), Spangler and Mickle (1956), and proposals of Terzaghi (1954), Newmark (1942), Gol'dshtein (1981) and others indicate that lateral pressures can be computed for various types of surcharges by using modified forms of the theory of elasticity equations [1]. Here, the earth-retaining structure is considered to be rigid and the modified forms of the theory of elasticity equations are used in accordance with Gol'dshtein's solutions which consider type of soil, degree of consolidation, densities and stratification.

The various surcharges along retaining walls may include vehicle loads; railroad, earth embankment, or building which is parallel to the retaining structure, grain storage, etc. Lateral pressures may be developed from ice formation due to pore water freezing. Vibration of ground along retaining wall due to earthquakes or longer-duration vibrations from reciprocating machinery may increase the design lateral pressure for walls of normal height. Expansive clays placed behind retaining walls may develop significant lateral pressure when they become wet. In general, these various surcharges may be idealized as point loads, line loads, and strip loads acting on soil surface at a given distance from the earth-retaining structure.

DETERMINATION OF TOTAL THRUST ON EARTH-RETAINING STRUCTURE

A. Point Load

Lateral stress at a depth of z due to a point load P acting on soil surface at a distance of r from the given structure is given by [2]:

$$\sigma_x = BP\frac{\cos^{m-2}\alpha\sin^2\alpha}{R} \tag{1}$$

For medium stiff clay:

$m = 3$ *and* $B_1 = 2/\pi = 0.64$

For medium dense sand:

$m = 5$ *and* $B_2 = 0.85$

where P= Concentrated load, ton
m= Stress concentration factor
B= f(m) - Stress concentration influence factor

$R = \sqrt{r^2 + z^2}$, (see Fig. 1).

Inserting these terms in Eq. (1) and simplifying, we obtain pressures per unit length for medium stiff clay and medium dense sand, respectively:

$$\sigma_{x1} = \frac{B_1 P\cos\alpha\sin^2\alpha}{R} = \frac{B_1 Pzr^2}{R^4} = B_1 \Pr^2\frac{z}{\left(r^2+z^2\right)^2} \tag{2}$$

$$\sigma_{x2} = \frac{B_2 P\cos^3\alpha\sin^2\alpha}{R} = \frac{B_2 P}{R}\times\frac{z^3}{R^3}\times\frac{r^2}{R^2} = B_2 \Pr^2\frac{z^3}{\left(r^2+z^2\right)^3} \tag{3}$$

We shall further consider two cases.

Case 1: Medium Stiff Clay

The total thrust on an earth-retaining structure due to a point load acting on clay soil surface at a distance of r from the structure can be expressed as:

$$E_{w1} = B_1 \Pr^2\int_0^H\frac{z\,dz}{\left(r^2+z^2\right)^2} = \frac{B_1 P}{2}\left[\frac{1}{1+\left(r/H\right)^2}\right] \tag{4}$$

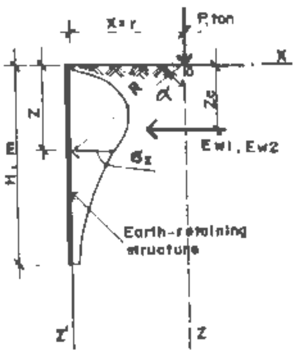

Figure 1 Lateral pressure against rigid wall due to a point load

Denoting that $n=r/H$ and $m_1=f(n=r/H)=\frac{0.64}{2}\left[\frac{1}{1+n^2}\right]$, we obtain

$$E_{w1}=m_1P \tag{5}$$

The total thrust acts at a distance of Z_c from X-axis as shown in Fig. 1.

Case 2: Medium Dense Sand

The total thrust in this case can be expressed as:

$$E_{w2}=B_2Pr^2\int_0^H\frac{z^3dz}{(r^2+z^2)^3}=\frac{B_2P}{2}\left[0.5+\frac{(r/H)^4}{2\left[(r/H)^2+1\right]^2}-\frac{(r/H)^2}{(r/H)^2+1}\right] \tag{6}$$

Analogous to Eq. (5), the total thrust simplifies to

$$E_{w2}=m_2P \tag{7}$$

Where

$$m_2=f(n=r/H)=\frac{0.85}{2}\left[0.5+\frac{n^4}{2(n^2+1)^2}-\frac{n^2}{n^2+1}\right]$$

Table 1 lists values of the influence factors m_1 and m_2 for various ratios of r/H.

The above formulae can be used for line load acting on soil surface at a distance of r from an earth-retaining structure per unit length, except that the values of E_{w1} and E_{w2} are doubled for this case as recommended by Terzaghi [1]. Doubling of the value for the case of a line load is due to the effect of "mirror load", placed symmetrically in front of a rigid wall for zero lateral displacement implied by the rigidity of the wall.

B. Uniform Load

Lateral stress at a depth of z due to a uniform pressure q acting on a strip area of width b and infinite length, the center of which is defined by x_o is given by: [1,5]

$$\sigma_{x3}=\frac{q}{\pi}(\alpha-\sin\alpha\cos2\beta) \tag{8}$$

Where α is in radians and the other terms are as shown in Fig. 2.

For convenience, Eq. (8) can be rearranged as:

$$\sigma_{x3}=\frac{-q}{2\pi}\left[2(\alpha_2-\alpha_1)-\sin2\alpha_2+\sin2\alpha_1\right] \tag{9}$$

Where α_1 and α_2 are also as shown in Fig. 2.
Eq. (9) will be used since it is equivalent to Eq. (8). This can be easily proved from Fig. 2, taking into consideration that:

$$\alpha_1=\alpha+\beta',\ \beta=\alpha/2+\beta' \text{ or } \alpha=2(\beta-\beta') \text{ and } \alpha_2=\beta'$$

and inserting these terms in Eq. (9) to derive Eq. (8).

From Fig. 2, it is important to note that

$(x_o-b/2)>0$ therefore, $b(x_o/b-0.5)>0$

It implies that, $x_o/b>0.5$ (10)

This restriction is applicable only in the presence of earth-retaining structure. We shall further perform some operations in the trigonometric functions of Eq. (9).

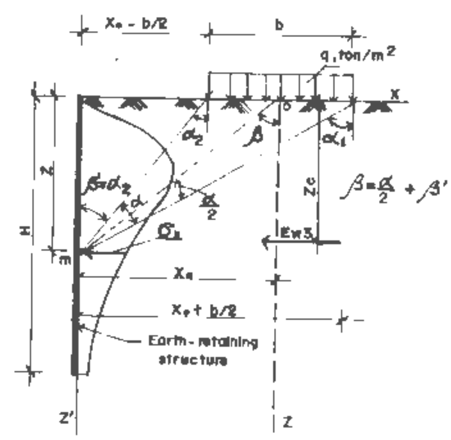

Figure 2 Lateral pressure against rigid wall due to a uniform strip load

$$Sin\, 2a_2 = 2\sin a_2 \cos a_2 = \frac{2\tan\alpha_2}{1+\tan^2\alpha_2} = \frac{\frac{(2x_o-b)}{z}}{1+\left(\frac{x_o-b/2}{z}\right)^2}$$

$$Sin\, 2\alpha_1 = 2\sin\alpha_1 \cos\alpha_1 = \frac{2\tan\alpha_1}{1+\tan^2\alpha_1} = \frac{\frac{(2x_o+b)}{z}}{1+\left(\frac{x_o+b/2}{z}\right)^2}$$

From Fig. 2 $\cot\alpha_2 = \frac{z}{x_o-b/2}$, $\cot\alpha_1 = \frac{z}{x_o+b/2}$

Then, $\alpha_2 = arccot\frac{z}{x_o-b/2}$, $\alpha_1 = arccot\frac{z}{x_o+b/2}$

Substituting the above in Eq. (9) we obtain

$$\sigma_{x3} = \frac{-q}{2\pi}\left[2\left(arc\cot\frac{z}{x_o-b/2} - arc\cot\frac{z}{x_o+b/2}\right) - \frac{\frac{2x_o-b}{z}}{1+\left(\frac{x_o-b/2}{z}\right)^2} + \frac{\frac{2x_o+b}{z}}{1+\left(\frac{x_o+b/2}{z}\right)^2}\right] =$$

$$\frac{-q}{\pi}\left[arc\cot\frac{z}{x_o-b/2} - arc\cot\frac{z}{x_o+b/2} - \frac{x_o-b/2}{z}\times \frac{1}{1+\left(\frac{x_o-b/2}{z}\right)^2} + \left(\frac{x_o+b/2}{z}\right)\times\frac{1}{1+\left(\frac{x_o+b/2}{z}\right)^2}\right]$$

Let $a_1 = x_0 + b/2 = Const$

$a_2 = x_0 - b/2 = Const$

Substituting these terms in the above equation,

$$\sigma_{x3} = \frac{-q}{\pi}\left[arc\cot\frac{z}{a_2} - arc\cot\frac{z}{a_1} - a_2\left(\frac{z}{a_2^2+x^2}\right) + a_1\left(\frac{z}{a_1^2+z^2}\right)\right] \tag{11}$$

The total thrust on an earth-retaining structure due to a uniform strip load condition will therefore be:

$$E_{w3} = \int_0^H \sigma_{x3}\, dz = \frac{qH}{\pi}\left[\arctan\frac{H}{a_2} - \arctan\frac{H}{a_1}\right]$$

Substituting for a_1 and a_2,

$$E_{w3} = \frac{qH}{\pi}\left[\arctan\frac{H}{x_o-b/2} - \arctan\frac{H}{x_o+b/2}\right] = \frac{qH}{\pi}\left[\arctan\frac{H}{b(x_o/b-1/2)} - \arctan\frac{H}{b(x_o/b+1/2)}\right]$$

Denoting the ratios $H/b = m$ and $x_o/b = n$

$$E_{w3} = m_3\, qH \tag{12}$$

Where, $m_3 = f(n = x_o/b,\ m = H/b) = \frac{1}{\pi}\left[\arctan\frac{m}{n-0.5} - \arctan\frac{m}{n+0.5}\right]$

Terzaghi and Bowles recommended to double the lateral stress value σ_{x3} for the case of a uniform strip load [1].

Hence:

$$E_{w3} = 2m_3 qH \tag{13}$$

Table 2 list values of the influence factor m_3 for various ratios of $n = x_o/b$ and $m = H/b$.

C. Total Thrust For Stratified Soils due to Point Load

Total thrust on earth-retaining structure due to point load P acting on the surface of stratified soil at a distance of r from the given structure can be written as [see Fig. (3)]:

a) Stratification in 'Clay - Sand' series

$$E_{w1,2} = \int_0^{H_1}\sigma_{x1}dz + \int_{H_1}^{H_2}\sigma_{x2}dz + \ldots \tag{14}$$

b) Stratification in 'Sand - Clay' Series

$$E_{w2,1} = \int_0^{H_1}\sigma_{x2}dz + \int_{H_1}^{H_2}\sigma_{x1}dz + \ldots \tag{15}$$

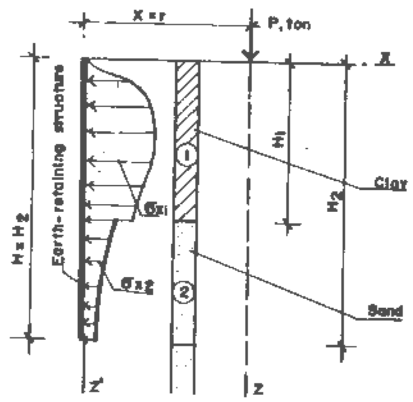

Figure 3 Lateral pressure against rigid wall due to point load for stratified soil layers

Note that we have considered only the fist two layers for convenience.

Substituting for σ_{x1} and σ_{x2} in Eq. (14) and Eq. (15),

$$
\begin{aligned}
E_{w1,2} &= B_1 \mathrm{P} r^2 \int_0^{H_1} \frac{z\,dz}{\left(r^2+z^2\right)^2} + B_2 P r^2 \int_{H_1}^{H_2} \frac{z^3 dz}{\left(r^2+z^2\right)^3} + \ldots \\
&= P\left[\frac{B_1}{2}\left\{\frac{1}{1+(r/H_1)^2}\right\} + \frac{B_2}{2}\left\{-\frac{r^2}{H_2^2\left((r/H_2)^2+1\right)} + \right.\right. \\
&\quad \frac{r^2}{H_1^2\left((r/H_1)^2+1\right)} + \frac{r^4}{2H_2^4\left((r/H_2)^2+1\right)^2} - \\
&\quad \left.\left.\frac{r^4}{2H_1^4\left((r/H_1)^2+1\right)^2}\right\} + \ldots\right]
\end{aligned} \tag{16}
$$

Let $r/H_1 = n_1$ *and* $r/H_2 = n_2$, then

$$
\begin{aligned}
E_{w1,2} &= P\left[\frac{B_1}{2}\left\{\frac{1}{1+n_1^2}\right\} + \frac{B_2}{2}\left\{\frac{n_1^2}{1+n_1^2} - \frac{n_2^2}{1+n_2^2} + \right.\right. \\
&\quad \left.\left.\frac{n_2^4}{2\left(1+n_2^2\right)^2} - \frac{n_1^4}{2\left(1+n_1^2\right)^2}\right\} + \ldots\right] = \\
&\quad P\left[\frac{B_1}{2}\left\{\frac{1}{1+n_1^2}\right\} + \frac{B_2}{2}\left\{\frac{n_1^2}{1+n_1^2}\left(1 - \frac{n_1^2}{2\left(1+n_1^2\right)}\right) + \right.\right. \\
&\quad \left.\left.\frac{n_2^2}{1+n_2^2}\left(\frac{n_2^2}{2\left(1+n_2^2\right)} - 1\right)\right\} + \ldots\right] = m_{1,2} P
\end{aligned} \tag{17}
$$

Considering the values of influence factors given by Eqs. (5) & (7)

$m_{1,2} = \underbrace{\left(m_1^1 - m_0^1\right)}_{clay} + \underbrace{\left(m_2^2 - m_1^2\right)}_{sand} + \ldots =$ the sum of the difference of influence factors for each soil layer.

Similarly,

$$
\begin{aligned}
E_{w2,1} &= B_2 \mathrm{P} r^2 \int_0^{H_1} \frac{z^3 dz}{\left(r^2+z^2\right)^3} + B_1 \mathrm{P} r^2 \int_{H_1}^{H_2} \frac{z\,dz}{\left(r^2+z^2\right)^2} + \ldots \\
&= P\left[\frac{B_2}{2}\left\{0.5 + \frac{r^4}{H_1^4}\frac{1}{2\left((r/H_1)^2+1\right)^2} - \frac{r^2}{H_1^2}\frac{1}{\left((r/H_1)^2+1\right)}\right\} + \right. \\
&\quad \left.\frac{B_1}{2}\left\{\frac{r^2}{H_1^2}\frac{-1}{\left((r/H_2)^2+1\right)} + \frac{r^2}{H_1^2}\frac{1}{\left((r/H_1)^2+1\right)}\right\} + \ldots\right]
\end{aligned} \tag{18}
$$

Letting the ratios $r/H_1 = n_1$ *and* $r/H_2 = n_2$, one obtains

$$
\begin{aligned}
E_{w2,1} &= P\left[\frac{B_2}{2}\left\{0.5 + \frac{n^4}{2\left(n_1^2+1\right)^2} - \frac{n_1^2}{1+n_1^2}\right\} + \right. \\
&\quad \left.\frac{B_1}{2}\left\{\frac{n_1^2}{1+n_1^2} - \frac{n_2^2}{1+n_2^2}\right\} + \ldots\right] = m_{2,1} P
\end{aligned} \tag{19}
$$

And similarly we obtain $m_{21} = \underbrace{\left(m_1^1 - m_0^1\right)}_{sand} + \underbrace{\left(m_2^2 + m_1^2\right)}_{clay} + \ldots$

From Eqs. (16) - (19), it can be easily deduced that for stratified soil of j layers (Fig. 4), the total thrust will be

$$
E_w = P \sum_{i=0,k=1}^{j-1,j} \left(m_{i+1}^k - m_i^k\right) = mP \tag{20}
$$

Where $m = \sum_{i=0,k=1}^{j-1,j} \left(m_{i+1}^k - m_i^k\right) =$ the sum of the difference of influence factors for k-th soil layer, limited between i-th and $(i+1)$-th boundaries, where, $i = 0,1,2,3,\ldots j-1$.

The influence factors can be determined from Table 1 for the given soil type.

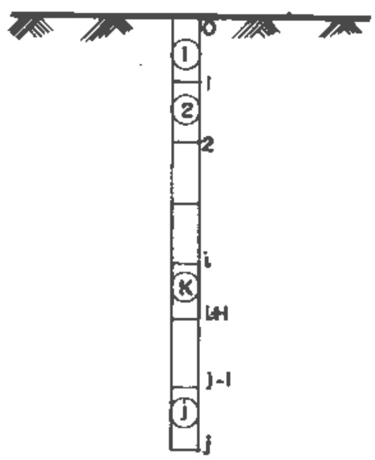

Figure 4 Stratified Soil Layers and Limits

LOCATION OF THE CENTROID OF TOTAL STRESS AREA

The centroid of the total stress area (Fig.5) is given by

$$Z_c = \frac{S_x}{A} = \frac{1}{A}\int_A z\,dA \tag{21}$$

Where,

S_x = Moment of the area which covers the entire stress area (static moment about the x-axis)

A = Total stress area of Fig. 5 (i.e. the total thrust).

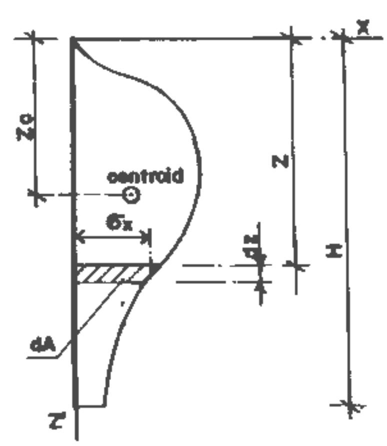

Figure 5 Centroid of total stress area

Thus, moment of the area of Fig. 5 is expressed as the sum of the moments of the elemental areas such as dA about X-axis as given in Eq. (21).

From Fig. 5, the elemental area $dA = \sigma_x dz$

Substituting this in Eq. (21) and inserting limits

$$Z_c = \frac{1}{A}\int_0^H z\sigma_x dz \tag{22}$$

Further, the centroid of Fig. 5 for different cases using Eq. (22) shall be determined.

A. Point Load

Case 1: Medium Stiff Clay

The value of σ_x in Eq. (22) can be expressed as given by Eq. (2) for the above case.

The centroid (i.e. the point of application of the total thrust) is therefore,

$$Z_c = \frac{1}{A}\int_0^H z\sigma_{x1}dz = \frac{B_1 Pr^2}{A}\int_0^H \frac{z^2 dz}{\left(r^2+z^2\right)^2} =$$

$$= \frac{B_1 Pr^2}{A}\left[-\frac{H}{2\left(r^2+H^2\right)} + \frac{1}{2r}\arctan\frac{H}{r}\right] \tag{23}$$

$$= \frac{B_1 PH}{A}\left[-\frac{r^2}{2H^2\left((r/H)^2+1\right)} + \frac{r}{2H}\arctan\frac{1}{r/H}\right]$$

The total stress area $A = E_{w1} = m_1 P$ [See Eq. (5)].

Let the ratio $r/H = n$.

Substituting these in Eq. (23),

$$Z_c = \alpha_c H \tag{24}$$

Where, $\alpha_c = f(n = r/H) = \frac{B_1}{m_1}\left[\frac{n}{2}\arctan\frac{1}{n} - \frac{n^2}{2\left(1+n^2\right)}\right]$

Table 3 lists values of α_c for various ratios of $n = r/H$.

Case 2: Medium Dense Sand

The value of σ_x in Eq.(22) can be expressed as given by Eq. (3) for the above case.

The centroid is found in a similar way and becomes

$$Z_c = \frac{1}{A}\int_0^H z\sigma_{x2}dz = \frac{B_2 Pr^2}{A}\int_0^H \frac{z^4 dz}{\left(r^2+z^2\right)^3} \tag{25}$$

For convenience, disintegrating the function under integral of Eq. (25)

$$\frac{z^4}{\left(r^2+z^2\right)^3}=\frac{\left(z^4-r^4\right)+r^4}{\left(r^2+z^2\right)^3}=\frac{\left(z^2+r^2\right)+\left(z^2-r^2\right)+r^4}{\left(r^2+z^2\right)^3}$$

$$=\frac{z^2}{\left(r^2+z^2\right)^2}-r^2\frac{1}{\left(r^2+z^2\right)^2}+r^4\frac{1}{\left(r^2+z^2\right)^3}$$

Hence:

$$Z_c=\frac{B_2Pr^2}{A}\left[\int_0^H\frac{z^2dz}{\left(r^2+z^2\right)^2}-r^2\int_0^H\frac{dz}{\left(r^2+z^2\right)^2}+\right.$$
$$\left.r^4\int_0^H\frac{dz}{\left(r^2+z^2\right)^3}\right]=\frac{B_2Pr^2}{A}\left[\frac{1}{2}\left(\frac{1}{r}\arctan\frac{H}{r}-\frac{H}{r^2+H^2}\right)\right.$$
$$-\frac{r^2}{2}\left(\frac{H}{r^2\left(r^2+H^2\right)}+\frac{1}{r^3}\arctan\frac{H}{r}\right)+\frac{r^4}{4}\left(\frac{H}{r^2\left(r^2+H^2\right)^2}\right.$$
$$\left.\left.+\frac{3H}{2r^4\left(r^2+H^2\right)}+\frac{3}{2r^5}\arctan\frac{H}{r}\right)\right]=\frac{B_2P}{A}\left[\frac{3r}{8}\arctan\frac{H}{r}+\right.$$
$$\left.\frac{r^4H}{4\left(r^2+H^2\right)^2}-\frac{5r^2H}{8\left(r^2+H^2\right)}\right]=\frac{B_2HP}{A}\left[\frac{3r}{8H}\arctan\frac{1}{r/H}+\right.$$
$$\left.+\frac{r^4}{4H^4\left(1+r^2/H^2\right)^2}-\frac{5r^2}{8H^2\left(1+r^2/H^2\right)}\right] \qquad (26)$$

The total stress area $A=E_{w2}=m_2P$, [see Eq. (7)].

Let the ratio $r/H=n$.

Substituting the above in Eq. (26), the centroid of the total stress area for case 2 will be,

$$Z_c=\alpha_s H \qquad (27)$$

where,

$$\alpha_s=f(n=r/H)=\frac{B_2}{m_2}\left[\frac{3n}{8}\arctan\frac{1}{n}+\frac{n^4}{4\left(1+n^2\right)^2}-\frac{5n^2}{8\left(1+n^2\right)}\right]$$

Table 3 lists value of α_s for various ratios of $n=r/H$. Eqs. (24) & (27) can be used for the case of line load without doubling the value of Z_c since the effect of doubling the stress value σ_x and the total stress area A gives a unity ratio for Eq. (21).

B. Uniform Load - Both Types of Soil

A uniform pressure q acts on a strip area of width b and infinite length, the centre of which is defined by x_o from the earth-retaining structure as per Fig. 2.

The value of σ_x in Eq. (22) can be expressed as given by Eq. (11).

The centroid is therefore,

$$Z_c=\frac{1}{A}\int_0^H z\sigma_{x3}dz=\frac{-q}{\pi A}\left[\int_0^H\left(z\,\mathrm{arccot}\frac{z}{a_2}\right)dz-\right.$$
$$\left.\int_0^H\left(z\,\mathrm{arccot}\frac{z}{a_1}\right)dz-a_2\int_0^H\frac{z^2dz}{a_2^2+z^2}+a_1\int_0^H\frac{z^2dz}{a_1^2+z^2}\right]=$$
$$=\frac{-q}{\pi A}\left[\left(\frac{1}{2}\left(z^2+a_2^2\right)\mathrm{arccot}\frac{z}{a_2}+\frac{a_2z}{2}\right)_0^H-\right.$$
$$\left(\frac{1}{2}\left(z^2+a_1^2\right)\mathrm{arccot}\frac{z}{a_1}+\frac{a_1z}{2}\right)_0^H-a_2\left(z-a_2\arctan\frac{z}{a_2}\right)_0^H+$$
$$\left.a_1\left(z-a_1\arctan\frac{z}{a_1}\right)_0^H\right]$$

From trigonometry; $\mathrm{arccot}\,x=\frac{\pi}{2}-\arctan x$

Substituting this, the equation simplifies to:

$$Z_c=\frac{-q}{\pi A}\left[\frac{H^2}{2}\left(\arctan\frac{H}{a_1}-\arctan\frac{H}{a_2}\right)+\frac{1}{2}a_2^2\arctan\frac{H}{a_2}\right.$$
$$\left.-\frac{a_1^2}{2}\arctan\frac{H}{a_1}+\frac{H}{2}\left(a_1-a_2\right)\right]=\frac{qH^2}{2\pi A}\left[\frac{a_1^2}{H^2}\arctan\frac{H}{a_1}-\right.$$
$$\left.\left(\arctan\frac{H}{a_1}-\arctan\frac{H}{a_2}\right)-\frac{a_2^2}{H^2}\arctan\frac{H}{a_2}+\frac{1}{H}\left(a_2-a_1\right)\right]=$$
$$\frac{qH^2}{2\pi A}\left[\left(\frac{a_1^2}{H^2}-1\right)\arctan\frac{H}{a_1}+\left(1-\frac{a_2^2}{H^2}\right)\arctan\frac{H}{a_2}+\right.$$
$$\left.\frac{1}{H}\left(a_2-a_1\right)\right] \qquad (28$$

Substituting for a_1 and a_2

$a_1=x_0+b/2$ and $a_2=x_0-b/2$,

and noting that the total stress area

$A=E_{w3}=m_3\,qH$, [see Eq. (12)].

$$Z_c = \frac{qH^2}{2\pi\, m_3\, qH}\left[\left\{\left(\frac{x_o + b/2}{H}\right)^2 - 1\right\} arctan\frac{H}{x_o + b/2} + \right.$$

$$\left.\left\{1 - \left(\frac{x_o - b/2}{H}\right)^2\right\} arctan\frac{H}{x_o - b/2} - b/H\right]$$

Let $x_o/b = n$ and $H/b = m$, then the centroid will be

$$Z_c = \frac{H}{2\pi\, m_3}\left[\left\{\left(\frac{n+0.5}{m}\right)^2 - 1\right\} arctan\frac{m}{n+0.5} + \right.$$

$$\left.\left\{1 - \left(\frac{n-0.5}{m}\right)^2\right\} arctan\frac{m}{n-0.5} - \frac{1}{m}\right] \qquad (29)$$

Denoting that $\alpha = f(n = x_o/b,\ m = H/b) =$

$$\frac{1}{2\pi\, \mathrm{m}_3}\left[\left\{\left(\frac{n+0.5}{m}\right)^2 - 1\right\} arctan\ \frac{m}{n+0.5} + \right.$$

$$\left.\left\{1 - \left(\frac{n-0.5}{m}\right)^2\right\} arctan\ \frac{m}{n-0.5} - \frac{1}{m}\right]$$

Then, $Z_c = \alpha H \qquad (30)$

Table 4 lists the influence factor $\alpha = f(n = x_o/b, m = H/b)$.

C. Centroid for Stratified Soils Due to a Point Load

We shall consider stratified j soil layers, formation of which is in 'clay-sand' and 'sand-clay' series as per Fig. 6(a) and 6(b), respectively.

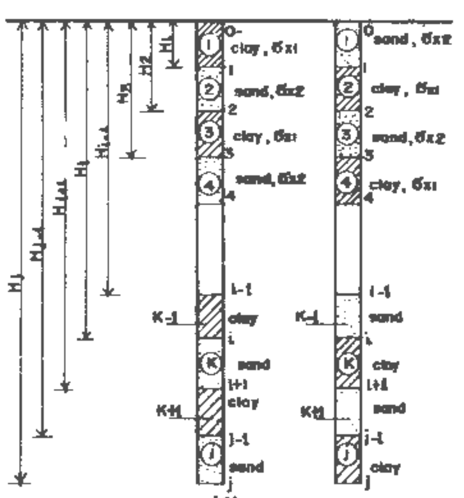

Figure 6 Stratified soil: (a) *'clay-sand'* series
(b) *'Sand-clay'* series

Case 1: *'Clay-Sand' series*

The centroid of the entire stress area due to concentrated load P acting on stratified j soil layers, the formation of which is in 'clay-sand' series can be expressed as [See Fig.6(a)].

$$Z_c = \frac{S_x}{A} = \frac{1}{A}\int_A z\,dA = \frac{1}{A}\left[\int_0^{H_1} z\sigma_{z1}dz + \int_{H_1}^{H_2} z\sigma_{z2}dz + \right.$$

$$\left.\int_{H_2}^{H_3} z\sigma_{z1}dz + \ldots + \int_{H_{i-1}}^{H_i} z\sigma_{z1}dz + \int_{H_i}^{H_{i+1}} z\sigma_{z2}dz + \ldots\right] =$$

$$\frac{1}{A}\left[\sum_{i=1,k=1}^{j,j}\left(\int_{H_{i-1}}^{H_i} z\sigma_{z1}dz + \int_{H_i}^{H_i+1} z\sigma_{z2}dz\right)\right] =$$

$$\frac{1}{A}\left[\sum_{i=1,k=1}^{j,j} \mathrm{Pr}^2\left(B_1\int_{H_{i-1}}^{H_i}\frac{z^2dz}{\left(r^2+z^2\right)^2} + B_2\int_{H_i}^{H_{i+1}}\frac{z^4dz}{\left(r^2+z^2\right)^3}\right)\right] =$$

$$\frac{P}{A}\left[\sum_{j=1,k=1}^{j,j}\left(B_1H_i\left\{\frac{r}{2H_i}\mathrm{arctan}\,\frac{1}{r/H_i} - \frac{r}{2H_i}\mathrm{arctan}\,\frac{1}{r/H_{i-1}}\right.\right.\right.$$

$$\left. - \frac{r^2}{2H_i^2\left(1+r^2/H_i^2\right)} + \frac{H_{i-1}}{H_i}\frac{r^2}{2H_{i-1}^2\left(1+r^2/H_{i-1}^2\right)}\right\} +$$

$$B_2H_{i+1}\left\{\frac{3r}{8H_{i+1}}\mathrm{arctan}\,\frac{1}{r/H_{i+1}} - \frac{3r}{8H_{i+1}}\mathrm{arctan}\,\frac{1}{r/H_i} + \right.$$

$$\frac{r^4}{4H_{i+1}^4\left(1+r^2/H_{i+1}^2\right)^2} - \frac{H_i}{H_{i+1}}\ \frac{r^4}{4H_i^4\left(1+r^2/H_i^2\right)^2} +$$

$$\left.\left.\left.\frac{5H_i}{8H_{i+1}}\ \frac{r^2}{H_i^2\left(1+r^2/H_i^2\right)} - \frac{5}{8}\ \frac{r^2}{H_{i+1}^2\left(1+r^2/H_{i+1}^2\right)}\right\}\right)\right] \qquad (31)$$

The total stress area

$$A = P\sum_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right), \qquad i = 0,1,2,3,\ldots,j-1$$

$$k = 1,2,3,\ldots,j$$

Let $n_{i-1} = \dfrac{r}{H_{i-1}}$, $n_i = \dfrac{r}{H_i}$, $n_{i+1} = \dfrac{r}{H_{i+1}}$

Substituting the above ratios and the value of total stress area in Eq. (31), centroid will be

$$
\begin{aligned}
Z_c = {} & \frac{1}{\sum\limits_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right)}\left[\sum_{i=1,k=1}^{j,j}\left(B_1 H_i\left\{\frac{n_i}{2}\arctan\frac{1}{n_i} - \right.\right.\right. \\
& \frac{n_i}{2}\arctan\frac{1}{n_{i-1}} - \frac{n_i^2}{2\left(1+n_i^2\right)} + \frac{H_{i-1}}{H_i}\frac{n_{i-1}^2}{2\left(1+n_{i-1}^2\right)}\Bigg\} + \\
& B_2 H_{i+1}\left\{\frac{3n_{i+1}}{8}\arctan\frac{1}{n_{i+1}} - \frac{3n_{i+1}}{8}\arctan\frac{1}{n_i} + \right. \\
& \frac{n_{i+1}^4}{4\left(1+n_{i+1}^2\right)^2} - \frac{H_i}{H_{i+1}}\frac{n_i^4}{4\left(1+n_i^2\right)^2} + \frac{5H_i}{8H_{i+1}}\frac{n_i^2}{\left(1+n_i^2\right)} - \\
& \left.\left.\left.\frac{5}{8}\frac{n_{i+1}^2}{\left(1+n_{i+1}^2\right)}\right\}\right)\right]
\end{aligned}
\tag{32}
$$

Multiplying Z_c by H_j/H_j for the 'clay sand' series,

$$
\begin{aligned}
Z_c = {} & \frac{H_j}{\sum\limits_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right)}\left[\sum_{i=1,k=1}^{j,j}\left(B_1\left\{\frac{H_i}{H_j}\frac{n_i}{2}arctan\frac{1}{n_i} - \right.\right.\right. \\
& \frac{H_i}{H_j}\frac{n_i}{2}arctan\frac{1}{n_{i-1}} - \frac{H_i}{H_j}\frac{n_i^2}{2\left(1+n_i^2\right)} + \frac{H_{i-1}}{H_j}\frac{n_{i-1}^2}{2\left(1+n_{i-1}^2\right)}\Bigg\} + \\
& B_2\left\{\frac{H_{i+1}}{H_j}\frac{3n_{i+1}}{8}arctan\frac{1}{n_{i+1}} - \frac{H_{i+1}}{H_j}\frac{3n_{i+1}}{8}arctan\frac{1}{n_i} + \right. \\
& \frac{H_{i+1}}{H_j}\frac{n_{i+1}^4}{4\left(1+n_{i+1}^2\right)^2} - \frac{H_i}{H_j}\frac{n_i^4}{4\left(1+n_i^2\right)^2} + \frac{5H_i}{8H_j}\frac{n_i^2}{\left(1+n_i^2\right)} - \\
& \left.\left.\left.\frac{5H_{i+1}}{8H_j}\frac{n_{i+1}^2}{\left(1+n_{i+1}^2\right)}\right\}\right)\right]
\end{aligned}
\tag{33}
$$

Here, $j = 1,2,3,\ldots$ = Number of soil layers
Note also that,

$$
arctan\frac{1}{n_o} = 0, \text{ since } \frac{1}{r/H_0} = 0 \text{ for } H_0 = 0!
$$

Case 2: *'Sand-Clay' series*

The centroid of the entire stress area due to concentrated load P acting on stratified j soil layers, formation of which is in 'Sand Clay' series can be expressed as [See Fig. 6(b)].

$$
\begin{aligned}
Z_c = {} & \frac{S_x}{A} = \frac{1}{A}\int_A z\,dA = \frac{1}{A}\left[\int_0^{H_1} z\sigma_{x2}dz + \int_{H_1}^{H_2} z\sigma_{x1}dz + \right. \\
& \left.\int_{H_2}^{H_3} z\sigma_{x2}dz + \ldots + \int_{H_{j-1}}^{H_i} z\sigma_{x2}dz + \int_{H_i}^{H_{i+1}} z\sigma_{x1}dz + \ldots\right] = \\
& \frac{1}{A}\left[\sum_{i=1,k=1}^{j,j}\left(\int_{H_{i-1}}^{H_i} z\sigma_{x2}dz + \int_{H_i}^{H_{i+1}} z\sigma_{x1}dz\right)\right] = \\
& \frac{1}{A}\left[\sum_{i=1,k=1}^{j,j} Pr^2\left(B_2\int_{H_{i-1}}^{H_i}\frac{z^4dz}{\left(r^2+z^2\right)^3} + B_1\int_{H_i}^{H_{i+1}}\frac{z^2dz}{\left(r^2+z^2\right)^2}\right)\right] = \\
& \frac{P}{A}\left[\sum_{i=1,k=1}^{j,j}\left(B_2H_i\left\{\frac{3r}{8H_i}arctan\frac{1}{r/H_i} - \frac{3r}{8H_i}arctan\frac{1}{r/H_{i-1}} + \right.\right.\right. \\
& \frac{r^4}{4H_i^4\left(1+r^2/H_i^2\right)^2} - \frac{H_{i-1}}{H_i}\frac{r^4}{4H_{i-1}^4\left(1+r^2/H_{i-1}^2\right)^2} + \\
& \frac{5H_{i-1}}{8H_i}\frac{r^2}{H_{i-1}^2\left(1+r^2/H_{i-1}^2\right)} - \frac{5}{8}\frac{r^2}{H_i^2\left(1+r^2/H_i^2\right)}\Bigg\} + \\
& B_1H_{i+1}\left\{\frac{r}{2H_{i+1}}arctan\frac{1}{r/H_{i+1}} - \frac{r}{2H_{i+1}}arctan\frac{1}{r/H_i} - \right. \\
& \left.\left.\left.\frac{r^2}{2H_{i+1}^2\left(1+r^2/H_{i+1}^2\right)} + \frac{H_i}{H_{i+1}}\frac{r^2}{2H_i^2\left(1+r^2/H_i^2\right)}\right\}\right)\right]
\end{aligned}
\tag{34}
$$

The total stress area

$$
A = P\sum_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right), \quad \text{where} \quad i = 0,1,2,3,\ldots,j\text{-}1
$$

$$
\text{Let } n_{i-1} = \frac{r}{H_{i-1}}, \; n_i = \frac{r}{H_i}, \; n_{i+1} = \frac{r}{H_{i+1}}
$$

Substituting the above ratios and the value of total stress area in Eq. (34),

$$
\begin{aligned}
Z_c = {} & \frac{1}{\sum\limits_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right)}\left[\sum_{i=1,k=1}^{j,j}\left(B_2H_i\left\{\frac{3n_i}{8}arctan\frac{1}{n_i} - \right.\right.\right. \\
& \frac{3n_i}{8}arctan\frac{1}{n_{i-1}} + \frac{n_i^4}{4\left(1+n_i^2\right)^2} - \frac{H_{i-1}}{H_i}\frac{n_{i-1}^4}{4\left(1+n_{i-1}^2\right)^2} + \\
& \frac{5H_{i-1}}{8H_i}\frac{n_{i-1}^2}{\left(1+n_{i-1}^2\right)} - \frac{5}{8}\frac{n_i^2}{\left(1+n_i^2\right)}\Bigg\} + B_1H_{i+1}\left\{\frac{n_{i+1}}{2}arctan\frac{1}{n_{i+1}}\right. \\
& \left.\left.\left.-\frac{n_{i+1}}{2}arctan\frac{1}{n_i} - \frac{n_{i+1}^2}{2\left(1+n_{i+1}^2\right)} + \frac{H_i}{H_{i+1}}\frac{n_i^2}{2\left(1+n_i^2\right)}\right\}\right)\right]
\end{aligned}
\tag{35}
$$

Similarly, for the 'Sand-clay' series,

$$
\begin{aligned}
Z_c = \frac{H_j}{\sum\limits_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right)} \Bigg[\sum_{i=1,k=1}^{j,j} \Bigg(B_2 \Bigg\{ & \frac{H_i}{H_j}\frac{3n_i}{8}\arctan\frac{1}{n_i} - \\
& \frac{H_i}{H_j}\frac{3n_i}{8}\arctan\frac{1}{n_{i-1}} + \frac{H_i}{H_j}\frac{n_i^4}{4\left(1+n_i^2\right)^2} - \frac{H_{i-1}}{H_j}\frac{n_{i-1}^4}{4\left(1+n_{i-1}^2\right)^2} + \\
& \frac{5H_{i-1}}{8H_j}\frac{n_{i-1}^2}{\left(1+n_{i-1}^2\right)} - \frac{5}{8}\frac{H_i}{H_j}\frac{n_i^2}{\left(1+n_i^2\right)} \Bigg\} + B_1 \Bigg\{ \frac{H_{i+1}}{H_j}\frac{n_{i+1}}{2}\arctan\frac{1}{n_{i+1}} - \\
& \frac{H_{i+1}}{H_j}\frac{n_{i+1}}{2}\arctan\frac{1}{n_i} - \frac{H_{i+1}}{H_j}\frac{n_{i+1}^2}{2\left(1+n_{i+1}^2\right)} + \frac{H_i}{H_j}\frac{n_i^2}{2\left(1+n_i^2\right)} \Bigg\} \Bigg) \Bigg]
\end{aligned}
\tag{36}
$$

For one layer of clay soil, in Eq. (33),

$$\sum_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right) = m_1^1 = m_1$$

because, $k = 1$ and $i_{max} = j-1 = 1-1 = 0$ and thus $m_0^1 = 0$

Again for a single layer, $H_j = H_1 = H$

and for the expression $\sum\limits_{i=1,k=1}^{j,j}\left[B_1\{\ldots\right.$, $i = i_{max} = j = 1$, $k =$

$$\text{Therefore } Z_c = \frac{H}{m_1}\left[B_1\left\{\frac{H_1}{H}\frac{n_1}{2}\arctan\frac{1}{n_1} - \frac{H_1}{H}\frac{n_1}{2}\arctan\frac{1}{n_0} - \frac{H_1}{H}\frac{n_1^2}{2\left(1+n_1^2\right)} + \frac{H_0}{H}\frac{n_0^2}{2\left(1+n_0^2\right)}\right\}\right] = H\left[\alpha_1 - \alpha_0\right]$$

Here, $H_1 = H$, $H_0 = 0$ and $arctan\frac{1}{n_0} = arctan\frac{1}{r/H_o} = 0$

Then, $\alpha_0 = 0$ and $\alpha_1 = \alpha_c$. Thus,

$$Z_c = \frac{B_1 H}{m_1}\left[\frac{n_1}{2}arctan\frac{1}{n_1} - \frac{n_1^2}{2\left(1+n_1^2\right)}\right] = \alpha_c H, \because \text{ same as Eq.(24).}$$

Note that, $\sum\limits_{i=1,k=1}^{j,j}$ is equivalent to $\sum\limits_{i=0,k=1}^{j-1,j}$

From the above summary, it can be concluded that the centroid of the entire stress area for stratified j soil layers in any order of the soil formation is given by:

$$Z_c = H\sum_{i=0,k=1}^{j-1,j}\left(\alpha_{i+1}^k - \alpha_i^k\right) = \alpha' H \tag{37}$$

Where, $\alpha' = \sum\limits_{i=0,k=1}^{j-1,j}\left(\alpha_{i+1}^k - \alpha_i^k\right)$

Table 1: Influence factors m_1 and m_2 for total thrust on earth-retaining structures due to a point load

r/H	m_1	m_2	r/H	m_1	m_2	r/H	m_1	m_2	r/H	m_1	m_2	r/H	m_1	m_2
0.00	0.3200	0.2125	0.50	0.2560	0.1360	1.00	0.1600	0.0531	1.50	0.0985	0.0201	2.00	0.0640	0.0085
0.02	0.3199	0.2123	0.52	0.2519	0.1317	1.02	0.1568	0.0510	1.52	0.0967	0.0194	2.10	0.0591	0.0073
0.04	0.3195	0.2118	0.54	0.2478	0.1274	1.04	0.1537	0.0490	1.54	0.0949	0.0187	2.20	0.0548	0.0062
0.06	0.3189	0.2110	0.56	0.2436	0.1231	1.06	0.1507	0.0471	1.56	0.0932	0.0180	2.30	0.0509	0.0054
0.08	0.3180	0.2098	0.58	0.2394	0.1190	1.08	0.1477	0.0453	1.58	0.0915	0.0174	2.40	0.0473	0.0047
0.10	0.3168	0.2083	0.60	0.2353	0.1149	1.10	0.1448	0.0435	1.60	0.0899	0.0168	2.50	0.0441	0.0040
0.12	0.3155	0.2065	0.62	0.2311	0.1109	1.12	0.1419	0.0418	1.62	0.0883	0.0162	2.60	0.0412	0.0035
0.14	0.3138	0.2044	0.64	0.2270	0.1069	1.14	0.1392	0.0402	1.64	0.0867	0.0156	2.70	0.0386	0.0031
0.16	0.3120	0.2020	0.66	0.2229	0.1031	1.16	0.1364	0.0386	1.66	0.0852	0.0151	2.80	0.0362	0.0027
0.18	0.3100	0.1994	0.68	0.2188	0.0994	1.18	0.1338	0.0371	1.68	0.0837	0.0145	2.90	0.0340	0.0024
0.20	0.3077	0.1965	0.70	0.2148	0.0957	1.20	0.1311	0.0357	1.70	0.0823	0.0140	3.00	0.0320	0.0021
0.22	0.3052	0.1933	0.72	0.2107	0.0922	1.22	0.1286	0.0343	1.72	0.0808	0.0136	3.20	0.0285	0.0017
0.24	0.3026	0.1900	0.74	0.2068	0.0887	1.24	0.1261	0.0330	1.74	0.0795	0.0131	3.40	0.0255	0.0013
0.26	0.2997	0.1864	0.76	0.2028	0.0854	1.26	0.1237	0.0317	1.76	0.0781	0.0127	3.60	0.0229	0.0011
0.28	0.2967	0.1827	0.78	0.1990	0.0821	1.28	0.1213	0.0305	1.78	0.0768	0.0122	3.80	0.0207	0.0009
0.30	0.2936	0.1789	0.80	0.1951	0.0[illegible]90	1.30	0.1190	0.0294	1.80	0.0755	0.0118	4.00	0.0188	0.0007
0.32	0.2903	0.1749	0.82	0.1913	0.0760	1.32	0.1167	0.0283	1.82	0.0742	0.0114	4.20	0.0172	0.0006
0.34	0.2868	0.1707	0.84	0.1876	0.0730	1.34	0.1145	0.0272	1.84	0.0730	0.0110	4.40	0.0157	0.0005
0.36	0.2833	0.1665	0.86	0.1840	0.0702	1.36	0.1123	0.0262	1.86	0.0718	0.0107	4.80	0.0133	0.0004
0.38	0.2796	0.1623	0.88	0.1803	0.0675	1.38	0.1102	0.0252	1.88	0.0706	0.0103	5.50	0.0102	0.0002
0.40	0.2759	0.1579	0.90	0.1768	0.0647	1.40	0.1081	0.0243	1.90	0.0694	0.0100	8.00	0.0049	0.0001
0.42	0.2720	0.1535	0.92	0.1733	0.0623	1.42	0.1061	0.0234	1.92	0.0683	0.0097	10.00	0.0032	-
0.44	0.2681	0.1492	0.94	0.1699	0.0599	1.44	0.1041	0.0225	1.94	0.0672	0.0094	15.00	0.0014	-
0.46	0.2641	0.1448	0.96	0.1665	0.0575	1.46	0.1022	0.0217	1.96	0.0661	0.0091	20.00	0.0008	-
0.48	0.2601	0.1404	0.98	0.1632	0.0553	1.48	0.1003	0.0209	1.98	0.0650	0.0088	50.00	0.0001	-

Note: 1) Intermediate values of the factors can be determined by linear interpolation.
2) The influence factors: m_1 – for clay
m_2 – for sand

Table 2: Influence factor m_3 for total thrust on earth-retaining structures due to a uniform load on a strip area

$m = \frac{H}{b}$	Value of $n = x_0/b$											
	0.51	0.75	1.0	1.25	1.50	2.0	2.5	3.0	3.5	5.0	7.5	10
0.00	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.25	0.4100	0.1872	0.0950	0.0573	0.0384	0.0213	0.0136	0.0090	0.0066	0.0032	0.0014	0.0008
0.50	0.3473	0.2313	0.1476	0.0986	0.0696	0.0396	0.0254	0.0177	0.0130	0.0064	0.0028	0.0016
0.75	0.2925	0.2256	0.1653	0.1211	0.0906	0.0548	0.0362	0.0256	0.0190	0.0094	0.0042	0.0024
1.00	0.2484	0.2073	0.1652	0.1299	0.1024	0.0660	0.0452	0.0325	0.0244	0.0124	0.0056	0.0032
1.25	0.2138	0.1872	0.1578	0.1306	0.1074	0.0736	0.0521	0.0384	0.0293	0.0151	0.0069	0.0039
1.50	0.1865	0.1685	0.1476	0.1268	0.1080	0.0780	0.0572	0.0431	0.0334	0.0177	0.0082	0.0047
1.75	0.1648	0.1523	0.1370	0.1211	0.1060	0.0800	0.0607	0.0468	0.0368	0.0200	0.0094	0.0054
2.00	0.1473	0.1382	0.1268	0.1146	0.1024	0.0804	0.0628	0.0495	0.0396	0.0221	0.0106	0.0061
2.50	0.1210	0.1159	0.1092	0.1016	0.0936	0.0780	0.0641	0.0526	0.0433	0.0256	0.0128	0.0075
3.00	0.1023	0.0992	0.0950	0.0901	0.0847	0.0736	0.0628	0.0533	0.0452	0.0282	0.0147	0.0089
3.50	0.0885	0.0865	0.0837	0.0804	0.0766	0.0686	0.0603	0.0526	0.0456	0.0300	0.0163	0.0099
4.00	0.0779	0.0766	0.0746	0.0723	0.0696	0.0636	0.0572	0.0510	0.0452	0.0311	0.0177	0.0110
4.50	0.0696	0.0686	0.0672	0.0655	0.0635	0.0590	0.0540	0.0490	0.0441	0.0317	0.0188	0.0119
5.00	0.0628	0.0621	0.0611	0.0598	0.0583	0.0548	0.0509	0.0468	0.0428	0.0319	0.0196	0.0128
5.50	0.0572	0.0567	0.0559	0.0549	0.0538	0.0510	0.0479	0.0446	0.0412	0.0317	0.0203	0.0135
6.00	0.0526	0.0521	0.0515	0.0507	0.0498	0.0477	0.0452	0.0424	0.0396	0.0313	0.0207	0.0141
6.50	0.0486	0.0483	0.0477	0.0471	0.0464	0.0447	0.0426	0.0404	0.0380	0.0308	0.0210	0.0146
7.00	0.0452	0.0449	0.0445	0.0440	0.0434	0.0420	0.0403	0.0384	0.0364	0.0301	0.0212	0.0150
7.50	0.0422	0.0420	0.0416	0.0412	0.0407	0.0396	0.0382	0.0366	0.0348	0.0294	0.0212	0.0153
8.00	0.0396	0.0394	0.0392	0.0388	0.0384	0.0374	0.0362	0.0349	0.0334	0.0286	0.0212	0.0155
8.50	0.0373	0.0371	0.0369	0.0366	0.0363	0.0355	0.0344	0.0333	0.0320	0.0278	0.0211	0.0157
9.00	0.0352	0.0351	0.0349	0.0347	0.0344	0.0337	0.0328	0.0318	0.0307	0.0270	0.0209	0.0158
9.50	0.0334	0.0333	0.0331	0.0329	0.0327	0.0321	0.0313	0.0305	0.0295	0.0262	0.0206	0.0159
10.00	0.0317	0.0316	0.0315	0.0313	0.0311	0.0306	0.0299	0.0292	0.0283	0.0255	0.0204	0.0159
10.50	0.0302	0.0301	0.0300	0.0299	0.0297	0.0292	0.0287	0.0280	0.0273	0.0247	0.0201	0.0159
11.00	0.0289	0.0288	0.0287	0.0286	0.0284	0.0280	0.0275	0.0269	0.0263	0.0240	0.0198	0.0158
11.50	0.0276	0.0276	0.0274	0.0273	0.0272	0.0269	0.0264	0.0259	0.0253	0.0233	0.0194	0.0158
12.00	0.0265	0.0264	0.0264	0.0262	0.0261	0.0258	0.0254	0.0250	0.0244	0.0226	0.0191	0.0157
12.50	0.0254	0.0254	0.0253	0.0252	0.0251	0.0247	0.0245	0.0241	0.0236	0.0219	0.0187	0.0155
13.00	0.0244	0.0244	0.0244	0.0243	0.0242	0.0239	0.0236	0.0232	0.0228	0.0213	0.0184	0.0154
13.50	0.0236	0.0235	0.0235	0.0234	0.0233	0.0231	0.0228	0.0225	0.0221	0.0207	0.0180	0.0152
14.00	0.0227	0.0227	0.0226	0.0226	0.0225	0.0223	0.0220	0.0217	0.0214	0.0202	0.0177	0.0151
14.50	0.0219	0.0219	0.0218	0.0218	0.0217	0.0215	0.0213	0.0210	0.0207	0.0196	0.0173	0.0149
15.00	0.0212	0.0211	0.0211	0.0210	0.0210	0.0208	0.0206	0.0204	0.0201	0.0191	0.0170	0.0147
15.50	0.0205	0.0205	0.0204	0.0204	0.0203	0.0202	0.0200	0.0198	0.0195	0.0186	0.0166	0.0145
16.00	0.0199	0.0199	0.0198	0.0198	0.0197	0.0196	0.0194	0.0192	0.0190	0.0181	0.0163	0.0143
17.00	0.0187	0.0187	0.0187	0.0186	0.0186	0.0185	0.0183	0.0182	0.0180	0.0172	0.0157	0.0139
20.00	0.0159	0.0159	0.0159	0.0159	0.0158	0.0158	0.0157	0.0156	0.0154	0.0150	0.0140	0.0127
30.00	0.0106	0.0106	0.0106	0.0106	0.0106	0.0106	0.0105	0.0105	0.0105	0.0103	0.0100	0.0095
50.00	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0064	0.0063	0.0063	0.0063	0.0062	0.0061
100.00	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032	0.0032
200.00	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016

Note: Intermediate values of the factor can be determined by linear interpolation.

Table 3: Influence factors α_c and α_s for the centroid of entire stress area due to a point load

r/H	α_c	α_s	r/H	α_c	α_s	r/H	α_c	α_s	r/H	α_c	α_s	r/H	α_c	α_s
0.00	0.0000	0.0000	0.50	0.4420	0.5787	1.00	0.5708	0.7124	1.50	0.6165	0.7555	2.00	0.6365	0.7736
0.02	0.0306	0.0456	0.52	0.4505	0.5881	1.02	0.5736	0.7151	1.52	0.6176	0.7565	2.10	0.6390	0.7759
0.04	0.0597	0.0881	0.54	0.4586	0.5970	1.04	0.5762	0.7176	1.54	0.6187	0.7575	2.20	0.6413	0.7779
0.06	0.0874	0.1279	0.56	0.4664	0.6054	1.06	0.5788	0.7201	1.56	0.6198	0.7585	2.30	0.6433	0.7796
0.08	0.1136	0.1652	0.58	0.4738	0.6133	1.08	0.5813	0.7225	1.58	0.6208	0.7594	2.40	0.6451	0.7812
0.10	0.1386	0.2000	0.60	0.4808	0.6208	1.10	0.5836	0.7247	1.60	0.6218	0.7603	2.50	0.6467	0.7826
0.12	0.1623	0.2325	0.62	0.4875	0.6279	1.12	0.5859	0.7269	1.62	0.6227	0.7612	2.60	0.6481	0.7839
0.14	0.1848	0.2630	0.64	0.4939	0.6347	1.14	0.5881	0.7290	1.64	0.6237	0.7620	2.70	0.6494	0.7850
0.16	0.2061	0.2915	0.66	0.5000	0.6411	1.16	0.5902	0.7310	1.66	0.6246	0.7628	2.80	0.6505	0.7860
0.18	0.2264	0.3182	0.68	0.5058	0.6471	1.18	0.5922	0.7329	1.68	0.6254	0.7636	2.90	0.6516	0.7869
0.20	0.2457	0.3432	0.70	0.5114	0.6529	1.20	0.5942	0.7347	1.70	0.6263	0.7644	3.00	0.6525	0.7878
0.22	0.2640	0.3667	0.72	0.5167	0.6583	1.22	0.5961	0.7365	1.72	0.6271	0.7652	3.20	0.6542	0.7892
0.24	0.2813	0.3887	0.74	0.5217	0.6635	1.24	0.5979	0.7382	1.74	0.6279	0.7659	3.40	0.6555	0.7904
0.26	0.2978	0.4093	0.76	0.5266	0.6685	1.26	0.5996	0.7399	1.76	0.6287	0.7666	3.60	0.6567	0.7914
0.28	0.3135	0.4287	0.78	0.5312	0.6732	1.28	0.6013	0.7415	1.78	0.6294	0.7673	3.80	0.6577	0.7923
0.30	0.3283	0.4468	0.80	0.5356	0.6776	1.30	0.6030	0.7430	1.80	0.6302	0.7679	4.00	0.6585	0.7930
0.32	0.3425	0.4639	0.82	0.5399	0.6819	1.32	0.6045	0.7444	1.82	0.6309	0.7686	4.20	0.6593	0.7937
0.34	0.3559	0.4800	0.84	0.5439	0.6859	1.34	0.6061	0.7459	1.84	0.6316	0.7692	4.40	0.6599	0.7943
0.36	0.3687	0.4950	0.86	0.5478	0.6898	1.36	0.6075	0.7472	1.86	0.6323	0.7698	4.80	0.6610	0.7951
0.38	0.3808	0.5092	0.88	0.5515	0.6935	1.38	0.6089	0.7485	1.88	0.6329	0.7704	5.50	0.6623	0.7963
0.40	0.3923	0.5226	0.90	0.5551	0.6970	1.40	0.6103	0.7498	1.90	0.6335	0.7709	8.00	0.6646	0.7978
0.42	0.4032	0.5352	0.92	0.5585	0.7004	1.42	0.6116	0.7510	1.92	0.6342	0.7715	10.00	0.6653	0.7989
0.44	0.4137	0.5470	0.94	0.5618	0.7036	1.44	0.6129	0.7522	1.94	0.6348	0.7720	15.00	0.6661	0.8004
0.46	0.4236	0.5582	0.96	0.5649	0.7067	1.46	0.6142	0.7533	1.96	0.6353	0.7726	20.00	0.6663	0.8148
0.48	0.4330	0.5687	0.98	0.5679	0.7096	1.48	0.6154	0.7544	1.98	0.6359	0.7731	50.00	0.6666	0.8571

Note: 1) Intermediate values of the factors can be determined by linear interpolation.

2) The influence factors: α_c - for clay

α_s - for sand

Table 4: Influence factor α for the centroid of entire stress area due to a uniform load on a strip area

$m = \frac{H}{b}$	Value of $n = x_o/b$											
	0.51	0.75	1.0	1.25	1.5	2.0	2.5	3.0	3.5	5.0	7.5	10
0.00	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞
0.25	0.4837	0.6271	0.6520	0.6590	0.6619	0.6643	0.6652	0.6657	0.6659	0.6665	0.6658	0.6618
0.50	0.4427	0.5698	0.6190	0.6391	0.6488	0.6573	0.6609	0.6628	0.6638	0.6654	0.6660	0.6649
0.75	0.4046	0.5205	0.5813	0.6126	0.6298	0.6466	0.6541	0.6580	0.6604	0.6637	0.6653	0.6658
1.00	0.3693	0.4781	0.5445	0.5836	0.6075	0.6329	0.6451	0.6517	0.6557	0.6613	0.6642	0.6652
1.25	0.3375	0.4409	0.5101	0.5546	0.5838	0.6172	0.6343	0.6439	0.6499	0.6584	0.6630	0.6645
1.50	0.3095	0.4081	0.4784	0.5264	0.5597	0.6002	0.6221	0.6349	0.6430	0.6549	0.6614	0.6637
1.75	0.2850	0.3792	0.4494	0.4998	0.5361	0.5826	0.6090	0.6250	0.6354	0.6508	0.6595	0.6627
2.00	0.2636	0.3535	0.4231	0.4748	0.5134	0.5647	0.5953	0.6144	0.6270	0.6463	0.6574	0.6614
2.50	0.2285	0.3105	0.3776	0.4300	0.4712	0.5295	0.5669	0.5917	0.6087	0.6359	0.6524	0.6585
3.00	0.2011	0.2762	0.3399	0.3916	0.4337	0.4962	0.5387	0.5681	0.5890	0.6242	0.6465	0.6550
3.50	0.1794	0.2483	0.3085	0.3587	0.4007	0.4654	0.5114	0.5445	0.5688	0.6114	0.6398	0.6511
4.00	0.1617	0.2254	0.2821	0.3305	0.3718	0.4373	0.4856	0.5215	0.5486	0.5979	0.6325	0.6466
4.50	0.1472	0.2062	0.2597	0.3061	0.3463	0.4118	0.4614	0.4994	0.5287	0.5840	0.6246	0.6417
5.00	0.1350	0.1899	0.2404	0.2848	0.3239	0.3886	0.4390	0.4784	0.5095	0.5698	0.6162	0.6363
5.50	0.1246	0.1760	0.2237	0.2662	0.3040	0.3676	0.4182	0.4586	0.4909	0.5557	0.6074	0.6307
6.00	0.1157	0.1639	0.2092	0.2498	0.2863	0.3486	0.3990	0.4399	0.4732	0.5416	0.5984	0.6247
6.50	0.1080	0.1534	0.1963	0.2352	0.2705	0.3313	0.3813	0.4224	0.4564	0.5278	0.5892	0.6185
7.00	0.1012	0.1441	0.1850	0.2222	0.2562	0.3155	0.3648	0.4060	0.4405	0.5142	0.5798	0.6120
7.50	0.0953	0.1359	0.1748	0.2106	0.2433	0.3010	0.3496	0.3907	0.4253	0.5010	0.5704	0.6054
8.00	0.0900	0.1286	0.1657	0.2000	0.2317	0.2878	0.3356	0.3763	0.4111	0.4882	0.5609	0.5986
8.50	0.0852	0.1220	0.1575	0.1905	0.2210	0.2756	0.3225	0.3628	0.3976	0.4758	0.5515	0.5917
9.00	0.0809	0.1160	0.1501	0.1818	0.2113	0.2644	0.3103	0.3502	0.3848	0.4639	0.5422	0.5847
9.50	0.0770	0.1106	0.1433	0.1738	0.2024	0.2540	0.2990	0.3384	0.3728	0.4523	0.5329	0.5776
10.00	0.0735	0.1056	0.1371	0.1665	0.1942	0.2444	0.2885	0.3272	0.3613	0.4412	0.5238	0.5706
10.50	0.0703	0.1011	0.1314	0.1598	0.1866	0.2354	0.2786	0.3168	0.3506	0.4305	0.5148	0.5635
11.00	0.0674	0.0970	0.1261	0.1536	0.1796	0.2271	0.2693	0.3069	0.3403	0.4202	0.5059	0.5564
11.50	0.0647	0.0932	0.1213	0.1479	0.1731	0.2193	0.2607	0.2976	0.3306	0.4104	0.4972	0.5494
12.00	0.0622	0.0896	0.1168	0.1426	0.1670	0.2120	0.2525	0.2888	0.3215	0.4008	0.4887	0.5423
12.50	0.0598	0.0864	0.1127	0.1376	0.1613	0.2052	0.2448	0.2805	0.3127	0.3917	0.4804	0.5354
13.00	0.0577	0.0833	0.1088	0.1330	0.1560	0.1988	0.2376	0.2727	0.3044	0.3829	0.4723	0.5285
13.50	0.0557	0.0805	0.1051	0.1287	0.1511	0.1928	0.2308	0.2652	0.2966	0.3745	0.4643	0.5217
14.00	0.0538	0.0778	0.1018	0.1246	0.1464	0.1872	0.2243	0.2582	0.2891	0.3664	0.4566	0.5149
14.50	0.0521	0.0754	0.0986	0.1208	0.1420	0.1818	0.2182	0.2515	0.2819	0.3586	0.4490	0.5083
15.00	0.0504	0.0730	0.0956	0.1172	0.1379	0.1768	0.2124	0.2451	0.2751	0.3510	0.4416	0.5017
15.50	0.0489	0.0708	0.0928	0.1138	0.1340	0.1720	0.2069	0.2390	0.2686	0.3438	0.4344	0.4953
16.00	0.0475	0.0688	0.0901	0.1106	0.1303	0.1674	0.2017	0.2332	0.2623	0.3368	0.4274	0.4889
17.00	0.0448	0.0650	0.0852	0.1048	0.1235	0.1590	0.1919	0.2224	0.2507	0.3237	0.4140	0.4765
20.00	0.0384	0.0558	0.0733	0.0903	0.1068	0.1382	0.1676	0.1952	0.2211	0.2894	0.3776	0.4418
30.00	0.0260	0.0379	0.0500	0.0619	0.0736	0.0961	0.1177	0.1384	0.1582	0.2129	0.2895	0.3514
50.00	0.0158	0.0231	0.0306	0.0380	0.0455	0.0597	0.0737	0.0873	0.1006	0.1385	0.1955	0.2456
100.00	0.0079	0.0117	0.0155	0.0193	0.0231	0.0306	0.0380	0.0453	0.0526	0.0737	0.1072	0.1386
200.00	0.0040	0.0059	0.0078	0.0097	0.0117	0.0155	0.0193	0.0231	0.0269	0.0381	0.0562	0.0737

Note: Intermediate values of the factor can be determined by linear interpolation.

Numerical Examples

Comparing the total thrust values using '*Summation Method*' and the *Proposed Formulae.*

Example 1

Calculate total thrust on earth-retaining structure given the following data:

Soil type:- clay
$x = r = 1m,$
$H = 2.1m,$ P=7 ton (concentrated Load)

Solution

a) 'Summation Method'

$$\sigma_{x1} = B_1 \Pr^2 \frac{z}{\left(r^2+z^2\right)^2} = 0.64 \frac{Pz}{\left(1+z^2\right)^2}, \quad \sigma_i = \frac{\sigma_{j-1}+\sigma_j}{2}$$

Calculation interval, h_i= 0.2H = 0.2 x 2.1 = 0.42m
Calculation is accompanied by Table 5 and Fig. 7.

Table 5: Accompanying Table for total thrust computations

z, m	σ_{x1}, ton/m²	σ_i ton/m²	h_i m	$\sigma_i h_i$ ton
0	0			
0.21	0.863	0.432	0.21	0.091
0.42	1.360	1.112	0.21	0.234
0.84	1.294	1.327	0.42	0.557
1.26	0.843	1.069	0.42	0.449
1.68	0.515	0.679	0.42	0.285
2.10	0.321	0.418	0.42	0.176

$$E_{w1} = \sum_{i=1}^{6} \sigma_i h_i = 0.091 + 0.234 + 0.557 + 0.449 + 0.285 + 0.176 = 1.79 \; ton$$

Here, centroid may be obtained by sub-dividing the entire stress area into convenient geometrical figures.

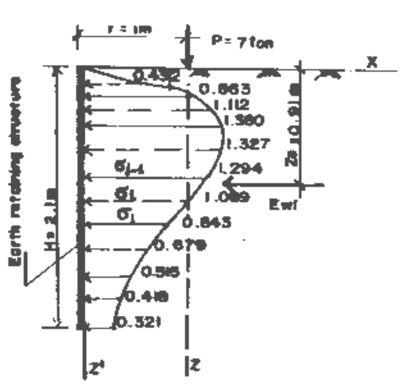

Figure 7 Accompanying figure for pressure computations

b) Using Eqs. (5) and (24)

$m_1 = f_1(\frac{r}{H} = \frac{1.0}{2.1} = 0.48) = 0.2601$ (See Table 1)
$E_{w1} = m_1 P = 0.2601 \times 7 = 1.82$ *ton*
$Z_c = \alpha_c H = 0.433 \times 2.1 = 0.91 m$ (See Table 3 for α_c)

Example 2

Calculate total thrust on earth-retaining structure given the following data:
Soil type: - sand
$x = r = 1m$
$H = 2.1\ m,$ P = 7 ton (concentrated Load)

Solution

a) 'Summation Method'

$$\sigma_{x2} = B_2 \Pr^2 \frac{z^3}{\left(r^2+z^3\right)^3} = 0.85 \frac{Pz^3}{\left(1+z^2\right)^3}, \quad \sigma_i = \frac{\sigma_{j-1}+\sigma_j}{2}$$

Calculation interval, $h_i = 0.2H = 0.2 \times 2.1 = 0.42\,m$
Calculation is accompanied by Table 6 and Fig. 8.

Table 6: Accompanying Table for total thrust computations

z, m	σ_{x2} ton/m²	σ_i ton/m²	h_i m	$\sigma_i h_i$, ton
0	0			
0.21	0.048	0.024	0.21	0.005
0.42	0.271	0.160	0.21	0.034
0.84	0.711	0.491	0.42	0.206
1.26	0.687	0.699	0.42	0.294
1.68	0.505	0.596	0.42	0.250
2.10	0.348	0.427	0.42	0.179

$$E_{w2} = \sum_{i=1}^{6} \sigma_i h_i = 0.005 + 0.034 + 0.206 + 0.294 + 0.250 + 0.179 = 0.97 \; ton$$

Centroid may be obtained as illustrated for Example 1, "Summation Method".

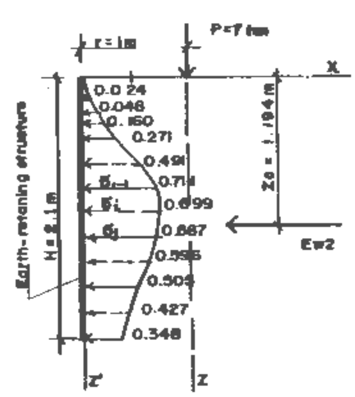

Figure 8 Accompanying figure for pressure computations

b) Using Eqs. (7) and (27)

$$m_2 = f_2\left(\frac{r}{H} = \frac{1.0}{2.1} = 0.48\right) = 0.1404 \qquad \text{(See Table 1)}$$

$$E_{w2} = m_2 P = 0.1404 \times 7 = 0.98\,ton$$

$$Z_c = \alpha_s H = 0.5686 \times 2.1 = 1.194m \qquad \text{(See Table 3 for } \alpha_s\text{)}$$

Example 3

Calculate total thrust on earth retaining structure if a uniform load of *2 ton/m²*, the centre of which is defined by $x_0 = 1m$ from the given structure acts on soil surface. The given load acts on a strip area of *1m* width and the height of the earth-retaining structure, $H = 2m$.

Solution

a) 'Summation Method'

$$\sigma_{x3} = \frac{2q}{\pi}(\alpha - \sin\alpha\cos 2\beta) = 2k_x q,$$

Where $k_x = f(x_o/b,\ z/b)$ – influence factor for lateral pressure due to uniform load, given by Tsytovich, 1979.

Calculation interval, $h_i = 0.125H = 0.125 \times 2 = 0.25m$

Average pressure for each elemental soil layer

$$\sigma_i = \frac{\sigma_{j-1} + \sigma_j}{2}$$

Calculation is for a unit length of the earth-retaining structure and is accompanied by Table 7 and Fig. 9.

Table 7: Accompanying Table for total thrust computations

z, m	k_x	$\sigma_x = 2k_x q$, ton/m²	σ_i ton/m²	h_i m	$\sigma_i h_i$ ton
0	0.00	0.00			
			0.34	0.25	0.085
0.25	0.17	0.68			
			0.76	0.25	0.190
0.50	0.21	0.84			
			0.86	0.25	0.215
0.75	0.22	0.88			
			0.74	0.25	0.185
1.00	0.15	0.60			
			0.52	0.25	0.130
1.25	0.11	0.44			
			0.34	0.25	0.085
1.50	0.06	0.24			
			0.22	0.25	0.055
1.75	0.05	0.20			
			0.14	0.25	0.035
2.00	0.02	0.08			

$$E_{w3} = \sum_{i=1}^{8} \sigma_i h_i = 0.98\ ton$$

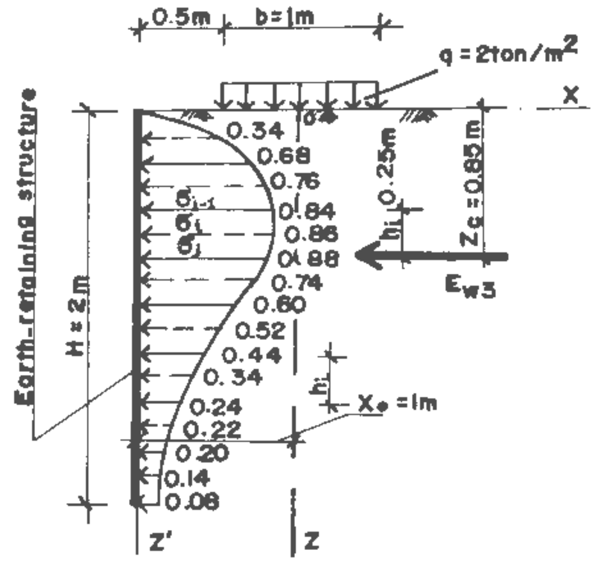

Figure 9 Accompanying figure for pressure computations

b) Using Eqs. (13) and (30)

$$m_3 = f_3(n = x_o/b = 1, m = H/b = 2) = 0.1268 \qquad \text{(See Table 2)}$$

$$E_{w3} = 2m_3 qH = 2 \times 0.1268 \times 2 \times 2 = 1.01\ ton$$

Location of the total thrust

$$Z_c = \alpha H = 0.4231 \times 2 = 0.846 \approx 0.85m \qquad \text{(See Table 4 for } \alpha\text{)}$$

Example 4

Calculate total thrust on earth-retaining structure given the following data.

Soil type:- *1st layer: Clay ($H_1 = 1m$)*
2nd layer: Sand (Continuous layer)
$x = r = 1m$
$H = H_2 = 2.1m$, $P = 7\ ton$

a) 'Summation method'

Calculation interval, $h_i = 0.2H = 0.2 \times 2.1 = 0.42\ m$

$$\sigma_{x1} = B_1 P r^2 \frac{z}{(r^2 + z^2)^2} = 0.64 \frac{Pz}{(1 + z^2)^2} \ \ldots\ldots\text{for clay}$$

$$\sigma_{x2} = B_2 P r^2 \frac{z^3}{(r^2 + z^2)^3} = 0.85 \frac{Pz^3}{(1 + z^2)^3} \ \ldots\text{for sand}$$

Calculation is accompanied by Table 8 and Fig. 10.

Table 8: Accompanying Table for total thrust computations

z, m	σ_z ton/m^2	σ_i ton/m^2	h_i m	$\sigma_i h_i$ ton
0	0			
		0.432	0.21	0.091
0.21	0.863			
		1.112	0.21	0.234
0.42	1.360			
		1.327	0.42	0.557
0.84	1.294			
		1.207	0.16	0.193
1.00	1.120			
1.00	0.744			
		0.716	0.26	0.186
1.26	0.687			
		0.596	0.42	0.250
1.68	0.505			
		0.427	0.42	0.179
2.10	0.348			

$$E_{w1,2}=\sum_{i=1}^{7}\sigma_i h_i = 1.69\ ton$$

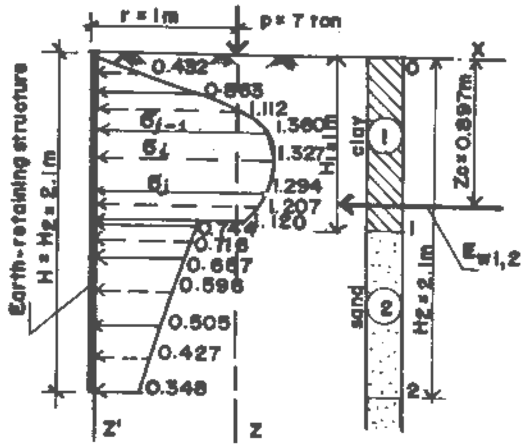

Figure 10 Accompanying figure for pressure computations

b) Using Eq. (17)

$$m_{1,2}=\frac{B_1}{2}\left[\frac{1}{1+n_1^2}\right]+\frac{B_2}{2}\left[\frac{n_1^2}{1+n_1^2}\left(1-\frac{n_1^2}{2\left(1+n_1^2\right)}\right)+\frac{n_2^2}{1+n_2^2}\left(\frac{n_2^2}{2\left(1+n_2^2\right)}-1\right)\right]$$

$n_1 = r/H_1 = 1/1 = 1,\quad n_2 = r/H_2 = 1/2.1 = 0.48$

$$m_{1,2}=\frac{0.64}{2}\left[\frac{1}{1+1^2}\right]+\frac{0.85}{2}\left[\frac{1^2}{1+1^2}\left(1-\frac{1^2}{2\left(1+1^2\right)}\right)+\frac{0.48^2}{1+0.48^2}\left(\frac{0.48^2}{2\left(1+0.48^2\right)}-1\right)\right]=0.2473$$

$E_{w1,2}=m_{1,2}P = 0.2473\times 7 = 1.73$ ton

c) Using Eq. (20) and Table 1

$i=0$, $k=1$, clay: $\quad m_0^1 = 0,\quad m_1^1 = f_1(r/H_1=1)=0.1600$

$i=1$, $k=2$, sand: $\quad m_1^2 = f_2(r/H_1=1)=0.0531,\quad m_2^2=f_2(r/H_2=0.48)=0.1404$

$$m=\sum_{i=0,k=1}^{1,2}\left(m_{i+1}^k-m_i^k\right)=\left(m_1^1-m_0^1\right)+\left(m_2^2-m_1^2\right)=$$

$0.1600-0+0.1404-0.0531=0.2473$

$E_{w1,2}=mP=0.2473\times 7=1.73\,ton$

Location of the centroid by Eq. (32)

$$Z_c=\frac{1}{\sum_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k-m_i^k\right)}\left[B_1H_1\left\{\frac{n_1}{2}\arctan\frac{1}{n_1}-\frac{n_1}{2}\arctan\frac{1}{n_0}-\frac{n_1^2}{2\left(1+n_1^2\right)}+\frac{H_0}{H_1}\frac{n_0^2}{2\left(1+n_0^2\right)}\right\}+B_2H_2\left\{\frac{3n_2}{8}\arctan\frac{1}{n_2}-\frac{3n_2}{8}\arctan\frac{1}{n_1}+\frac{n_2^4}{4\left(1+n_2^2\right)^2}-\frac{H_1}{H_2}\frac{n_1^4}{4\left(1+n_1^2\right)^2}+\frac{5H_1}{8H_2}\frac{n_1^2}{\left(1+n_1^2\right)}-\frac{5}{8}\frac{n_2^2}{\left(1+n_2^2\right)}\right\}\right]$$

Here, $j=2$, $i_{max}=j-1=1$ and $i=0,1$

$$Z_c=\frac{1}{0.2473}\left[0.64\times1\left\{\frac{1}{2}arctan\frac{1}{1}-0-\frac{1^2}{2\left(1+1^2\right)}+0\right\}+0.85\times 2.1\left\{\frac{3\times0.48}{8}arctan\frac{1}{0.48}-\frac{3\times0.48}{8}arctan\frac{1}{1}+\frac{0.48^4}{4\left(1+0.48^2\right)^2}-\frac{1}{2.1}\times\frac{0.48\times1^4}{4\left(1+1^2\right)^2}+\frac{5\times1}{8\times2.1}\times\frac{1^2}{\left(1+1^2\right)}-\frac{5}{8}\frac{0.48^2}{\left(1+0.48^2\right)}\right\}\right]=0.897m$$

Using Eq. (37) and Table 3:

$i=0$, $k=1$, clay: $\quad a_0^1=0,\quad a_1^1=f(r/H_1=1)=0.5708$

$i=0$, $k=2$, sand: $\quad a_1^2=f(r/H_1=1)=0.7124,\quad a_2^2=f(r/H_2=0.48)=0.5687$

$$\alpha' = \sum_{i=0,k=1}^{1,2}\left(\alpha_{i+1}^{k} - \alpha_{i}^{k}\right) = \left(\alpha_{1}^{1} - \alpha_{0}^{1}\right) + \left(\alpha_{2}^{2} - \alpha_{1}^{2}\right) =$$
$$(0.5708 - 0) + (0.5687 - 0.7124) = 0.4271$$

$$Z_c = \alpha' H = 0.4271 \times 2.1 = 0.897m$$

Example 5

Calculate total thrust on earth-retaining structure given the following data: -

Soil type: - 1^{st} layer: sand ($H_1 = 1m$)
2^{nd} layer: clay(continuous layer)
$x = r = 1m$
$H = H_2 = 2.1m$, $P = 7$ ton

a) 'Summation Method'

Calculation interval, $h_i = 0.2H = 0.2 \times 2.1 = 0.42m$

$$\sigma_{x2} = B_2 P r^2 \frac{z^3}{\left(r^2 + z^2\right)^3} = 0.85 \frac{Pz^3}{\left(1 + z^2\right)^3} \qquad \text{for sand}$$

$$\sigma_{x1} = B_1 P r^2 \frac{z}{\left(r^2 + z^2\right)^2} = 0.64 \frac{Pz}{\left(1 + z^2\right)^2} \qquad \text{for clay}$$

Calculation is accompanied by Table 9 and Fig. 11.

Table 9 Accompanying Table for total thrust computations

Z, m	$\sigma_x, ton/m^2$	$\sigma_i, ton/m^2$	h_i, m	$\sigma_i h_i, ton$
0	0			
		0.024	0.21	0.005
0.21	0.048			
		0.160	0.21	0.034
0.42	0.271			
		0.491	0.42	0.206
0.84	0.711			
		0.728	0.16	0.11
1.00	0.744			
1.00	1.120			
		0.982	0.26	0.255
1.26	0.843			
		0.679	0.42	0.285
1.68	0.515			
		0.418	0.42	0.176
2.10	0.321			

$$E_{w2,1} = \sum_{i=1}^{7} \sigma_i h_i = 1.08\ ton$$

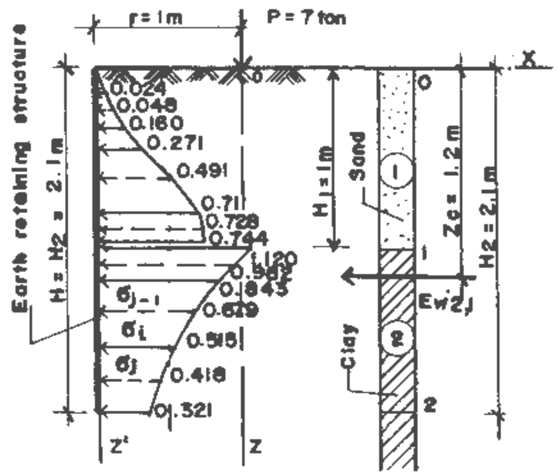

Figure 11 Accompanying figure for pressure computations

b) Using Eq. (19)

$$m_{2,1} = \frac{B_2}{2}\left[0.5 + \frac{n_1^4}{2\left(1 + n_1^2\right)^2} - \frac{n_1^2}{1 + n_1^2}\right] + \frac{B_1}{2}\left[\frac{n_1^2}{1 + n_1^2} - \frac{n_2^2}{1 + n_2^2}\right]$$

$n_1 = r/H_1 = 1/1 = 1$, $n_2 = r/H_2 = 1/2.1 = 0.48$

$$m_{2,1} = \frac{0.85}{2}\left[0.5 + \frac{1^4}{2\left(1 + 1^2\right)^2} - \frac{1^2}{1 + 1^2}\right] + \frac{0.64}{2}\left[\frac{1^2}{1 + 1^2} - \frac{0.48^2}{1 + 0.48^2}\right] = 0.1532$$

$$E_{w2,1} = m_{2,1} P = 0.1532 \times 7 = 1.07 ton$$

c) Using Eq. (20) and Table 1:

$i = 0$, $k = 1$, sand: $m_0^1 = 0$; $m_1^1 = f_2(r/H_1 = 1) = 0.0531$

$i = 0$, $k = 2$, clay: $m_1^2 = f_1(r/H_1 = 1) = 0.1600$; $m_2^2 = f_1(r/H_2 = 0.48) = 0.2601$

$$m = \sum_{i=0,k=1}^{1,2}\left(m_{i+1}^{k} - m_{i}^{k}\right) = \left(m_{1}^{1} - m_{0}^{1}\right) + \left(m_{2}^{2} - m_{1}^{2}\right) =$$
$$0.0531 - 0 + 0.2601 - 0.1600 = 0.1532$$
$$E_{w2,1} = mP = 0.1532 \times 7 = 1.07\ ton$$

Location of the centroid using Eq.(35),

$$Z_c = \frac{1}{\sum_{i=0,k=1}^{j-1,j}\left(m_{i+1}^k - m_i^k\right)}\left[B_2H_1\left\{\frac{3n_1}{8}arctan\frac{1}{n_1} - \frac{3n_1}{8}arctan\frac{1}{n_0} + \right.\right.$$

$$\left.\frac{n_1^4}{4\left(1+n_1^2\right)^2} - \frac{H_0}{H_1}\frac{n_0^4}{4\left(1+n_0^2\right)^2} + \frac{5H_0}{8H_1}\frac{n_0^2}{\left(1+n_0^2\right)} - \frac{5}{8}\frac{n_1^2}{\left(1+n_1^2\right)}\right\} +$$

$$\left.B_1H_2\left\{\frac{n_2}{2}arctan\frac{1}{n_2} - \frac{n_2}{2}arctan\frac{1}{n_1} - \frac{n_2^2}{2\left(1+n_2^2\right)} + \frac{H_1}{H_2}\frac{n_1^2}{2\left(1+n_1^2\right)}\right\}\right]$$

Here, $j = 2$, $i_{max} = j\text{-}1=2\text{-}1=1$ and $i = 0,1$

$$Z_c = \frac{1}{0.1532}\left[0.85\times 1.0\left\{\frac{3\times 1}{8}arctan\frac{1}{1} - 0 + \frac{1^4}{4\left(1+1^2\right)^2} - 0\right.\right.$$

$$\left. + 0 - \frac{5\times 1^2}{8\left(1+1^2\right)}\right\} + 0.64\times 2.1\left\{\frac{0.48}{2}arctan\frac{1}{0.48} - \right.$$

$$\left.\left.\frac{0.48}{2}arctan\frac{1}{1} - \frac{0.48^2}{2\left(1+0.48^2\right)} + \frac{1}{2.1}\times\frac{1^2}{2\left(1+1^2\right)}\right\}\right] = 1.2m$$

Using Eq. (37) Table 3:

$$\begin{array}{ll} i=0 & \\ k=1 & \\ \text{sand} & \end{array}\left\{\begin{array}{l} \alpha_0^1 = 0 \\ \\ \alpha_1^1 = f\left(r/H_1 = 1\right) = 0.7124 \end{array}\right.$$

$$\begin{array}{ll} i=1 & \\ k=2 & \\ \text{clay} & \end{array}\left\{\begin{array}{l} \alpha_1^2 = f\left(r/H_1 = 1\right) = 0.5708 \\ \\ \alpha_2^2 = f\left(r/H_2 = 0.48\right) = 0.4330 \end{array}\right.$$

$$\alpha' = \sum_{i=0,k=1}^{1,2}\left(\alpha_{i+1}^k - \alpha_i^k\right) = \left(\alpha_1^1 - \alpha_0^1\right) + \left(\alpha_2^2 - \alpha_1^2\right) =$$

$$\left(0.7124 - 0\right) + \left(0.4330 - 0.5708\right) = 0.5746$$

$$Z_c = \alpha' H = 0.5746\times 2.1 = 1.2m$$

CONCLUSION

The formulas derived for computing total thrust and its point of application are much simpler, quick and are also more accurate compared to the *'Summation Method'* as demonstrated in the illustrative examples.

The formulas derived are also more practical since they consider the nature of soil and the type of the superimposed load when compared to the conventional way of considering surcharges through equivalent height in the design of earth-retaining structures.

For design purpose, the equations derived for medium stiff clay can be applied to cohesive soils and the equations derived for medium dense sand can be applied to non-cohesive soils.

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