PREDICTING SAND BEHAVIOUR USING A STATE PARAMETER MODEL

Samuel Tadesse
Department of Construction Technology and Management
Addis Ababa University

ABSTRACT

In this paper a comparison between predicted and experimental stress-strain behaviour and volumetric strains of sand are studied using a simple constitutive model. The model used a state parameter, that combines the influence of density and stress level with reference to a steady state; a hyperbolic hardening rule for representing the global stress-strain behaviour and finally a link between volumetric and deviatoric plastic strain increments through a simple dilatancy rule.

The calibration and the comparison of the simulations with experimental results show that the model is sufficient to reproduce with a very good accuracy the drained behaviour of sand over a wide range of density and confining pressure, using simple variations of model constants.

INTRODUCTION

The behaviour of soil is influenced by a number of factors such as soil type, density, initial stress, drainage condition, grain shape and size, stress history etc. A number of constitutive models are being developed considering the effect of one or more of the above mentioned factors. However, in most of the models softening of dense sand after reaching a peak value has not been adequately considered. In this paper, a model developed by Wood et al. [1] has been used to simulate the behaviour of loose to dense sands. The model proposed by Wood et al. [1] is based on the state parameter, ψ, and it is able to represent the mechanical behaviour of granular soils over a wide range of void ratios and mean stress level.

BASIC CONCEPTS

The basic concepts of the model in the triaxial q: p' space, e: p' space and definition of some key quantities are described below. q, $p'$and e are deviatoric stress $(\sigma_1 - \sigma_3)$, effective mean stress $\left[\frac{1}{3}(\sigma_1 + 2\sigma_3)\right]$ and void ratio respectively.

Steady state is the basic theoretical framework. The steady state, at which deformation continues for constant stress and zero volumetric strain rate is attained when the stress ratio $\eta = q/p'$ equals M_C, the critical value in triaxial compression. The corresponding void ratio, e, equals e_s, the void ratio at the steady state, which is a unique function of p'. The state parameter, ψ, is defined in e:p' space as the difference in void ratio between the present and the steady state at the same effective mean principal stress.

$$\psi = e - e_s \tag{1}$$

A typical soil response, as obtained from triaxial test results [2], is illustrated in Figs. 1 and 2. If a state initially denser than steady state, represented by point a in Fig. 1 is subjected to conventional drained triaxial compression, it will first contract and then dilate until it reaches point a' on the steady state line. Simultaneously, in Fig.2 the stress path moves towards the critical line with the slope of 1:3.

For a state looser than steady state, as in point b of Fig. 1, the response is different. Under drained conventional triaxial loading the soil contracts and moves to the steady state line, in general without dilation and softening. Here also the stress path moves towards the critical line with a slope of 1:3.

The analytical form of the model presented here is adopted from Wood [1]. Virtual peak or bounding stress ratio, M_p, is assumed to be related to the critical stress ratio, M_C, by way of ψ.

$$M_p = M_C - \kappa\psi \tag{2}$$

Where κ is a material parameter.

A hyperbolic relation is used to relate the current stress ratio, η, to distortional strain, ε_q, and provides the basis for a hardening law.

$$\eta = \frac{M_p \varepsilon_q}{(B + \varepsilon_q)} \tag{3}$$

In which B is a material parameter and

$$\varepsilon_q = \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right)$$

By derivation of Eq. (3) one can obtain the rate of change of stress ratio with distortional strain at small strains.

$$\frac{\delta\eta}{\delta\varepsilon_q} = \frac{M_p B}{\left(B + \varepsilon_q\right)^2} \qquad (4)$$

Which gives the small strain threshold stiffness by inserting $\varepsilon_q = 0$ in Eq. (4) and shows that the stiffness is controlled by B. Further more volumetric and distortional strain increments are linked by the following simple flow rule

$$\frac{\delta\varepsilon_p}{\delta\varepsilon_q} = A(M_c - \eta) \qquad (5)$$

Where A is a material parameter controlling the amount of plastic dilatancy or contractancy during shear and $\delta\varepsilon_p = \delta\varepsilon_1 + 2\delta\varepsilon_3$.

In drained triaxial test the void ratio of the sample changes continuously during shearing. Therefore the void ratio at every stage of shearing should be calculated in order to update the state parameter, ψ. The change in void ratio and the current void ratio can be computed from following relationships respectively.

$$\Delta e = \Delta\varepsilon_p\,(1+e_o) \qquad (6)$$

$$e = e_o - \Delta e = e_o - \Delta\varepsilon_p\,(1+e_o) \qquad (7)$$

For conventional drained triaxial test isotropically consolidated to p_o', the effective mean stress corresponding to the current void ratio is given by

$$P' = \frac{3P_o'}{3-\eta} \qquad (8)$$

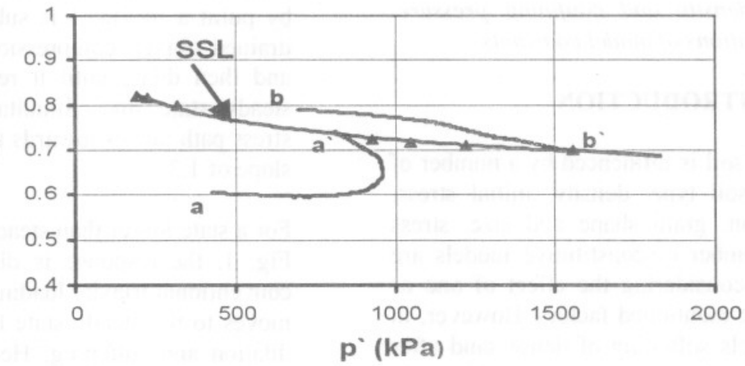

Figure1 Schematic illustration of drained paths in e:p` space for a state denser and looser than the steady state.

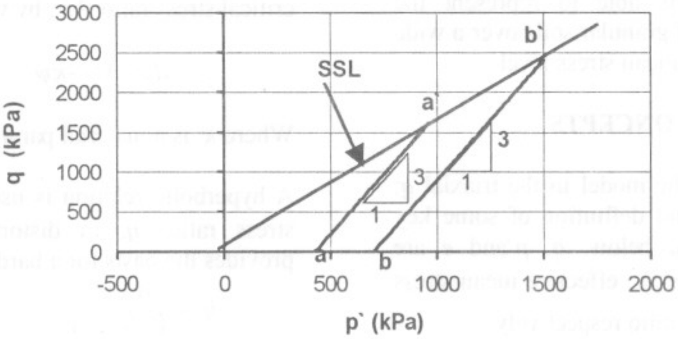

Figure 2 Schematic illustration of drained paths in q:p`space

CONSTANT DETERMINATION AND SIMULATION

The model constants have been determined based on the results of drained triaxial compression test conducted on Hokksund sand [2]. The sand has the following index properties. G_s = 2.71, D_{60} = 0.5mm, C_u =2.04, e_{max} = 0.949 and e_{min} = 0.572. Specimens were prepared in membrane-lined split moulds mounted on the lower platen of the triaxial apparatus. Moist compacted samples were tamped into the mould at the desired void ratio in six layers. Specimens with height equal to the diameter and smooth pressure heads were used in all tests. The specimens were saturated by flushing with de-aired water under vacuum and by increasing the backpressure until a B value greater than 0.96 was obtained. All the tests were conducted on isotropically consolidated specimens.

Using the test results of the sand tested as described above, the model constants were determined and presented in Table 1. All constants are dimensionless. As it was expected all the model parameters depend on the relative density (D_r). The dependency is illustrated in Fig. 3. This figure shows that parameters A and κ increase linearly for the range of relative density covered in the study. Parameter B that is the measure of stiffness is decrease as the relative density increases. Best-fit linear equations for the calculated data points of the parameters were obtained by regression.

Figures 4 to 8 show the results of conventional drained triaxial tests on which predicted behaviour has been superimposed; the agreement can be seen to be good. Overall, the model replicate the experimental results of Hokksund sand and capture the influence of initial relative density and confining pressure on the constitutive behaviour. Contractancy, dilatancy, peak strength, softening are modelled well.

Table1: Model constants

A	$0.02\,D_r - 0.25$
B	$-4E\text{-}05\,D_r + 0.0122$
κ	$0{,}022D_r + 2{,}0548$

D_r is relative density after consolidation in %

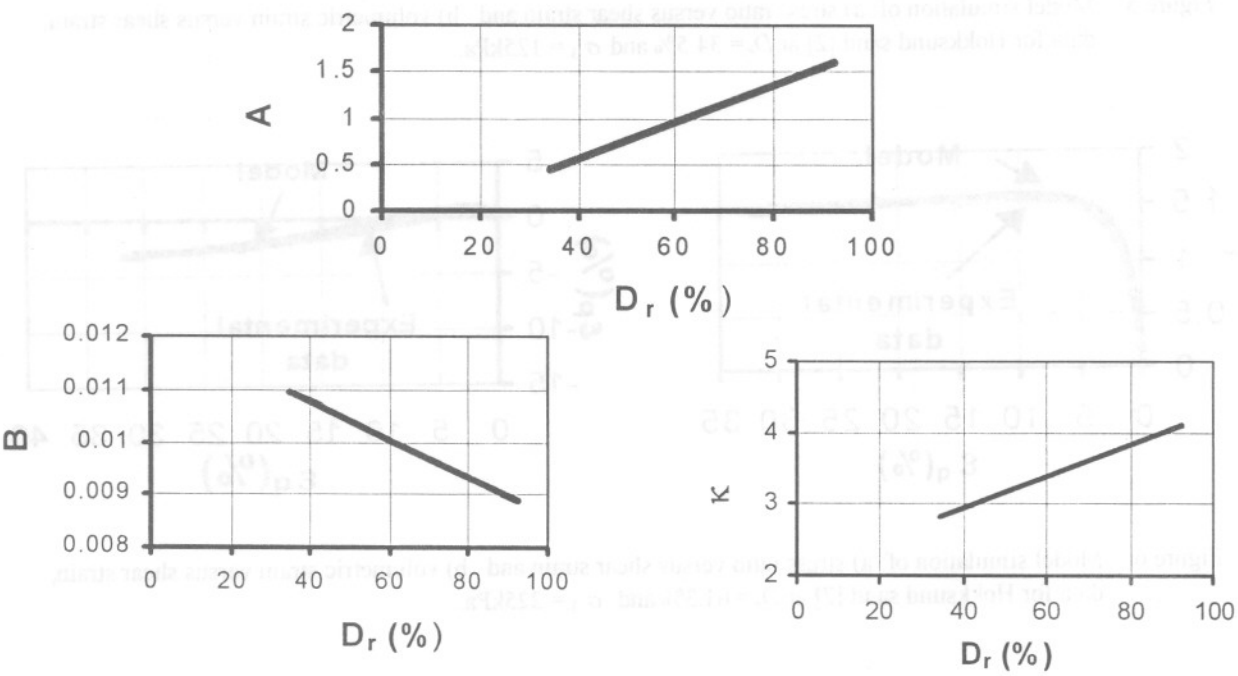

Figure 3 Variation of model constants with relative density

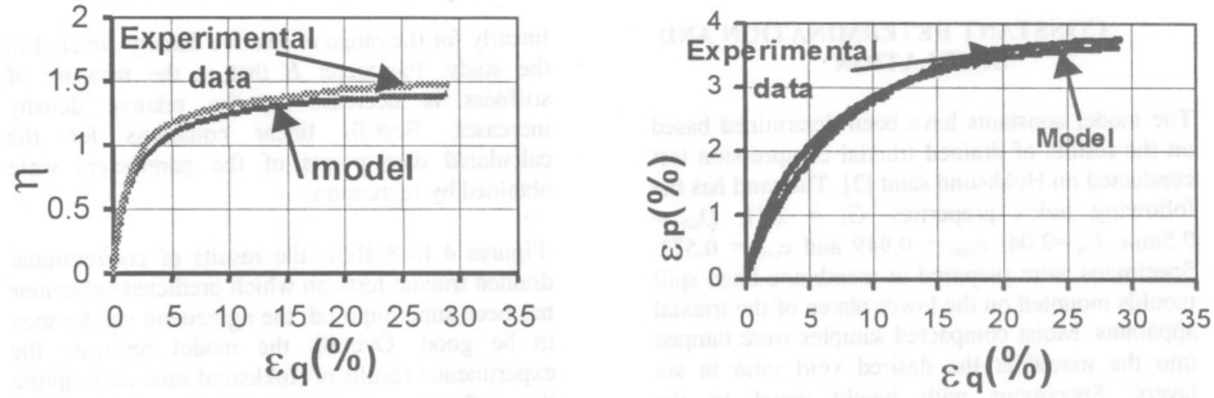

Figure 4 Model simulation of: a) stress ratio versus shear strain and b) volumetric strain versus shear strain, data for Hokksund sand [2] at $D_r = 42.2\%$ and $\sigma'_3 = 700$kPa.

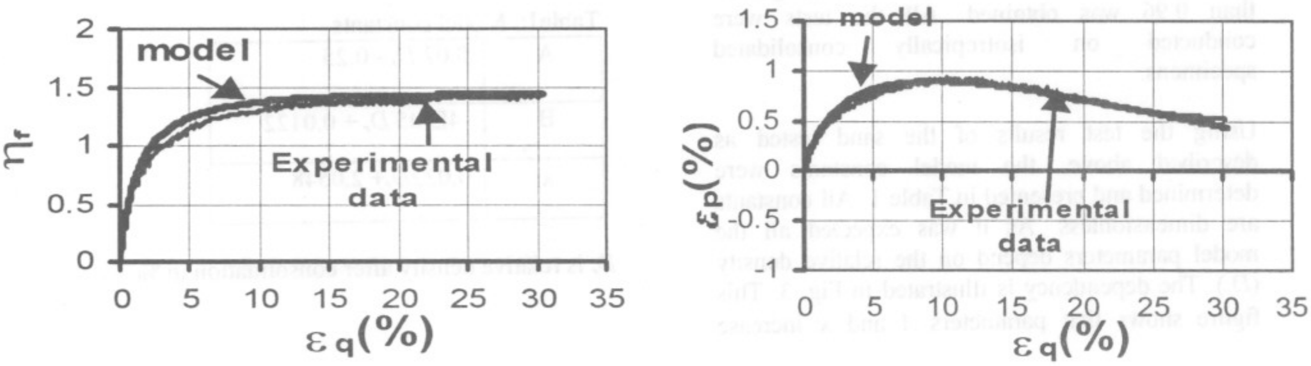

Figure 5 Model simulation of: a) stress ratio versus shear strain and b) volumetric strain versus shear strain, data for Hokksund sand [2] at $D_r = 34.5\%$ and $\sigma'_3 = 125$kPa.

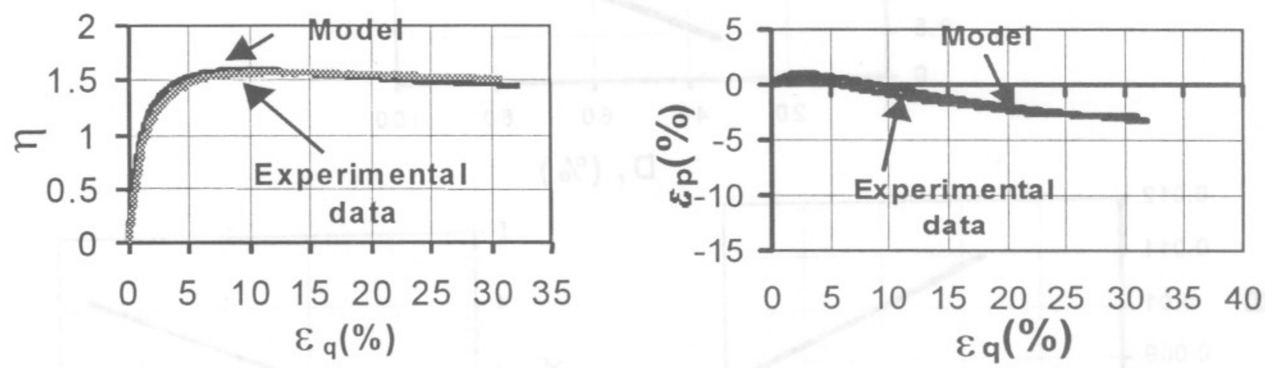

Figure 6 Model simulation of: a) stress ratio versus shear strain and b) volumetric strain versus shear strain, data for Hokksund sand [2] at $D_r = 61.3\%$ and $\sigma'_3 = 225$kPa.

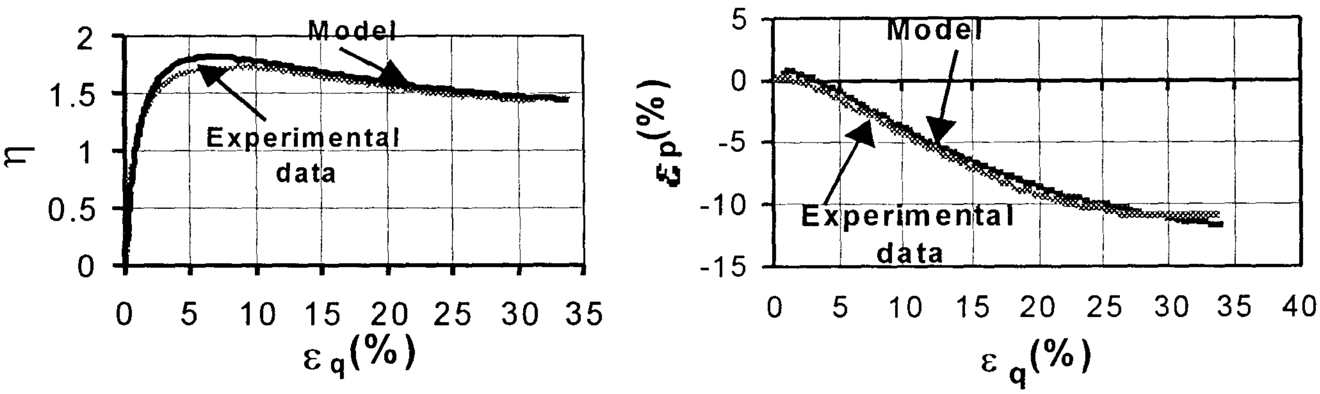

Figure 7 Model simulation of: a) stress ratio versus shear strain and b) volumetric strain versus shear strain, data for Hokksund sand [2] at $D_r = 88.9\%$ and $\sigma'_3 = 125$kPa.

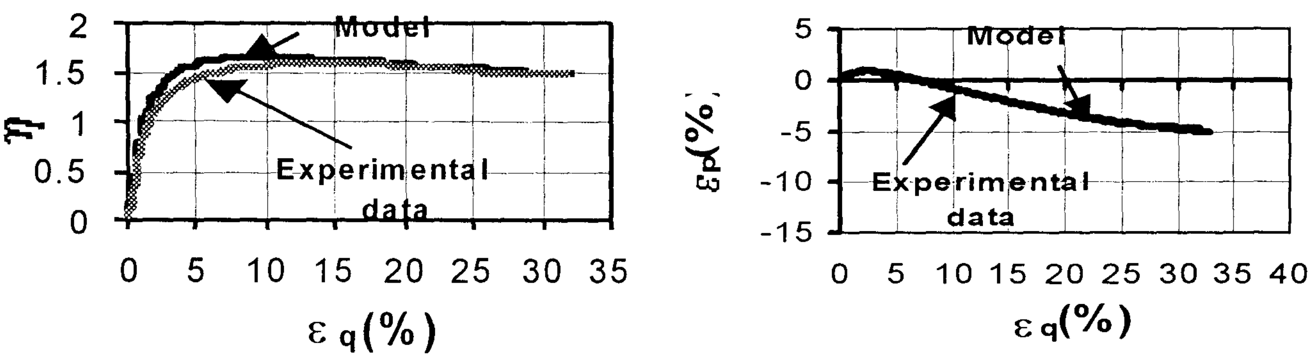

Figure 8 Model simulation of: a) stress ratio versus shear strain and b) volumetric strain versus shear strain, data for Hokksund sand [2] at $D_r = 92.6\%$ and $\sigma'_3 = 700$kPa.

CONCLUSION

The model developed by Wood et al. [1] has been used to predict the real behaviour of sand. The calibration and the validation of the model through the comparison of the simulations with experimental results, show that the simple concepts on which the model is built are sufficient to reproduce with a very good accuracy the drained behaviour of sand over a wide range of density and confining pressures, using simple variations of model constants.

REFERENCES

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Additional References

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